Quiz 4.

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$$f_X(x) = \begin{cases} \dfrac{\alpha\, x_m^{\alpha}}{x^{\alpha+1}} & x \geq x_m \\ 0 & x < x_m \end{cases}$$

a)

$$L = \prod_{i=1}^{n} \frac{\alpha \cdot x_m^{\alpha}}{x_i^{\alpha+1}}$$

$$\ln(L) = n \cdot \ln(\alpha) + n\alpha \cdot \ln(x_m) - (\alpha+1)\sum_{i=1}^{n} \ln(x_i)$$

$$\frac{\partial}{\partial\alpha} \ln(L) = 0$$

$$\therefore \frac{n}{\alpha} + n\ln(x_m) - \sum_{i=1}^{n} \ln(x_i) = 0.$$

$$\therefore \frac{1}{\alpha} = \frac{1}{n}\sum_{i=1}^{n} \ln(x_i) - \ln(x_m)$$

$$\therefore \hat{\alpha}_{ML} = \left(\frac{1}{n}\sum_{i=1}^{n} \ln(x_i) - \ln(x_m)\right)^{-1} = \frac{n}{\sum_{i=1}^{n} \ln\left(\frac{x_i}{x_m}\right)} \quad (\text{for } x \geq x_m)$$

b) If x_m is unknown, the max-likelihood estimate of this parameter is the minimum observed value of X.

$$\because \ln(L) = n \cdot \ln(\alpha) + n\alpha \ln(x_m) - (\alpha+1)\sum_{i=1}^{n} \ln(x_i)$$

To maximize α, the max of ~~max~~ x_m can be is the minimum of the observation of X.

Probability of having $\min(X) = x_m$ is large given a set of observation $X = \{x_1, x_2, \cdots x_N\}$

$\therefore$ Max-likelihood estimate of x_m is $x_m = \min(X)$

c) $$\hat{\alpha}_{ML} = \frac{n}{\sum_{i=1}^{n} \ln\left(\frac{x_i}{\min(X)}\right)}$$