$$x[n] = 2\delta[n] + \delta[n-1] + \delta[n-3]$$

a) $X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \qquad 0 \le k \le (N-1)$

$\qquad = \sum_{n=0}^{4} x[n] W_5^{kn} \qquad 0 \le k \le 4.$

$\qquad = \sum_{n=0}^{4} (2\delta[n] + \delta[n-1] + \delta[n-3]) W_5^{kn}$

$\qquad = \sum_{n=0}^{4} 2\delta[0] W_5^{0} + W_5^{k} \sum_{n=0}^{4} \delta[n-1] + W_5^{3k} \sum_{n=0}^{4} \delta[n-3]$

$\qquad = 2 + W_5^{k} + \cancel{W} W_5^{3k}$

b) $Y[k] = X[k]^2 = (2 + W_5^{k} + W_5^{3k})^2$

$\qquad = 4 + 4W_5^{k} + 4W_5^{3k} + W_5^{2k} + 2W_5^{4k} + W_5^{6k} \quad \nearrow W_5^{k}$

$\qquad = 4 + 5W_5^{k} + W_5^{2k} + 4W_5^{3k} + 2W_5^{4k}$

c) $y[n] = \frac{1}{N} \sum_{k=0}^{N-1} Y[k] e^{j(\frac{2\pi}{N})kn} = \frac{1}{5} \sum_{k=0}^{4} Y[k] e^{j(\frac{2\pi}{5})kn}$

$y[n] = 4\delta[n] + 5\delta[n-1] + \delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$

d) $4 + 4 - 1 = 7$

$\therefore N = 7$

e)

a) $X[k] = 2 + W_7^k + W_7^{3k}$

b) $Y[k] = 4 + 4W_7^k + W_7^{2k} + 4W_7^{3k} + 2W_7^{4k} + W_7^{6k}$

c) $y[n] = 4\delta[n] + 4\delta[n-1] + \delta[n-2] + 4\delta[n-3] + 2\delta[n-4] + \delta[n-6]$