ECE-302 - Stoch

Project 5 - MMSE_FIR

Allister Liu

Amy Leong

Part 1

$$R_{\cancel{rr}}[n] = R_{dd}[n] + R_{yy}[n]$$

$$\therefore R_{rr}[n] = R_{dd}[n] + R_{ss}[n] * R_{cc}[n]$$

$$\because \text{ for } n \neq 0, \quad R_{dd}[n] = 0$$

$$\therefore R_{rr}[n] = \underbrace{R_{ss}(n)}_{\text{Impulse.}} * R_{cc}[n] = \bar{R}_{cc}[n]$$

$$\therefore \bar{R}_{cc}[n] = \sum_{n=-\infty}^{\infty} C[n+m]\, C[n]$$

$$R_{rs}[n] = c[n] * R_{ss}[n] = c[n] \Rightarrow R_{sr} = c[-n]$$

For $N = 4$:

$$\begin{bmatrix} R_{rr}[0] & R_{rr}[-1] & R_{rr}[-2] & R_{rr}[-3] \\ R_{rr}[1] & R_{rr}[0] & R_{rr}[-1] & R_{rr}[-2] \\ R_{rr}[2] & R_{rr}[1] & R_{rr}[0] & R_{rr}[-1] \\ R_{rr}[3] & R_{rr}[2] & R_{rr}[1] & R_{rr}[0] \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \\ h[2] \\ h[3] \end{bmatrix} = \begin{bmatrix} R_{\hat{s}r}[0] \\ R_{\hat{s}r}[1] \\ R_{\hat{s}r}[2] \\ R_{\hat{s}r}[3] \end{bmatrix}$$

$$R_{\hat{s}r} = R_{sr}$$

$$\begin{bmatrix} 1.2 & 0.28 & 0.4 & 0 \\ 0.28 & 1.2 & 0.28 & 0.4 \\ 0.4 & 0.28 & 1.2 & 0.28 \\ 0 & 0.4 & 0.28 & 1.2 \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ h(2) \\ h(3) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow h = \begin{bmatrix} 0.99 \\ -0.20 \\ -0.31 \\ 0.14 \end{bmatrix}$$

```
clear; close all; clc;

% c[n]
c = [1 .2 .4];

% simulation for N = 4, 6, 10
N = [4,6,10];

% array to store MSE of each N
MSE = zeros(1,length(N));

% signal length
len_N = 1000;

% random signal (+/-1)
s = randi(2, 1, len_N);
s(s == 2) = -1;

[Rss, lags] = xcorr(s); % index 1000 is time delay = 0;

% output of 1st filter c[n]
y = filter(c, 1, s);

% standard deviation of noise
sd = 0.5;

% y + d[n] -> input of 2nd filter h[n]
r = y + normrnd(0, sd, 1, length(y));

% simulate for each N
for m = 1:length(N)
    % Rrr -> autocorrelation
    Rrr = xcorr(r);
    Rrr_mid = (length(Rrr)+1)/2;
    % Rsr
    Rsr = xcorr(s,r);
    Rsr_mid = (length(Rsr)+1)/2;
    Left = zeros(N(m));
    for i = 1:N(m)
        Left(i,:) = transpose(Rrr(Rrr_mid - i+1 : Rrr_mid - i+N(m)));
    end
    right = transpose(Rsr(Rsr_mid : Rsr_mid + N(m)-1));

    % slove for h[n]
    h = Left\right;

    % output of 2nd filter h[n]
    s_hat = filter(transpose(h), 1, r);

    % MSE
    MSE(m) = sum((s_hat-s).^2)/len_N;
end
```

```
table(MSE(1), MSE(2), MSE(3), 'VariableNames', ["N = 4", "N = 6", "N =
 10"],...
 'RowNames', "MSE")
```

```
ans =

  1×3 table

              N = 4      N = 6      N = 10
              ______    _______    _______

    MSE       0.2442    0.23558    0.23393
```

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