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```
clc;
clear;
close all;
```

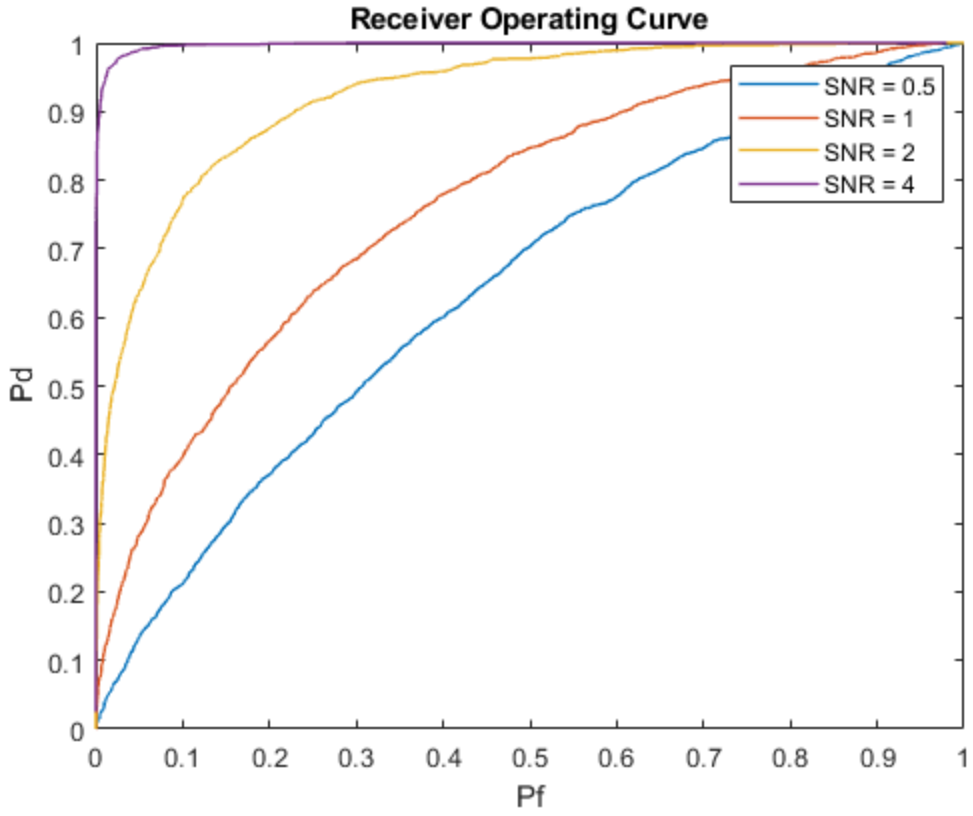
Part 1

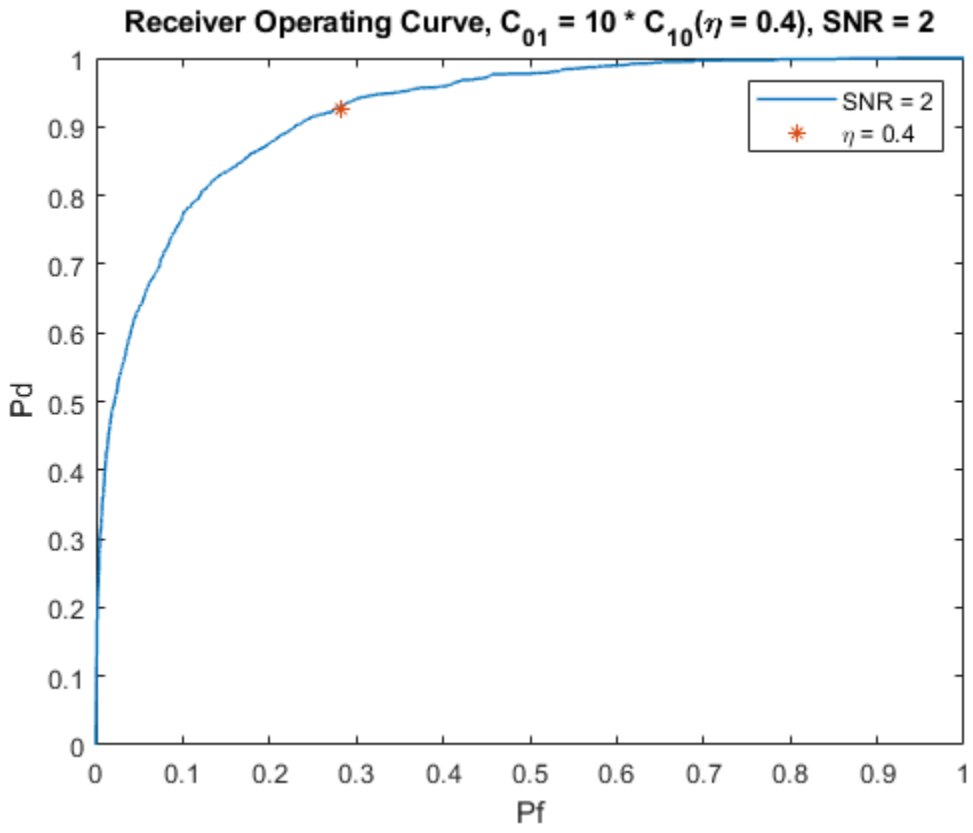
```
% a)

% number of iterations
N = 10000;
% probability of target being there
p0 = 0.8;
% probability of target not being there
p1 = 1 - p0;

eta_1a = p0/p1;

% variance
var_1a = 1;
% standard deviation
sigma_1a = sqrt(var_1a);
% mean difference
a_1a = 1;
% signal to noise ratio
SNR_1a = a_1a/var_1a;

target_1a = (rand(N,1) > p0);
A_1a = a_1a * double(target_1a);
X_1a = sqrt(var_1a) * rand(N,1);

Y_1a = A_1a + X_1a;

% MAP decision boundary
% f(eta|H0) * P0 = f(eta|H1) * P1
%    (2 * var * log(P0/P1) + a^2) / (2 * a)
%  = a/2 + var * log(P0/P1)
gamma_1a = a_1a ./ 2 + var_1a * log(eta_1a) ./ (a_1a);

% theoretical probability of error
p1_0 = 1 - normcdf(gamma_1a, 0, sigma_1a);
p0_1 = normcdf(gamma_1a, a_1a, sigma_1a);
theoretical_err = p0 * p1_0 + p1 * p0_1;
```

```
% compare with the target to find the experimental probability of
 error
experimental_err = 1 - sum(or(and(Y_1a > gamma_1a, target_1a), ...
                              and(Y_1a <= gamma_1a, ~target_1a))) / N;

disp("Perr_theoretical = " + theoretical_err);
disp("Perr_experimental = " + experimental_err);

%b)

% same process as part a
a_1b = [0.5, 1, 2, 4];
eta_1b = logspace(-7, 7, N);
var_1b = 1;

pf = zeros(length(a_1b), 1, N);
pd = zeros(length(a_1b), 1, N);
SNR = zeros(length(a_1b), 1, N);

for i = 1:length(a_1b)
    % same process as part a
    target_1b = (rand(N,1) > p0);
    A_1b = a_1b(i) * double(target_1b);
    X_1b = sqrt(var_1b) * randn(N,1);

    Y_1b = A_1b + X_1b;

    gamma_1b = a_1b(i)/2 + var_1b * log(eta_1b) / a_1b(i);

    % find false positive probability pf and true positive probability
 pd
    pf(i, :, :) = sum(and(Y_1b >gamma_1b, ~target_1b)) /
 sum(~target_1b);
    pd(i, :, :) = sum(and(Y_1b >gamma_1b, target_1b)) ./
 sum(target_1b);

    SNR(i) = a_1b(i) / var_1b;
end

% plotting
figure;
for j = 1:length(a_1b)
    plot(reshape(pf(j, :, :), [1,N]), reshape(pd(j, :, :), [1,N]), ...
        'DisplayName', ['SNR = ', num2str(SNR(j))], 'linewidth', 1)
    hold on
end

xlabel('Pf'),ylabel('Pd'),title('Receiver Operating Curve'),legend;

% c)

% (C0_1 - C1_1) * P1 * f(y|H1) = (C1_0 - C0_0) * P0 * f(y|H0)
% solve for eta
```

```
% eta = (C1_0 - C0_0) * P0 / ((C0_1 - C1_1) * P1)
eta_1c = (0.1) * p0 /p1;
a_1c =2;
var_1c = 1;

% same stuff as part b
% just a different eta
target_1c = (rand(N,1) > p0);
A_1c = a_1c * double(target_1c);
X_1c = sqrt(var_1c) * randn(N,1);

Y_1c = A_1c + X_1c;

gamma_1c = a_1c/2 + var_1c * log(eta_1c) / a_1c;

% find false positive probability pf and true positive probability pd
pf_1c = sum(and(Y_1c > gamma_1c, ~target_1c)) / sum(~target_1c);
pd_1c = sum(and(Y_1c > gamma_1c, target_1c)) ./ sum(target_1c);

figure;
% using a = 2, first plot the corresponding ROC with a = 2, SNR = 2
plot(reshape(pf(3, :, :), [1,N]), reshape(pd(3, :, :), [1,N]), ...
        'DisplayName', ['SNR = ',num2str(SNR(3))], 'linewidth', 1)
hold on
% mark the point
plot(pf_1c, pd_1c, '*', 'DisplayName', '\eta = 0.4')
xlabel('Pf'),ylabel('Pd')
title(['Receiver Operating Curve, C_{01} = 10 * C_{10}' ...
        '(\eta = 0.4), SNR = ', num2str(SNR(3))])
legend;
```

Perr_theoretical = 0.18616
Perr_experimental = 0.1802

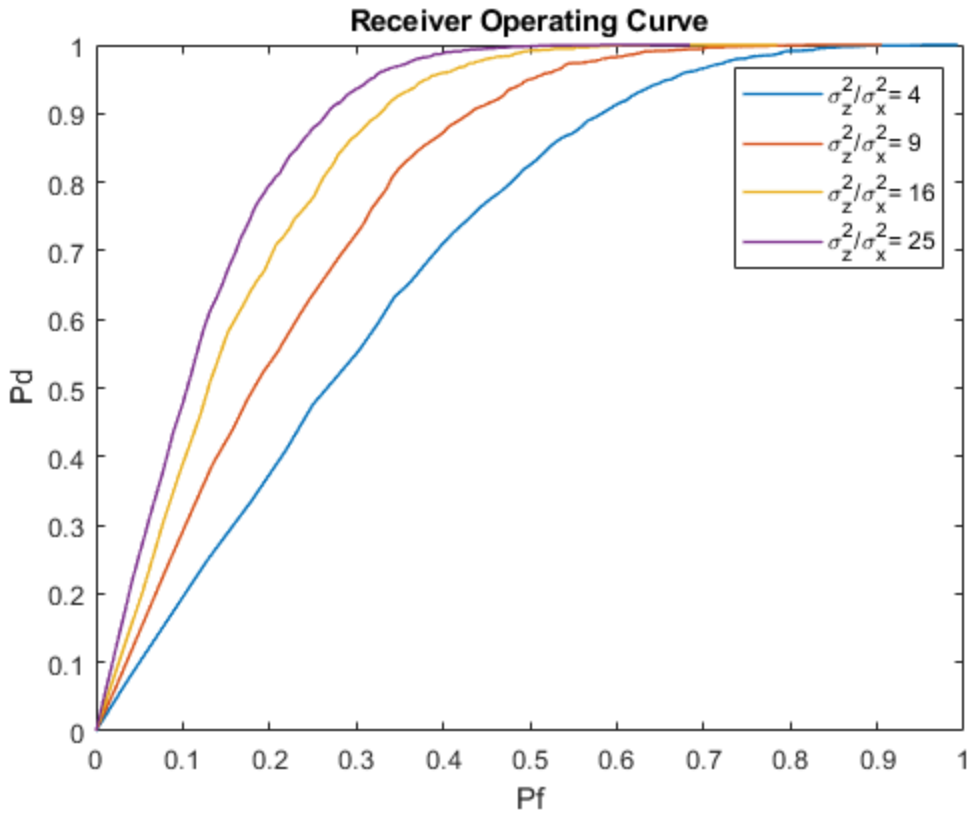

Receiver Operating Curve
SNR = 0.5
SNR = 1
SNR = 2
SNR = 4
Pd
Pf
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Receiver Operating Curve, C01 = 10 * C10(η = 0.4), SNR = 2
SNR = 2
η = 0.4
Pd
Pf
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

e)

```
clear;

% a)
% retyping values
N = 10000;
p0 = 0.8;
p1 = 1 - p0;
eta = p0/p1;
var_x = 1;
var_z = 25;
sigma_x = sqrt(var_x);
sigma_z = sqrt(var_z);
a = 1;

target = (rand(N,1) > p0);
A = a * double(target);
X = sigma_x * randn(N,1);
Z = sigma_z * randn(N,1);

% new definition of Y here
Y = a + X .* target + Z .* (~target);


% theoretical probability of error
gamma = sqrt(2 * ((var_x * var_z)/(var_x - var_z)) * ...
                log(eta * sqrt(var_x/var_z)));

p1_0 = normcdf(gamma, 0, sigma_z) - normcdf(-gamma, 0, sigma_z);
p0_1 = 2 * (1 - normcdf(gamma, 0, sigma_x));

theoretical_err = p1_0 * p0 + p0_1 * p1;

% experimental probability of error
experimental_err = sum(or(...
 and( ...
  (p1 * (1 / sqrt(var_x * 2 * pi)) * exp(-((Y - a).^2) / (2 *
 var_x)))>=...
  (p0 * (1 / sqrt(var_z * 2 * pi)) * exp(-((Y - a).^2) / (2 *
 var_z)))...
    , target),...
  and(...
  (p1 * (1 / sqrt(var_x * 2 * pi)) * exp(-((Y - a).^2) / (2 *
 var_x)))>=...
  (p0 * (1 / sqrt(var_z * 2 * pi)) * exp(-((Y - a).^2) / (2 *
 var_z)))...
  , ~target))) / N;
disp("Perr_theoretical = " + theoretical_err);
disp("Perr_experimental = " + experimental_err);

% b)
var_z = [4, 9, 16, 25];
```

```
sigma_z = sqrt(var_z);

eta = logspace(-5,3,500);

pf = zeros(length(var_z), 1, 500);
pd = zeros(length(var_z), 1, 500);
sigZ_to_sigX = zeros(length(var_z), 1, N);

for i = 1:length(var_z)
    target = (rand(N,1) > p0);
    A = a * double(target);
    X = sigma_x * randn(N,1);
    Z = sigma_z(i) * randn(N,1);
    Y = a + X .* target + Z .* (~target);

    % receiver operating value
    pf(i,:,:) = sum(and(...
     (p1 * (1/sqrt(var_x *2*pi)) * exp(-((Y - a).^2) / (2 * var_x)))
 >= ...
     (p0*(1/sqrt(var_z(i)*2*pi)) * exp(-((Y-a).^2)/
(2*var_z(i))))*eta...
     , ~target))/sum(~target);
    pd(i,:,:) = sum(and(...
     (p1 * (1/sqrt(var_x *2*pi)) * exp(-((Y - a).^2) / (2 * var_x)))
 >= ...
     (p0*(1/sqrt(var_z(i)*2*pi)) * exp(-((Y-a).^2)/
(2*var_z(i))))*eta...
     , target))/sum(target);
    % \sigma_z^2 / \sigma_x^2 ratio
    sigZ_to_sigX(i) = var_z(i) / var_x;
end

% plotting
figure;
for j = 1:length(var_z)
    plot(reshape(pf(j, :, :), [1,500]), reshape(pd(j, :, :),
 [1,500]), ...
        'DisplayName', ['\sigma_z^2/\sigma_x^2=
 ',num2str(sigZ_to_sigX(j))], ...
        'linewidth', 1)
    hold on
end

xlabel('Pf'),ylabel('Pd'),title('Receiver Operating Curve'),legend;

% Thanks to Mark Koszykowski for his help with part 1 of this project.

Perr_theoretical = 0.18584
Perr_experimental = 0.1857
```

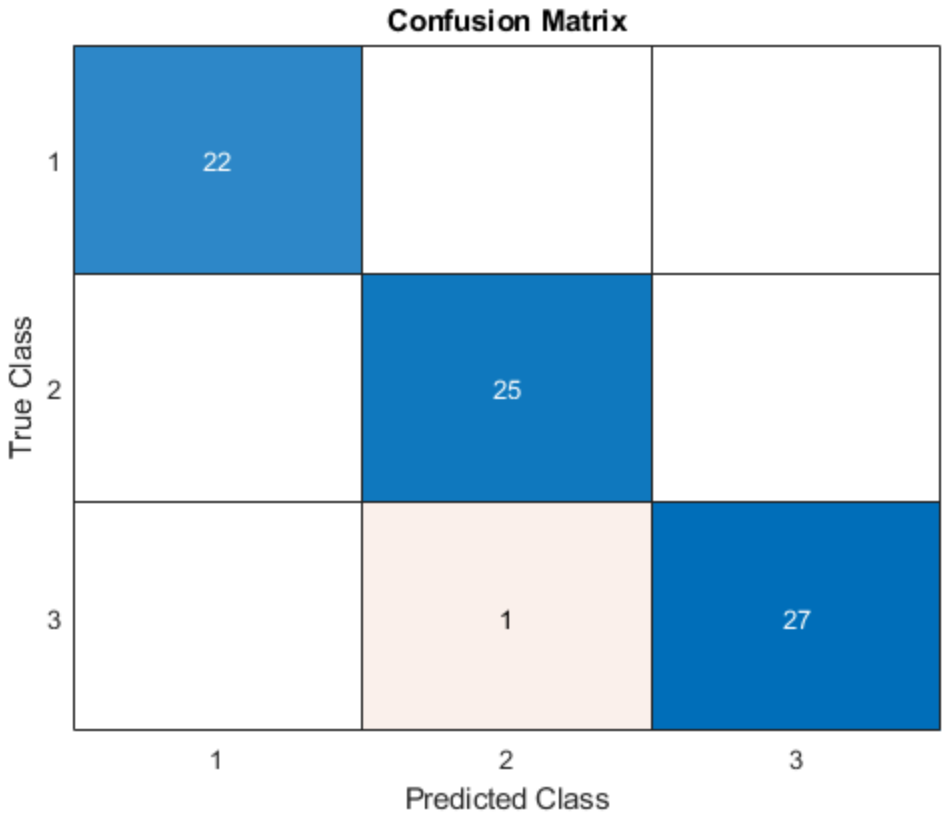


Part 2

```
clear;
load('Iris.mat');

%given/set-up
num_samples = size(features, 1);
num_classes = 3;

%splitting data
size_split= num_samples/2;
shuffling=randperm(num_samples,num_samples);
training=transpose(shuffling);
training=training(76:150);
testing=training(1:75);

%training
train_f = features(training, :);
train_l = labels(training, :);

%testing
test_f = features(testing, :);
test_l= labels(testing, :);

% calculating priors
```

```
priors = histcounts(test_l) / length(test_l);

%locating where the labels are for each class (1,2,3)
track_1=train_l==1;
track_2=train_l==2;
track_3=train_l==3;
feat1=train_f(track_1,:);
feat2=train_f(track_2,:);
feat3=train_f(track_3,:);

%calculating the mean
mu1=mean(feat1);
mu2=mean(feat2);
mu3=mean(feat3);

%calculating the variances
cov1=cov(feat1);
cov2=cov(feat2);
cov3=cov(feat3);

%compute likelihood
result(:,1) = mvnpdf(test_f, mu1, cov1) * priors(1);
result(:,2) = mvnpdf(test_f, mu2, cov2) * priors(2);
result(:,3) = mvnpdf(test_f, mu3, cov3) * priors(3);

% get the maximum
[~, est] = max(result, [], 2);

%calculate the error
prob_err = 1- mean(est == test_l);
disp("The probability of error: "+prob_err);

%confusion matrix
cm=confusionmat(est, test_l);

figure;
confusionchart(cm);
title('Confusion Matrix');
%disp(cm);
```

The probability of error: 0.013333



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