ECE-302

Quiz 1

Allister Liu

1. a) Possible situations:

$64

$63 - 1 = 62$	$+2$	$\frac{1}{2}$
$62 - 2 = 60$	$+4$	$\frac{1}{4}$
$60 - 4 = 56$	$+8$	$\frac{1}{8}$
$56 - 8 = 48$	$+16$	$\frac{1}{16}$
$48 - 16 = 32$	$+32$	$\frac{1}{32}$
$32 - 32 = \$0$	$+64$	$\frac{1}{64}$

$\therefore$ Sample space: $\{0, 64\}$

b) Let X be the outcome (the amount of money you have when you walk out of the casino)

Pmf:

$$P_X(X) = \begin{cases} \frac{63}{64}, & X = 64 \\ \frac{1}{64}, & X = 0. \end{cases}$$

c) $E[X] = 64\left(\frac{63}{64}\right) + 0\left(\frac{1}{64}\right) = \$63.$

d) This is not a good strategy if you want to win money. On average, you would walk out with the same amount of money you ~~started~~ started with, so you probably should not do this every day because it's just a waste of time.

2. $X = -\ln(1-U)$

$F_X(x) = P(X \le x) = P(-\ln(1-U) \le x) = P(U \le 1-e^{-x})$

$\therefore F_X(x) = F_U(1-e^{-x})$

$\because U$ is uniform random variable

$\therefore U \in [0,1]$

$\therefore X \in [0, \infty)$

cdf: $\therefore F_X(x) = \begin{cases} 0 & x < 0 \\ 1-e^{-x} & x \ge 0 \end{cases}$

pdf: $f_X(x) = \frac{d}{dx} F_X(x) = \begin{cases} 0 & x < 0 \\ e^{-x} & x \ge 0 \end{cases}$