Mathematical Analysis I VERSION A January, 12 2010 at 1:30 P.M.

Surname, Name, Matriculation number:

Degree Course: Surname of Professor:

Warning: *The written exam is composed of 3 exercises. The solutions (computations, explanations and conclusions) must be written in detail on separate sheets of paper, that must be handed in at the end, with the candidate's name written on top. The present question sheet must be also returned.*

Exercise 1. (12 marks) Consider the function

$$f(x) = \log\left(\left|x^3 - 8\right| + \frac{1}{4}\right),$$

a) Find the domain of the function.

b) Find the zeroes.

c) Compute the limits as x tends to the extreme points of the domain.

d) Find all possible asymptotes of f, if they exist.

e) Find all possible points where f is not differentiable, if they exist.

f) Find the monotonicity intervals and possible maximum and minimum points of f.

g) Draw a qualitative graph of f.

Exercise 2. *(9 marks)* Let $\beta \neq 0$, and consider the function

$$f(x) = \left(1 + \frac{2}{\beta}x\right)^{\sinh(\beta x^2)}.$$

a) Compute the MacLaurin expansion of order 4 of f. [Recall: $a^b = e^{b \log a}$].

b) Set $g(x) = f(x) - 2x^3$ and find the values of $\beta \in \mathbf{R}$ for which the function g admits a local minimum at $x = 0$.

Exercise 3. *(9 marks)* Solve the Cauchy problem

$$\begin{cases} y' = 4 - 2e^{-y} - 2e^{y} \\ \\ y(0) = \log(\frac{3}{2}) \end{cases}$$

Mathematical Analysis I VERSION B January, 12 2010 at 1:30 P.M.

Surname, Name, Matriculation number:

Degree Course: Surname of Professor:

Warning: *The written exam is composed of 3 exercises. The solutions (computations, explanations and conclusions) must be written in detail on separate sheets of paper, that must be handed in at the end, with the candidate's name written on top. The present question sheet must be also returned.*

Exercise 1. (12 marks) Consider the function

$$f(x) = \log\left(\left|27 + x^3\right| + \frac{1}{3}\right),$$

a) Find the domain of the function.

b) Find the zeroes.

c) Compute the limits as x tends to the extreme points of the domain.

d) Find all possible asymptotes of f, if they exist.

e) Find all possible points where f is not differentiable, if they exist.

f) Find the monotonicity intervals and possible maximum and minimum points of f.

g) Draw a qualitative graph of f.

Exercise 2. *(9 marks)* Let $\alpha \neq 0$, and consider the function

$$f(x) = \left(1 - \frac{4}{\alpha}x\right)^{\sin(\alpha x^2)}.$$

a) Compute the MacLaurin expansion of order 4 of f. [Recall: $a^b = e^{b \log a}$].

b) Set $g(x) = f(x) + 4x^3$ and find the values of $\alpha \in \mathbf{R}$ for which the function g admits a local minimum at $x = 0$.

Exercise 3. *(9 marks)* Solve the Cauchy problem

$$\begin{cases} y' = 6 - 3(e^{-y} + e^{y}) \\ \\ y(0) = \log(\frac{4}{3}) \end{cases}$$

Mathematical Analysis I VERSION C January, 12 2010 at 1:30 P.M.

Surname, Name, Matriculation number:

Degree Course: Surname of Professor:

Warning: *The written exam is composed of 3 exercises. The solutions (computations, explanations and conclusions) must be written in detail on separate sheets of paper, that must be handed in at the end, with the candidate's name written on top. The present question sheet must be also returned.*

Exercise 1. (12 marks) Consider the function

$$f(x) = \log\left(\left|x^3+8\right| + \frac{1}{4}\right),$$

a) Find the domain of the function.

b) Find the zeroes.

c) Compute the limits as x tends to the extreme points of the domain.

d) Find all possible asymptotes of f, if they exist.

e) Find all possible points where f is not differentiable, if they exist.

f) Find the monotonicity intervals and possible maximum and minimum points of f.

g) Draw a qualitative graph of f.

Exercise 2. *(9 marks)* Let $\beta \neq 0$, and consider the function

$$f(x) = \left(1 - \frac{2}{\beta}x\right)^{\sinh(\beta x^2)}.$$

a) Compute the MacLaurin expansion of order 4 of f. [Recall: $a^b = e^{b\log a}$].

b) Set $g(x) = f(x) + 2x^3$ and find the values of $\beta \in \mathbf{R}$ for which the function g admits a local minimum at $x = 0$.

Exercise 3. *(9 marks)* Solve the Cauchy problem

$$\begin{cases} y' = 8 - 8e^{-y} - 2e^{y} \\ y(0) = \log(\frac{5}{2}) \end{cases}$$

Mathematical Analysis I VERSION D January, 12 2010 at 1:30 P.M.

Surname, Name, Matriculation number:

Degree Course: Surname of Professor:

Warning: *The written exam is composed of 3 exercises. The solutions (computations, explanations and conclusions) must be written in detail on separate sheets of paper, that must be handed in at the end, with the candidate's name written on top. The present question sheet must be also returned.*

Exercise 1. (12 marks) Consider the function

$$f(x) = \log\left(\left|x^3 - 27\right| + \frac{1}{3}\right),$$

a) Find the domain of the function.
b) Find the zeroes.
c) Compute the limits as x tends to the extreme points of the domain.
d) Find all possible asymptotes of f, if they exist.
e) Find all possible points where f is not differentiable, if they exist.
f) Find the monotonicity intervals and possible maximum and minimum points of f.
g) Draw a qualitative graph of f.

Exercise 2. *(9 marks)* Let $\alpha \neq 0$, and consider the function

$$f(x) = \left(1 + \frac{4}{\alpha}x\right)^{\sin(\alpha x^2)}.$$

a) Compute the MacLaurin expansion of order 4 of f. [Recall: $a^b = e^{b \log a}$].

b) Set $g(x) = f(x) - 4x^3$ and find the values of $\beta \in \mathbf{R}$ for which the function g admits a local minimum at $x = 0$.

Exercise 3. *(9 marks)* Solve the Cauchy problem

$$\begin{cases} y' = 12 - 3e^{y} - 12e^{-y} \\ \\ y(0) = \log(\frac{7}{3}) \end{cases}$$