Verifying Placeholder on Bitcoin via Procedurally-Predicated FH-FE Covenants

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Abstract

Zero-Knowledge Proof (ZKP) verification on the Bitcoin has long been considered unfeasible due to the limitations of the existing Bitcoin Script language. Specifically, the absence of covenants, such as CAT, CTV, $CSFS$, has prevented the implementation of Merkle tree paths computations required for FRI/LPC-alike commitment schemes commitments verification. Despite various proposals (e.g. BIP-420) to re-enable or introduce new covenant opcodes, these changes have not been adopted (yet?), leaving ZKP verification on Bitcoin an unresolved challenge.

This paper proposes an approach to enable a hash-based commitment scheme (LPC) proof system (Placeholder[1]) proofs verification on Bitcoin via (i) emulating abscent covenants through the use of Function Hiding Functional Encryption (FH-FE) and (ii) by introducing FH-FE predicate-defined Placeholder verification (effectively introducing the Placeholder verification covenant opcode with this). By leveraging FH-FE, we construct covenants that can efficiently verify Placeholder proofs without the need for `OP_CAT` or any other protocol upgrades. The method proposed involves generating unique keys and signatures that are conditionally valid based on the satisfaction of Placeholder proof conditions. This approach not only overcomes the current limitations of Bitcoin Script but also opens up new possibilities for implementing conditional transfers (via FH-FE covenants) and true Bitcoin zkRollups.

1   Introduction

Zero-Knowledge Proof (ZKP) verification on the Bitcoin has long been considered unfeasible due to the limitations of the existing Bitcoin Script language. Specifically, the absence of covenants, such as CAT, CTV, $CSFS$, has prevented the implementation of Merkle tree paths verification required for FRI/LPC-alike commitment schemes commitments verification. Despite various proposals (e.g. BIP-420) to re-enable or introduce new covenant opcodes, these changes have not been adopted, leaving ZKP verification on Bitcoin an unresolved challenge.

The obvious solution to this problem is to upgrade Bitcoin's protocol, which leads to the necessity to achieve social consensus, which is a quite complicated process. This means that the next solution in line is to emulate covenants necessary for the verification to happen.

This paper proposes an approach to enable a hash-based commitment scheme (LPC) proof system (Placeholder[1]) proofs verification on Bitcoin via (i) emulating abscent covenants through the use of Single Argument Functional Encryption (SA-FE) and (ii) by introducing SA-FE predicate-defined Placeholder verification (effectively introducing the Placeholder verification covenant opcode with this). By leveraging SA-FE, we construct covenants that can efficiently verify Placeholder proofs without the need for `OP_CAT` or any other protocol upgrades. The method proposed involves generating unique keys and signatures that are conditionally valid based on the satisfaction of Placeholder proof conditions. This approach not only overcomes the current limitations of Bitcoin Script but also opens up new possibilities for implementing conditional transfers (via SA-FE covenants) and true Bitcoin zkRollups.

2 Preliminaries

2.1 Covenants

Covenants are restrictions on Bitcoin transactions. They define rules about where and how Bitcoin can be spent, adding a layer of programmability to Bitcoin transactions. Covenants are not currently part of Bitcoin's native functionality and require the community to agree on and implement specific upgrades. Here are some key proposals:

1. **CheckTemplateVerify (CTV)**: CTV is a covenant which only allows the exact next transaction to be executed. It allows a user to commit to a specific transaction by ensuring that only the hash of that transaction matches a predefined value. It was proposed back in 2020 and got assigned BIP-119.

2. **Concatenation (CAT)**: CAT operation is crucial for more advanced covenants. It concatenates two data items on the stack, enabling more complex scripts and conditions. For instance, CAT can be used to implement covenants that check multiple conditions on the Bitcoin being spent, allowing for a broader range of transaction types and restrictions.

   CAT's significance lies in its ability to support complex Bitcoin Script operations that go beyond simple locking and unlocking scripts. This makes it possible to create more sophisticated covenants that can enforce a wide variety of spending conditions.

   In particular CAT can help construct Merkle trees for verifying commitments of hash-based commitment scheme-based proof systems proofs within Bitcoin script natively.

   There were a couple of BIPS introduced to reflect CAT: BIP-347 and a more recent (and a popular one) - BIP-420.

2.1.1 Covenants: CAT

The absence of concatenation covenant (expressed in an `OP_CAT` opcode) in Bitcoin has made certain operations cumbersome or impossible. `OP_CAT` is essential for efficient Merkle tree operations for verifying commitments. Without `OP_CAT`, simulating its functionality requires cumbersome workarounds that are often impractical. The re-enabling of `OP_CAT` would simplify these operations and make Bitcoin scripts more powerful and flexible.

The re-enabling of `OP_CAT` has been positively received by the Bitcoin community, with several proposals and discussions taking place to bring it back. The draft BIP-420 (`https://github.com/bip420/bip420`) for `OP_CAT` has undergone several iterations, and its implementation in Bitcoin Core is actively being discussed. Re-enabling `OP_CAT` would require a soft fork, which, if successful, would mark a significant enhancement in Bitcoin's scripting capabilities.

The progress towards re-enabling `OP_CAT` is promising, with discussions and reviews happening in the Bitcoin Core PR Review Club and other forums. The potential activation of `OP_CAT` would enable more advanced scripts and applications on Bitcoin, paving the way for Turing-complete applications and improved functionality.

2.1.2 Bitcoin-friendly Proof Systems

To verify zero-knowledge proofs on Bitcoin, the proof system must be efficient and fit within Bitcoin's constraints. A Bitcoin-friendly proof system should minimize the weight units used in the script, stay within stack limits, and utilize existing opcodes like hash functions to reduce computational costs. Hash-based commitment scheme-enabled proof systems, such as Placeholder, are more likely to be compatible with Bitcoin due to their reliance on hash functions and a more ofter usage of smaller prime fields.

By leveraging recursive verification and optimizing for Bitcoin's limitations, it is possible to create proof systems that are both efficient and practical for on-chain verification. The combination of `OP_CAT` and efficient proof systems can enable powerful and flexible covenants, enhancing Bitcoin's programmability and privacy features.

Unfortunately, `OP_CAT` being enabled will take time. This means to unlock Bitcoin native SNARK verification, we need to emulate missing covenants and use a hash-based commitment scheme-enabled proof system to enable the verificaiton without the upgrade.

2.2 Placeholder Proof System

Placeholder [1] is a zero-knowledge succinct non-interactive argument of knowledge based on $\mathcal{PlonK}$-style arithmetization. Placeholder's commitment scheme and types of arithmetization, are replaceable and configurable. Low-level Placeholder circuits can adapt to selected parameters, such as table size, date degree, and lookup options. These properties enable the flexible configuration of Placeholder with trade-offs between circuit parameters, trust assumptions, and efficiency of proof generation. Due to this flexibility, Placeholder can accommodate particular cases, consistently achieving efficient results.

zkSNARK is a type of zero-knowledge proof system that allows one to prove the authenticity of a statement to a verifier without revealing any additional information beyond the statement's validity. The "succinct" and "non-interactive" aspects of zk-SNARKs refer to the fact that the proof is short and does not require any interaction between the prover and verifier beyond the initial setup.

Conceptually, general SNARK construction contains three steps:

1. Translate the problem into a set of polynomials.

2. Commit the polynomials.

3. Prove some relations on the committed polynomials.

Placeholder follows this general SNARK construction and contains two main modules:

1. Arithmetization: Defines the arithmetic representation of the proving statement. Placeholder uses $\mathcal{PlonK}$-based representation with custom gates. The idea was introduced in the TurboPLONK paper [2] and modified later in other proof systems like Halo2 [3] and Kimchi.

2. Commitment Scheme: Placeholder uses the List Polynomial Commitment scheme ([4], [1]) for polynomials obtained from the arithmetization procedure.

Because of the use of List Polynomial Commitment (LPC), Placeholder is positioned as a perfect proof system to be verified on Bitcoin.

But the problem is that even this method doesn't guarantee the so-called "pessimistic" verification on Bitcoin because of commitments-only check being possible. To verify the circuit part, it is required to introduce `OP_MUL` and a larger acceptable script size which requires Bitcoin protocol upgrade. This means a different method should be introduced and it is required to emulate all the following at once:

1. Missing covenants (e.g. CAT)

2. Missing opcodes (e.g. `OP_MUL`)

3. Larger script size

2.3 Functional Encryption

Functional Encryption (FE) is a technique that allows computation over encrypted data to yield decrypted results. It supports restricted secret keys that enable a key holder to learn a specific function of the encrypted data, without learning anything else about the data. For example, given an encrypted program, the secret key may enable the key holder to learn the output of the program on a specific input without learning anything else about the program.

The concept of Functional Encryption was formally studied by Boneh, Sahai, and Waters in [5], who provided precise definitions and discussed its security challenges. The security of FE is non-trivial to define; a natural game-based definition is inadequate for some functionalities, leading to a simulation-based definition, which, while provably secure in the random oracle model, cannot be satisfied in the standard model.

In a functional encryption system, a decryption key allows a user to learn a function of the encrypted data. Briefly, in an FE system for functionality $F(\cdot, \cdot)$ (modeled as a Turing Machine), an authority holding a master secret key can generate a key sk_k that enables the computation of the function $F(k, \cdot)$ on encrypted data. More precisely, using sk_k, the decryptor can compute $F(k, x)$ from an encryption of x. The security of the system guarantees that one cannot learn anything more about x.

An FE scheme for a functionality F is a tuple of four polynomial-time algorithms: *setup*, *keygen*, *enc*, and *dec*, satisfying the following correctness condition for all k in the key space K and x in the plaintext space X:

1. $(pp, mk) \leftarrow setup(1^\lambda)$ (generate a public and master secret key pair)

2. $sk \leftarrow keygen(mk, k)$ (generate secret key for k)

3. $c \leftarrow enc(pp, x)$ (encrypt message x)

4. $y \leftarrow dec(sk, c)$ (use sk to compute $F(k, x)$ from c)

The output y should equal $F(k, x)$ with probability 1.

Standard public-key encryption is a simple example of functional encryption where $K = \{1, \epsilon\}$ and $F(k, x) = x$ if $k = 1$ and $F(k, x) = \text{len}(x)$ if $k = \epsilon$.

2.4 Function-Hiding Inner Product Encryption

Besides many different kinds of FE schemes out there, as reported in [6], in many real scenarios it is important to consider also the privacy of the computed function. The motivation behind this is the fact that a typical workflow of a Bitcoin transaction involves techniques around manipulating pre-signed transactions (or an encrypted signing key) which, if being revealed not in the right moment, would break the whole protocol. If the FE scheme in use does not guarantee any hiding of the function (which is the case for many existing FE schemes), then a hypothetical key sk_f might reveal the predicate functionality contents f, which is undesirable when f itself contains sensitive information (aka a pre-signed transaction or a private key). This has motivated the study of function privacy in FE, see for instance [7, 8, 9].

An IPE scheme is called function-hiding if the keys and ciphertexts reveal no additional information about the related vectors beyond their inner product. The fully function-hiding IPE achieves the most robust IND-based notion of both data and function privacy in the private-key setting in the standard model. The model of full function privacy is sketched hereas described in [9, 10]. Adversaries are allowed to interact with two left-or-right oracles $KeyGen_b(mk, \cdot, \cdot)$ and $Enc_b(mk, \cdot, \cdot)$ for a randomly chosen $b \in \{0, 1\}$, where $KeyGen_b$ takes two functions f_0 and f_1 as input and it returns a functional decryption key $sk_{f_b} = KeyGen(mk, f_b)$. The algorithm Enc_b takes two messages x_0 and x_1 as input and it outputs a ciphertext $c_{x_b} = Enc(mk, x_b)$. Adversaries can adaptively interact with oracles for any polynomial (a priori unbounded) number of queries. To exclude inherently inevitable attacks, there is a condition for adversarial queries that all pairs (x_0, x_1) and (f_0, f_1) must satisfy $f_0(x_0) = f_1(x_1)$.

Only two approaches have been proposed for (fully) function-private IPE schemes in the private-key setting. One is to employ the Brakerski-Segev general transformation from (non-function-private) FE schemes for general circuits [8]. The transformation itself is efficient since it simply combines symmetric key encryption with FE in a natural manner. Anyway, this approach requires computationally intensive cryptography tools, such as IND obfuscation, to realise non-function-private FE for general circuits, meaning it may be relatively inefficient overall. The other approach may be more practical. It directly constructs IPE schemes by using the dual-pairing vector spaces (DPVS) introduced by Okamoto and Takashima [11, 12].

In the last few years, there has been a flurry of works on the construction of function-hiding IPE, starting with the work of Bishop, Jain, and Kowalczyk [6]. They propose a function-hiding IPE scheme under the SXDH assumption, which satisfies an adaptive IND-based security definition. But the security model has one limitation: all ciphertext queries x_0, x_1 and all secret key queries y_0, y_1 are restrained by $\langle x_0, y_0 \rangle = \langle x_0, y_1 \rangle = \langle x_1, y_0 \rangle = \langle x_1, y_1 \rangle$. In [9], Datta et al. develop a full function-hiding IPE scheme built in the setting of asymmetric bilinear pairing groups of prime order. The security of the scheme is based on the well-studied SXDH assumption where the restriction on adversaries' queries is only $\langle x_0, y_0 \rangle = \langle x_1, y_1 \rangle$. Here, secret keys and ciphertexts of n-dimensional vectors consist of $4n + 8$ group elements. Tomida

et al., in [13], construct a more efficient function-hiding IPE scheme than that of [9] under the XDLIN assumption, where secret keys and ciphertexts consist of $2n + 5$ group elements. Kim et al., in [14], put forth a fully-secure function-hiding IPE scheme with smaller parameter sizes and run-time complexity than those in [6, 9]. The scheme is proved SIM-based secure in the generic model of bilinear maps. In [15], Zhao et al. present the first SIM-based secure secret-key IPE scheme under the SXDH assumption in the standard model. The authors claim that the scheme can tolerate an unbounded number of ciphertext queries and adaptive key queries. Zhao et al. in [16] propose a new version of the scheme, which is an improvement in terms of computational and storage complexity. In a very recent work [17], Liu et al. present a more efficient and flexible private-key IPE scheme with SIM-based security. To ensure correctness, the scheme requires that the computation of inner products is within a polynomial range, where the discrete logarithm of $g^{\langle x, y \rangle}$ can be found in polynomial time. In [17], the authors compare their proposed IPE scheme with those in [9, 13, 15, 16]. The performance of this scheme appears superior in both storage complexity and computation complexity. Moreover, secret keys and ciphertexts are shorter.

Although most aforementioned IPE schemes are efficient and based on standard assumptions, they all have one inconvenient property: they are bounded. The maximum length of vectors has to be fixed at the beginning, and afterward, one cannot handle vectors whose lengths exceed it. This could be inconvenient when it is hard to predict which data will be encrypted in the setup phase. One may think to solve the problem by setting the maximum length to a large value. However, the size of parameters expands at least linearly with the fixed maximum length, and such a solution incurs an unnecessary efficiency loss. In the context of IP-PE and ABE, there exist unbounded schemes (see, for instance, [18, 19, 20, 6]), whose public parameters do not impose a limit on the maximum length of vectors or number of attributes used in the scheme. In [13], Tomida and Takashima construct two concrete unbounded IPE schemes based on the standard SXDH assumption, both secure in the standard model: the first is a private-key IPE with fully function hiding, the second scheme is a public-key IPE with adaptive security. Concurrently and independently, in [21], Dufour-Sans and Pointcheval describe an unbounded IPE system supporting identity access control with succinct keys. Their construction is proved selectively IND-secure in the random oracle model based on the standard DBDH assumption. In [13], it is shown a comparison, in terms of efficiency, among private-key schemes that are fully function hiding [20, 19, 15] and public-key schemes with adaptive security in the standard model [22].

3 Proposal

Let's consider ways to verify Placeholder (defined as follows) within Bitcoin:

Algorithm 1 Verify

    Input: $\pi_{\mathsf{Placeholder}}$, preprocessed_data, transcript
    Output: true/false

1: Parse proof $\pi_{\mathsf{Placeholder}}$ into:

$\pi_{\mathsf{comm}} = \{\ \overline{\text{variable}},\ \overline{\text{V\_polynomials}},\ \overline{\text{lookup}^{\mathsf{perm}}},\ \overline{\text{quotient}},\ \overline{\text{fixed}}\}$

π_{eval} is evaluation proofs for

polynomial_evaluations $= \{$

    $w_i(y), w_i(\zeta^d \cdot y), i = 0, \ldots, \mathsf{N_{wt}} - 1, \quad s_i(y), s_i(\zeta^d \cdot y), i = 0, \ldots, \mathsf{N_{pi}} - 1$

        for all corresponding $d \in \mathbf{o}$,

    $V^\sigma(y), V^\sigma(\zeta \cdot y), a^{\mathrm{perm}}(y), a^{\mathrm{perm}}(\zeta^{-1} \cdot y), l^{\mathrm{perm}}(y), V_L(y), V_L(\zeta \cdot y),$

    $\{T_i(y)\}, i = 0, \ldots, N_T - 1$

    $c_i(y), c_i(\zeta^d \cdot y), i = 0, \ldots, \mathsf{N_{cn}} - 1, \quad l_i(y), l_i(\zeta^d \cdot y), i = 0, \ldots, \mathsf{N_{lk}} - 1, \quad q_i(y), q_i(\zeta^d \cdot y), i = 0, \ldots, \mathsf{N_{sl}} - 1$

        for all corresponding $d \in \mathbf{o}$,

    $q^{\mathrm{last}}(y), q^{\mathrm{pad}}(y), L_0(y)\}$

2: **Verify Permutation Argument:**
3:     Denote polynomials included in permutation argument as $f_0, \ldots, f_{\mathsf{N_{perm}}-1}$
4:     Get values $\{f_i(y)\}, \{S_i^e(y)\}, \{S_i^\sigma(y)\}, V^\sigma(y), V^\sigma(\zeta \cdot y), L_0(y), q^{\mathrm{last}}(y), q^{\mathrm{pad}}(y)$ from $\pi_{\mathsf{Placeholder}}$
5:     Calculate

$$F_0(y), F_1(y), F_2(y) = \mathsf{PermArgumentVerify}(y, V^\sigma(y), V^\sigma(\zeta \cdot y), \{f_i(y)\},$$
$$\{S_i^e(y)\}, \{S_i^\sigma(y)\}, q^{\mathrm{pad}}(y), q^{\mathrm{last}}(y), L_0(y),\ \mathsf{transcript})$$

6: **Verify Lookup Argument:**
7:     Denote polynomials included in lookup argument as $a_0, \ldots, a_{\mathsf{N_{lk}}-1}$
8:     Get values $\{a_i(y)\}, \{l_i(y)\}, a^{\mathrm{perm}}(y), a^{\mathrm{perm}}(\zeta^{-1} \cdot y), l^{\mathrm{perm}}(y), V_L(y), V_L(\zeta \cdot y), L_0(y), q^{\mathrm{last}}(y), q^{\mathrm{pad}}(y)$
    from $\pi_{\mathsf{Placeholder}}$
9:     Calculate:

$$F_3(y), F_4(y), F_5(y), F_6(y), F_7(y) = \mathsf{LookupArgumentVerify}($$
$$\overline{\text{lookup}^{\mathsf{perm}}}, \{a_i(y)\}, \{l_i(y)\},$$
$$a^{\mathrm{perm}}(y), a^{\mathrm{perm}}(\zeta^{-1} \cdot y), l^{\mathrm{perm}}(y),$$
$$V_L(y), V_L(\zeta \cdot y), L_0(y), q^{\mathrm{last}}(y), q^{\mathrm{pad}}(y),\ \mathsf{transcript})$$

10: transcript.$append(\overline{\text{V\_polynomials}})$
11: **Verify Basic Constraints:**
12:     For $i = 0, \ldots, \mathsf{N_{sl}} - 1$

$$g_i(X) = q_i(X) \cdot (\theta^{k_i - 1 + \nu_i} \mathcal{C}_{i_0}(X) + \cdots + \theta^{\nu_i} \mathcal{C}_{i_{k-1}}(X))$$

13:     Calculate a constraints-related numerator of the quotient polynomial $F_8(y) = \sum\limits_{0 \le i < \mathsf{N_{sl}}} (g_i(y))$

14: **Quotient polynomial check:**
15:     **if** $\sum\limits_{i=0}^{8} \alpha_i F_i(y) \neq Z(y) T(y)$ **then return** $false$
16:     Get challenges $\{\alpha_i \in \mathbb{F}\}_{i=0}^{8}, \theta \in \mathbb{F}, y \in \mathbb{F} \setminus H$ from transcript
17:     transcript.$append(\overline{\text{quotient}})$
18: **Evaluation proof check**
19:     **if** LPC.EvalVerify(polynomials_evaluations, π_{comm}, transcript) $= false$ **then return** $false$

And, specifically, commitment scheme verification procedures are defined as follows:

Algorithm 2 FRI.Verify

Input: FRI proof π, Merkle roots $\{\mathsf{T\_root}_k\}_{k=0}^{K-1}$, $\{U_k(X)\}_{k=0}^{l-1}$, $\{V_k(X)\}_{k=0}^{l-1}$, transcript

Output: verification result = true/false

1: $\{\mathsf{fri\_root}_0, \ldots, \mathsf{fri\_root}_{\mathsf{steps}_{\mathsf{FRI}}-1}, \pi^{(0)}, \ldots \pi^{(r_q-1)}, \mathsf{final\_polynomial}\} = \mathtt{parse}(\pi)$
2: $\mathcal{D}^{(0)} = \mathcal{D}, \quad \mathcal{D}^{(i+1)} = q_{r_i}\left(\mathcal{D}^{(i)}\right)$, for $i = 0, \ldots, \mathsf{steps}_{\mathsf{FRI}} - 2$
3: **for all** $k = 0, \ldots, L-1$ **do**
4:     $\mathsf{transcript}.append(\mathsf{T\_root}_k)$
5: **end for**
6: $\tau := \mathsf{transcript}.challenge(\mathbb{F})$
7: $t := 0$
8: **for all** $i := 0, \ldots, \mathsf{steps}_{\mathsf{FRI}} - 1$ **do**
9:     $\mathsf{transcript}.append(\mathsf{fri\_root}_i)$
10:     **for all** $step := 0, \ldots, r_i - 1$ **do**
11:         $\alpha_t := \mathsf{transcript}.challenge(\mathbb{F})$
12:         $t := t + 1$
13:     **end for**
14: **end for**
15: **for all** $query = 0, \ldots, r_q - 1$ **do**
16:     $\{\pi^*, \pi_0, \ldots, \pi_{\mathsf{steps}_{\mathsf{FRI}}-1}\} = \mathtt{parse}(\pi^{(round)})$
17:     $x^{(0)} = \mathsf{transcript}.challenge(\mathcal{D}_0)$
18:     $x^{(i+1)} = q_{r_i}(x^{(i)}), \ i = 0, \ldots, \mathsf{steps}_{\mathsf{FRI}} - 1$
19:     Construct cosets $S^{(i)} = \{s \in \mathcal{D}^{(i)} \mid q_{r_i}(s) = x^{(i+1)}\}$, for $i = 0, \ldots, \mathsf{steps}_{\mathsf{FRI}} - 1$ $\triangleright |S^{(i)}| = m^{r_i}$
20:     `Initial proof check`
21:     t := 0;
22:     **for all** $k := 0, \ldots, K - 1$ **do**
23:         **if** $\pi^*.\mathsf{auth}_k.\mathsf{root} \neq \mathsf{T\_root}_k$ **then return** $false$
24:         **if** $\mathsf{MT.Validate}(\pi^*.\mathsf{auth}_k, \{\pi^*.\mathsf{val}^{(t)}, \ldots, \pi^*.\mathsf{val}^{(t+l_k-1)}\}) = false$ **then return** $false$
25:         $t := t + l_k$
26:     **end for**
27:     Compute values of combined polynomial Q values val from $\pi_k.\mathsf{val}$

$$\mathsf{val} = \left\{ \prod_{k=0}^{l-1} \tau^{l-k-1} \frac{\pi^*.\mathsf{val}_s^{(k)} - U_k(s)}{V_k(s)} \right\}_{s \in S^{(0)}}$$

28:     `Round proofs check`
29:     $t := 0, S := S^{(0)}$
30:     **for all** $i := 0, \ldots, \mathsf{steps}_{\mathsf{FRI}} - 1$ **do**
31:         **if** $\pi_i.\mathsf{auth}.\mathsf{root} \neq \pi.\mathsf{fri\_root}_i$ **then return** $false$
32:         **if** $\mathsf{MT.Validate}(\pi_i.\mathsf{auth}, \mathsf{val}) = false$ **then return** $false$
33:         **for all** $step := 0, \ldots, r_i - 1$ **do**
34:             $S_{\mathsf{next}} := \{q(s)\}_{s \in S}$
35:             $\mathsf{interpolant}_s := \mathsf{lagrange\_interpolation}(\{s_j, \mathsf{val}_{s_j}\}_{q(s_j)=s})(\alpha_t)$ for $s \in S_{\mathsf{next}}$
36:             $t := t+1, S := S_{\mathsf{next}}, \mathsf{val} := \{\mathsf{interpolant}_s\}_{s \in S_{\mathsf{next}}}$ $\triangleright |S_{\mathsf{next}}| = r_i - step - 1$
37:         **end for**
38:         **if** $\mathsf{val} \neq \pi_i.\boldsymbol{y}_{(x^{(i+1)})}$ **then return** $false$ $\triangleright |\mathsf{val}| = 1$
39:         $\mathsf{val} := \pi_i.\boldsymbol{y}$
40:     **end for**
41:     **if** $\mathsf{final\_polynomial}(x^{(\mathsf{steps}_{\mathsf{FRI}})}) \neq \mathsf{val}$ **then return** $false$
42: **end for**
43: **return** true

Algorithm 3 LPC.EvalVerify

Input:
    proof $\mathcal{P}$,
    evaluation points $\{\xi^{(k)}\}_{k=0}^{l-1}$,
    roots of Merkle trees $\{\mathsf{root}_k\}_{k=0}^{K-1}$,
    transcript

Output: verification result = true/false

1: $\{\boldsymbol{z}^{(0)}, \ldots, \boldsymbol{z}^{(l-1)}, \pi\} = \mathsf{parse}(\mathcal{P})$
2: Interpolate polynomials $U_k(X) = \mathsf{lagrange\_interpolation}(\{\xi_j^{(k)}, \boldsymbol{z}_j^{(k)}\})$ for $0 \le k < l, 0 \le j < |\xi^{(k)}|$
3: Compute $V_k(X) = \prod_{j=0}^{|\xi^{(k)}|-1}(X - \xi_j^{(k)})$
4: **if** $\mathsf{FRI.Verify}(\pi, \{\mathsf{root}_k\}_{k=0}^K, \{U_k(X)\}_{k=0}^{l-1}, \{V_k(X)\}_{k=0}^{l-1}, \mathsf{transcript}) = \mathsf{false}$ **then return** false
5: **return** true

This means, to achieve direct verification of Placeholder proofs on Bitcoin practically two approaches can be considered:

1. Implement absent trivial covenants (e.g. CAT) within FH-FE predicates and to implement LPC/FRI commitments verification using FH-FE covenant signing as an equivalent of using `OP_CAT` itself.

   This approach would lead to the commitments being verified, but not the gates part (aka the circuit definition). This means the verification would still be some considered some kind of "optimistic" 'cause only commitments will be checked (i.e. $LPCVer$ and $FRIVer$) (no constraint-related checks will be made).

2. Implement the whole Placeholder verification procedure within the FE predicate using an FE scheme enabling circuits-based predicates (i.e. using non-bilinear FE scheme so arbitrary circuits would be expressable).

Considering the complexity of the verifier, both of these verification approaches would require to pick a functional encryption scheme, which would allow predicates to be defined with at least polynomial-size circuits (such schemes can be found among predicate encryption schemes) as $F(k, x)$ would need to define, among other primitives, the ECDSA signing algorithm as the encrypted transaction needs to be signed at the last stage of a predicate.

Some results have been obtained for PE schemes for circuits. In [23], Gorbunov et al. present a leveled PE scheme for all circuits (Boolean predicates of bounded depth), with succinct ciphertexts and secret keys independent of the size of the circuit. The achieved privacy notion is a selective SIM-based variant of attribute-hiding, assuming the hardness of the subexponential LWE problem. Recall that the strong variant notion (full attribute-hiding) is impossible to realize for many messages [24].

Some results have also been obtained for general-purpose FE, in which functions associated with secret keys can be any arbitrary circuits. In [25], Garg et al. give constructions for IND obfuscation (based on multilinear maps), and they use it to construct FE for all polynomial-size circuits. In [26], there are constructions directly based on multilinear maps. Further analysis can be found in [24] where Lin and Tessaro reduce the degree of the required multilinear map to 3.

It is also required to take into consideration that an obfuscated/encrypted signing entity private key is necessary to be baked into the covenant, which means the choice of a particular FE scheme would depend on if it supposes the presence of IO or function-hiding.

3.1   Function-hiding Inner Product Encryption

As mentioned, it is required to have the transaction signing key encrypted within the covenant. This means the FE scheme should be a function-hiding one. If the FE scheme in use does not guarantee any hiding of the function (which is the case for many existing FE schemes), then the key sk_f might reveal f, which is undesirable when f itself contains sensitive information (Secp256k1 signing key). As discussed in 2.4,

3.2 Functional Encryption Covenants

Emulating CAT via function-hiding functional encryption (similar to the proposal in [27]) would require to define, besides ECDSA signing procedure, an implementation of a circuit-expressed `OP_CAT`.

3.2.1 Setup

A setup of the overall scheme goes as follows:

Algorithm 4 $CovenantSetup$

Input:

    FE/PE Master Keys: $(m, M) \in K = \{0, 1\}_* \cup \{\epsilon\}$,

    Secp256k1 Keys: $(sk, pk) \in \mathcal{F}_p : y^2 = x^3 + 7 \ (mod \ p) \ \forall \ p = 2^{256} - 2^{32} - 977$,

Output: Ciphertext C_f

 1: $m : \ C_p \leftarrow Encrypt(M, p)$

 2: $PrivateKey(sk, \alpha, \beta) = Add(m, Concat(\alpha, \beta))$

 3: $Publickey(pk\alpha, \beta) = TweakAdd(pk, Concat(\alpha, \beta))$

 4: Functionality for $F(p, \alpha, \beta, tx) = ECDSASign(PrivateKey(p, \ \alpha, \beta), \ tx)$

 5: Compute the public key: $u \leftarrow PublicKey(p, \ \alpha, \beta)$

 6: Encrypt the function: $C_f = EncryptFunc(m, \ F)$

 7: Trusted party is supposed to erase m and p and the signing now can be arranged as follows:

$$\sigma \leftarrow C_f(C_p, \ Encrypt(M, tx))$$

From now on, σ can be used as a signature of u over tx. If m and p were erased, C_f can only sign in case CAT is "executed" correctly.

3.3 Placeholder Verification as a Covenant

As the circuit which defines predicate within the chosen PE scheme is possible to be arranged to represent a Turing-complete computation, a much more complicated covenant can be constructed. For example, the signing procedure can be prefixed/conditioned with the Placeholder verification procedure. This means that it is required to define the predicate functionality as follows at first:

Algorithm 5 $VerifySignPredicate$

Input:

    Secp256k1 Private Key: sk

    Public Input: $input$

    Transaction: tx

    Placeholder zkProof: $proof$

Output: Signature S

 1: **if** $\mathsf{Verify}(proof, input) = \mathsf{true}$ **then return** $ECDSASign(sk, \ tx)$

 2: **return** true

Algorithm 6 *CovenantSetup*

Input:

    FE/PE master keys: $(m, M) \in K = \{0, 1\}_* \cup \{\epsilon\}$,

    Secp256k1 Keys: $(sk, pk) \in \mathcal{F}_p : y^2 = x^3 + 7 \ (mod \ p) \ \forall \ p = 2^{256} - 2^{32} - 977$,

    Public Input: $input$

    Transaction: tx

Output: Functionality $C_f : K \times X \Leftrightarrow \{0, 1\}_*$

  1: $m : \ C_p \leftarrow Encrypt(M, sk)$

  2: $PrivateKey(sk, proof, input) = Add(m, \mathsf{Verify}(proof, input))$

  3: $Publickey(pk, proof, input) = TweakAdd(pk, \mathsf{Verify}(proof, input))$

  4: $F(sk, tx, proof) = VerifySignPredicate(sk, input, tx, proof)$

  5: Compute the public key: $u \leftarrow PublicKey(pk, \ proof, input)$

  6: Encrypt the function: $C_f = EncryptFunc(m, \ F)$

  7: Protocol initializer erases m and sk.

$$\sigma \leftarrow C_f(C_p, \ Encrypt(M, tx))$$

From now on, σ can be used as a signature of u over tx. If m and sk were erased, C_f can only sign in case the covenant functionality F is "executed" correctly.

3.3.1 Usage

The generic flow usage of such a FH-FE covenant would be defined as follows.

Such a covenant would still serve as a usual Bitcoin account, but the withdrawal from it would be restructed by the covenant binary.

This means sending to such an account would look like this:

1. Retrieve an encrypted opcode binary with covenant e and private key sk.

2. Select the covenant instance parameters i if needed.

3. Tweak sk by e_i.

4. Send funds to e_i.

Redeeming from the account would look as:

1. Prepare the state transition with a specific tx and collect all necessary verification data.

2. Pass the data and tx to the encrypted opcode binary.

3. Use the resulting signature.

4 Conclusion

This paper proves that a single private argument dual input functional encryption is enough to generate SA-FE predicates to express complex application-specific covenants and, as an example, to verify a hash-based commitment scheme-based proof system (Placeholder) proof, conditionally signing Bitcoin transaction with ECDSA signature, which can induce covenant-alike functionality within Bitcoin Script without any upgrades to the Bitcoin protocol/implementation.

Having covenants implemented this via SA-FE means:

1. Computation integrity verification happening outside of Bitcoin Script, while still being checked by it.

2. Fixed witness size 64 bytes signature, script size 32 bytes private key.

3. Composability of a signature with other opcodes in a script with other conditions.

4. Non-interactive protocol definition means no infrastructure is necessary to maintain liveness, which improves security assumptions to native Bitcoin L1-level ones.

5. Setup procedure, due to the nature of currently existing SA-FE schemes, supposes the initial covenant deployer to behave honest. This can be improved with traditional MPC-based trusted setup, so only one participant out of N generating master key would be required to be honest.

6. Trivial covenants remain bytewise and verification wise more efficient.

In the same time, having Placeholder verification implemented as an SA-FE covenant brings to the table:

1. Full proof verification, which means not only commitments are being verified (like it is with an `OP_CAT` case), but the verification also includes a circuit (gates) part into it which means security assumptions improve to the strongest ones (no challenger oracle is required to be present within the verification protocol).

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