BREAKWATER DESIGN USING HUDSON AND RELATED EQUATIONS

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Breakwater Design Using Hudson and Related Equations

Description

A rubble structure is often composed of several layers of random-shaped or random-placed stones, protected with a cover layer of selected armor units of either quarrystone or specially shaped concrete units. This ACES application provides estimates for the armor weight, minimum crest width, armor thickness, and the number of armor units per unit area of a breakwater using Hudson's and related equations. The material presented herein can be found in Chapter 7 of the SPM (1984).

Introduction

Until about 1930, design of rubble structures was based only on experience and general knowledge of site conditions. Empirical methods have been developed that, if used with care, will give satisfactory determination of the stability characteristics of these structures when under attack by storm waves.

General Assumptions and Limitations

Empirical formulas that were developed for the design of rubble-mound structures are generally expressed in terms of the stone weight required to withstand design wave conditions. These formulas have been largely derived from physical model studies. They are guides and must be used with experience and engineering judgment. Physical modeling is often a cost-effective measure to determine the final cross-section design for most rubble-mound structures.

Stability of Rubble Structures

A proposed breakwater may necessarily be designed for either nonbreaking or breaking waves depending upon positioning of the breakwater and severity of anticipated wave action during its economic life. Some local wave conditions may be of such severity that the protective cover layer must consist of specially shaped concrete armor units in order to provide economic construction of a stable breakwater. The following four sections describe empirical formulas used in the ACES package for design of rubble structures.

Weight of Primary Armor Unit

Comprehensive investigations were made by Hudson (1953, 1959, 1961a, 1961b) to develop a formula to determine the stability of armor units on rubble structures. The stability formula, based on the results of extensive small-scale model testing and some preliminary verification by large-scale model test, is

$$W = \frac{w_r H_i^3}{K_D (S_r - 1)^3 \cot\theta} \qquad (1)$$

where

W = weight of individual armor unit in the primary cover layer

w_r = unit weight of armor unit material

H_i = design wave height

K_D = armor unit stability coefficient (see Table A-1 of Appendix A)

$S_r = w_r / w_w$ = specific gravity of armor material

w_w = unit weight of water

θ = angle between seaward structure slope and horizontal

The dimensionless stability coefficient, K_D, accounts for factors other than structure slope, wave height, and the specific gravity of water at the site. The most important of these variables include

- Shape of armor units.
- Number of units comprising the thickness of the armor layer.
- Manner of placing armor units.
- Surface roughness and sharpness of edges of the armor units (degree of interlocking of the armor units).
- Type of wave attacking structure (breaking or nonbreaking).
- Part of the structure being attacked (trunk or head).
- Angle of incident wave attack.

These stability coefficients (Table A-1 of Appendix A) were derived from large- and small-scale tests that used many various shapes and sizes of both natural and artificial armor units. Values are reasonably definitive and are recommended for use in the design of rubble-mound structures, supplemented by physical model test results when economically warranted.

Crest Width

The width of the crest depends greatly on the degree of allowable overtopping; where there will be no overtopping, crest width is not critical. Little study has been made of crest width of a rubble structure subject to overtopping. As a general guide for overtopping conditions, the minimum crest width should equal the combined widths of three armor units (n = 3). The crest should be wide enough to accommodate any construction and maintenance equipment that may be operated from the structure. Crest width is obtained from the following equation:

$$B = n k_{\Delta} \left(\frac{W}{w_r} \right)^{1/3} \tag{2}$$

where

B = crest width

n = number of armor units (ACES application sets n = 3)

k_Δ = layer coefficient (see Table A-2 of Appendix A)

W = weight of individual armor unit in the primary cover layer

w_r = unit weight of armor unit material

Thickness of the Armor Layer

The thickness of the cover layer is determined from the following formula:

$$r = n k_\Delta \left(\frac{W}{w_r} \right)^{1/3} \tag{3}$$

where

r = average layer thickness

n = number of layers of armor units

Armor Unit Placement Density

The placing density is given by the following formula:

$$N_r = A n k_\Delta \left(1 - \frac{p}{100} \right) \left(\frac{w_r}{W} \right)^{\frac{2}{3}} \tag{4}$$

where

N_r = number of armor units for a given surface area

A = surface area (assumed as 1000)

p = average porosity of cover layer (see Table A-2 of Appendix A)

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Toe Protection Design

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TOE PROTECTION DESIGN

DESCRIPTION

Toe protection consists of armor for the beach or bottom material fronting a structure to prevent wave scour. This application determines armor stone size and width of a toe protection apron for *vertical* faced structures such as seawalls, bulkheads, quay walls, breakwaters, and groins. Apron width is determined by the geotechnical and hydraulic guidelines specified in Engineer Manual 1110-2-1614 (Headquarters, Department of the Army, 1985). Stone size is determined by a method (Tanimoto, Yagyu, and Goda, 1982) whereby a stability equation is applied to a single rubble unit placed at a position equal to the width of the toe apron and subjected to standing waves.

INTRODUCTION

Coastal structures rely upon the foundation material for vertical support. Some types of retaining walls also rely upon the bottom material for lateral support. Wave action resulting in loss of bottom material can cause damage and ultimate collapse of a protective structure. While a variety of methods for wave scour protection are employed in practice, this application addresses a simple toe protection design using an apron of armor stones fronting a structure with a vertical seaward face. Unbroken waves are assumed to be normally incident to the structure and are assumed to produce standing waves above the toe protection apron. Stone size is determined by consideration of the stability of a single stone subjected to the standing waves and situated at the seaward edge of the apron.

GENERAL ASSUMPTIONS AND LIMITATIONS

The methodology represented in this application is a composite of largely empirical guidance for the width of the toe apron, and a semi-empirical formulation for the toe stone weight. General assumptions include the following:

- Waves are normally incident to the structure.
- Standing waves form as a result of wave interaction with a vertical (seaward) face of the structure and remain unbroken in the water depth above the toe apron.
- Linear wave theory approximations are adequate to predict the standing wave properties.
- Rankine theory is adequate for evaluating the stability of the soil wedge beneath the toe apron.
- Rubble-mound material is used as the toe protection material.

In general, this application considers the stability of an armor stone at the seaward crest of the toe protection apron. It does not offer any guidance for preparation or detailed protection of foundation material at the dredge line.

The method for stone weight (Tanimoto, Yagyu, and Goda, 1982) is based upon irregular waves (characterized by significant wave height, H_s) acting on composite breakwaters. To be formally consistent with Tanimoto's approach, the normally incident wave train should use H_s values and structure configurations limited to those tested in the original research. Sheet-pile designs were not considered in the development of the method. In general, Tanimoto's method has not been verified by model tests within the Corps of Engineers. The empirical guidance used for estimating the toe protection width (Engineer Manual 1110-2-1614) has no physical coupling with Tanimoto's method for determining the toe stone weight, yet has significant impact on the formulas for stone weight.

In using Rankine theory as part of the guidance for determining toe protection width for pile structures, a common practice is to use the effective penetration depth of the pile which is less than the full driven depth. The actual computations will apply the user-specified value, and engineering judgment should be applied to adjust d_e as an input parameter.

Official guidance regarding toe protection is being revised to a less conservative design, and this interim methodology will be revised at a later date.

Width of Toe Apron

The width of the toe apron is estimated using hydraulic and geotechnical factors. Sketches depicting geotechnical and hydraulic factors considered in the estimation of the apron width for a typical sheet-pile wall and a gravity type breakwater are shown in Figures 4-2-1 and 4-2-2 respectively.

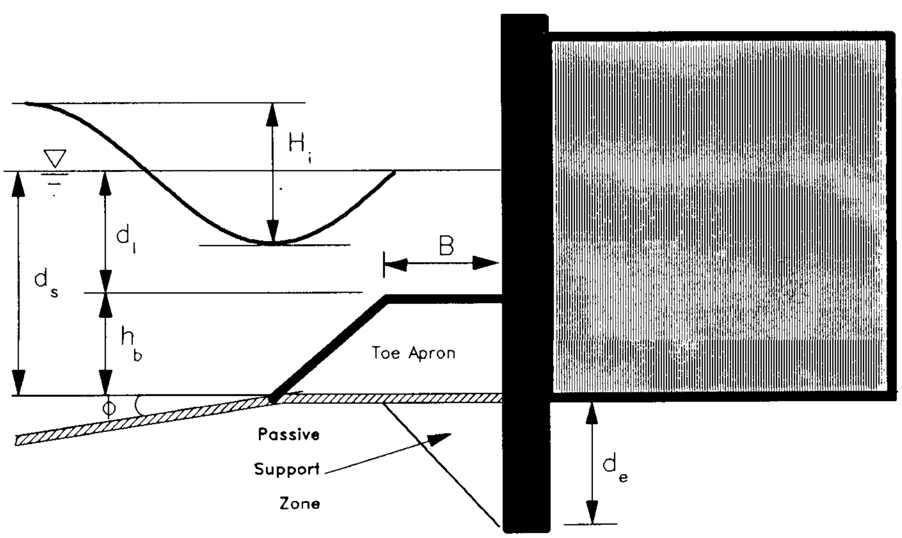

Figure 4-2-1. Typical Toe Apron for Sheet-Pile Walls

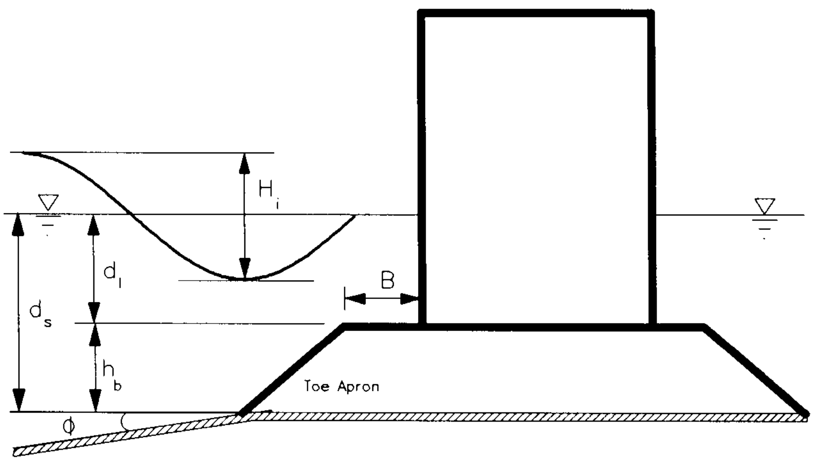

Figure 4-2-2. Typical Toe Apron for Breakwaters

The minimum width of the toe apron, B, from a geotechnical perspective is associated with structure-soil equilibrium considerations and is estimated using Rankine theory (Eckert, 1983 and Eckert and Callendar, 1987):

$$B_1 = K_p d_e \tag{1}$$

where

K_p = coefficient of passive earth pressure

d_e = sheet-pile penetration depth (0 if no pile)

The minimum width of the toe apron, B, from a hydraulics perspective is estimated from simple criteria stated as (EM 1110-2-1614):

$$B_2 = 2H_i \tag{2}$$

or

$$B_3 = 0.4 d_s \tag{3}$$

where

H_i = incident wave height

d_s = water depth at structure (in absence of toe protection)

The design width for the apron is taken as the largest value predicted by the various criteria considered in Equations *1-3*.

$$B = \max(B1, B2, B3) \tag{4}$$

Toe Stone Weight

The stability equation (SPM, 1984) used to determine the armor stone weight in the toe protection apron is cast in the form of the equation derived by Hudson (1959) for armor units for rubble-mound breakwaters.

$$W = \frac{w_r H_i^3}{N_s^3 (S_r - 1)^3} \tag{5}$$

where

W = weight of individual armor stone

w_r = unit weight of armor stone

N_s = stability number

$S_r = w_r / w_w$ = specific gravity of armor stone

w_w = unit weight of water

Stability Number

The stability number, N_s, is semi-empirically formulated on the basis of irregular wave tests conducted by Tanimoto, Yagyu, and Goda (1982). It is determined as:

$$N_s = \max\left\{ 1.3\left(\frac{1-K}{K^{\frac{1}{3}}}\right)\left(\frac{d_l}{H_i}\right) + 1.8e^{\left[-1.5\frac{(1-K)^2}{K^{\frac{1}{3}}}\frac{d_l}{H_i}\right]} \quad ; \quad 1.8 \right\} \tag{6}$$

where

K = parameter associated with the maximum horizontal velocity at the edge of the apron determined from standing waves as described by linear wave theory

$$= \frac{\frac{4\pi d_l}{L}}{\sinh\left(\frac{4\pi d_l}{L}\right)}\left(\sin\frac{2\pi B}{L}\right)^2$$

d_l = water depth (at top of toe protection)

L = wavelength at depth d_l predicted by linear wave theory

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NONBREAKING WAVE FORCES AT VERTICAL WALLS

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NONBREAKING WAVE FORCES AT VERTICAL WALLS

DESCRIPTION

This application provides the pressure distribution and resultant force and moment loading on a vertical wall caused by normally incident, *nonbreaking* regular waves. The results can be used to design vertical structures in protected or fetch-limited regions when the water depth at the structure is greater than about 1.5 times the maximum expected wave height. The application provides the same results as found using the design curves given in Chapter 7 of the SPM (1984).

INTRODUCTION

The pressure distribution on the seaward side of a vertical wall exposed to wave action is composed of two components, the hydrostatic pressure due to the depth of water at the wall and the wave-induced dynamic pressure caused by acceleration of the fluid particles. Estimates of wave-induced pressure are required to design vertical walls that will resist the applied loads without loss of functionality.

Design curves presented in the SPM (1984) provide a means of determining nonbreaking wave forces and moments as a function of water depth, water specific weight, incident wave height, and wave period. One set of curves applies to the case of complete reflection ($\chi = 1.0$) whereas the other set is for a slightly less ($\chi = 0.9$) reflective case.

The curves in the SPM represent a composite method using two solution methods for the wave-induced pressure distribution on a vertical wall. The Miche-Rundgren method provides better fit to laboratory data for steep, nonbreaking waves, but the theory begins to overpredict as the wavelength is increased. On the other hand, the Sainflou method provides better estimates for long, low-steepness waves, but it overpredicts as the waves become steeper. Therefore, the curves in the SPM use the Sainflou method for low-steepness waves and the Miche-Rundgren method for steeper waves. Transition from one method to the other is determined simply by whichever method provides the minimum force or moment for given values of wave steepness and wave height-to-depth ratio.

The major disadvantage of using the design curves in the SPM (other than inconvenience and potential for error) is determination of resultant forces and moments for values of wave height-to-depth ratio that fall between the curves. Additionally, the designer is restricted to using a reflection coefficient of either $\chi = 1.0$ or $\chi = 0.9$.

GENERAL ASSUMPTIONS AND LIMITATIONS

Hydrodynamic assumptions invoked in derivation of both the Sainflou and the Miche-Rundgren equations are typical for theoretical wave motion theories. These assumptions include:

- Nonviscous flow, i.e., ideal fluid in which there are no tangential stresses between adjacent water particles.

- Incompressible fluid of constant density.
- Irrotational flow in two dimensions.
- The absolute pressure at the surface is equal to atmospheric pressure everywhere (so surface pressure is defined as zero).

The above assumptions allow the wave motion problem at a reflecting vertical wall to be expressed in terms of potential flow theory.

Both methods are derived in the Lagrangian system, which follows individual water particles rather than remaining stationary. The primary advantage of Lagrangian coordinates is better representation of the hydrodynamic pressure above the still-water level when the crest of the standing wave is at the wall. Hydrodynamic pressure exerted on the vertical wall is hydrostatic pressure due to the depth of water, and wave pressure due to transformation of wave kinetic energy into pressure energy when wave motion acts on the wall. Most applications of the theories usually omit the hydrostatic pressure component of the total pressure, and this custom has been followed in this ACES application.

Both methods assume normally incident, monochromatic waves being reflected by a vertical wall. The waves are nonbreaking and of constant form, and it is assumed that no overtopping of the wall occurs. The bottom fronting the vertical wall is assumed horizontal. A definition sketch for a nonbreaking, normally incident, monochromatic wave being reflected from a vertical wall is shown on Figure 4-3-1.

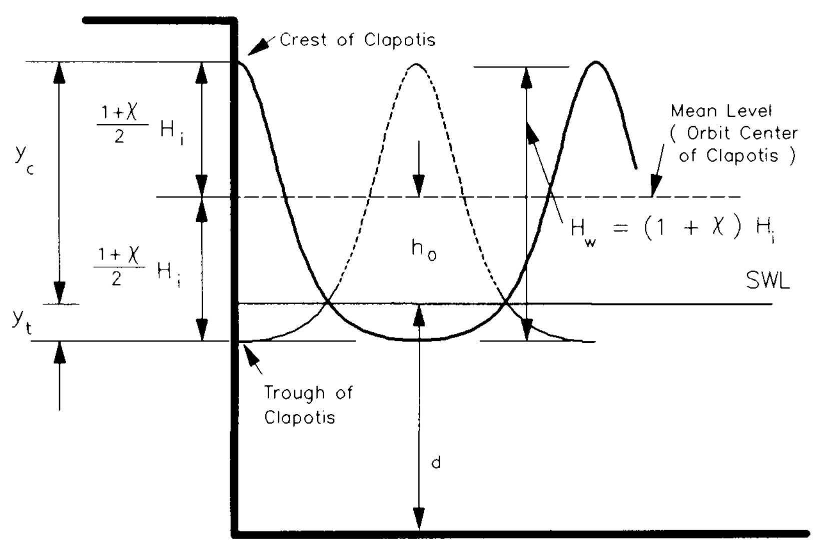

Figure 4-3-1. Definition Sketch

Primary drawbacks to using the Sainflou or Miche-Rundgren method of calculating wave-induced pressure distributions and resulting forces and moments are the facts that they are monochromatic theories and they do not give breaking wave forces. There may be waves in an *equivalent* irregular wave train that produce greater force loading than predicted using monochromatic waves. Also waves breaking directly on the wall can produce significantly higher forces than nonbreaking waves.

The only limitation in applying the Miche-Rundgren method is the restriction that the reflection coefficient can vary only over the range $0.9 < \chi < 1.0$. This range is based on recommendations given in the SPM. The Sainflou method accommodates only a reflection coefficient equal to unity $\chi = 1.0$. Consequently, predictions from this method may sometimes be conservative.

Sainflou Method

Sainflou (1928) published a theoretical solution for the pressure distribution at a vertical wall for a perfectly reflected monochromatic wave of incident height, H_i. His derivation was based on the classical hydrodynamic equations of continuity and momentum. Sainflou developed his equations as a simplification of second-order wave theory in which he omitted some of the second-order terms in the expressions. Consequently, his solution is partly of second-order in the parameter H/L. He assumed that ambient atmospheric pressure is constant everywhere on the free surface, and his expression for pressure due to the wave motion does not include any atmospheric pressure contribution.

Free Surface Elevation

In the Lagrangian coordinate system, Sainflou derived the following expressions for the vertical elevations of water particles of a standing wave at a vertical wall under the crest and trough:

Crest

$$y_{cr} = y_0 + H_i \frac{\sinh\left[\frac{2\pi}{L}(d + y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)} + \pi H_i \left(\frac{H_i}{L}\right) \frac{\sinh\left[\frac{2\pi}{L}(d + y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)} \frac{\cosh\left[\frac{2\pi}{L}(d + y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)} \tag{1}$$

Trough

$$y_{tr} = y_0 - H_i \frac{\sinh\left[\frac{2\pi}{L}(d + y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)} + \pi H_i \left(\frac{H_i}{L}\right) \frac{\sinh\left[\frac{2\pi}{L}(d + y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)} \frac{\cosh\left[\frac{2\pi}{L}(d + y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)} \tag{2}$$

where

H_i = incident wave height

L = wavelength

d = water depth

y_0 = initial vertical elevation of a water particle at rest

The free surface at rest is given by $y_0 = 0$. Therefore, the elevation of the free surface relative to still-water level (SWL) when the crest or trough of the standing wave is at the wall is found by substituting $y_0 = 0$ into Equations *1* and *2*, yielding:

Crest

$$\eta_{cr} = H_i + \pi H_i \left(\frac{H_i}{L}\right) \coth\left(\frac{2\pi d}{L}\right) \tag{3}$$

Trough

$$\eta_{tr} = -H_i + \pi H_i \left(\frac{H_i}{L}\right) \coth\left(\frac{2\pi d}{L}\right) \tag{4}$$

The first term is the first-order contribution, whereas the second term is a second-order component representing the elevation of the particle orbit center about SWL. The second-order term arises from the continuity requirement of constant water volume. Thus, the elevation of the orbit center (see Figure 4-3-1) is given as:

$$\frac{h_0}{H_i} = \pi \left(\frac{H_i}{L}\right) \coth\left(\frac{2\pi d}{L}\right) \tag{5}$$

Sainflou Pressure

The pressure (expressed as force per unit area) at any vertical elevation at the wall under a standing wave when the crest or trough is at the vertical wall was given by Sainflou as:

Crest

$$\frac{p_{cr}}{\gamma} = -y_0 - H_i \frac{\sinh\left(\frac{2\pi y_0}{L}\right)}{\sinh\left(\frac{2\pi d}{L}\right)\cosh\left(\frac{2\pi d}{L}\right)} \tag{6}$$

Trough

$$\frac{p_{tr}}{\gamma} = -y_0 + H_i \frac{\sinh\left(\frac{2\pi y_0}{L}\right)}{\sinh\left(\frac{2\pi d}{L}\right)\cosh\left(\frac{2\pi d}{L}\right)} \tag{7}$$

where

γ = specific weight of water

Miche-Rundgren Method

Miche (1944) used the classical hydrodynamic equations to derive a theoretical description for the pressure distribution at a vertical wall for perfectly reflected monochromatic waves of incident height, H_i. Miche's theory is expressed in the Lagrangian coordinate system (he also transformed the equations to Eulerian coordinates), and the equations are complete to second order in the parameter H/L. The assumption of constant atmospheric pressure on the free surface was also adopted by Miche, and the pressure equation does not include the contribution due to atmospheric pressure.

Rundgren (1958) further developed the equations of Miche (1944) by including provision for a reflection coefficient (χ) less than unity. Rundgren's equations reduce to those of Miche when the reflection coefficient of $\chi = 1.0$ is substituted.

Rundgren performed laboratory experiments and obtained pressure measurements that were compared with predictions from a number of theoretical methods expressed in both Lagrangian and Eulerian coordinates. He concluded that equations in the Lagrangian system offered the most complete description of the pressure distribution because they include the pressure contribution above SWL. Generally, he observed that the fully second-order expressions gave better comparisons than the Sainflou method, which in turn performed better than first-order theory.

Free Surface Elevation

Rundgren's equations in Lagrangian coordinates for the vertical elevations of standing wave water particles when the crest (Equation *8*) or the trough (Equation *9*) is at the vertical wall are given as:

Crest

$$\begin{aligned} y_{cr} &= y_0 + \frac{H_i}{2}(1+\chi)\frac{\sinh\left[\frac{2\pi}{L}(d+y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)} \\ &+ \frac{\pi H_i}{4}\left(\frac{H_i}{L}\right)\frac{\sinh\left[\frac{2\pi}{L}(d+y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)}\frac{\cosh\left[\frac{2\pi}{L}(d+y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)}\left[(1+\chi)^2\Theta_1+(1-\chi)^2\Theta_2\right] \end{aligned} \tag{8}$$

Trough

$$\begin{aligned} y_{tr} &= y_0 - \frac{H_i}{2}(1+\chi)\frac{\sinh\left[\frac{2\pi}{L}(d+y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)} \\ &+ \frac{\pi H_i}{4}\left(\frac{H_i}{L}\right)\frac{\sinh\left[\frac{2\pi}{L}(d+y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)}\frac{\cosh\left[\frac{2\pi}{L}(d+y_0)\right]}{\sinh\left(\frac{2\pi d}{L}\right)}\left[(1+\chi)^2\Theta_1+(1-\chi)^2\Theta_2\right] \end{aligned} \tag{9}$$

where

$$\Theta_1 = 1 + \frac{3}{4\sinh^2\left(\frac{2\pi d}{L}\right)} - \frac{1}{4\cosh^2\left(\frac{2\pi d}{L}\right)} \tag{10}$$

$$\Theta_2 = \frac{3}{4\sinh^2\left(\frac{2\pi d}{L}\right)} + \frac{1}{4\cosh^2\left(\frac{2\pi d}{L}\right)} \tag{11}$$

The free surface elevation (relative to SWL) when the crest or trough is at the wall is found by substituting $y_0 = 0$ into Equations *8* and *9*, resulting in the expressions:

Crest

$$\eta_{cr} = \frac{H_i}{2}(1+\chi) + \frac{\pi H_i}{4}\left(\frac{H_i}{L}\right)\coth\left(\frac{2\pi d}{L}\right)\left[(1+\chi)^2\Theta_1 + (1-\chi)^2\Theta_2\right] \tag{12}$$

Trough

$$\eta_{tr} = -\frac{H_i}{2}(1+\chi) + \frac{\pi H_i}{4}\left(\frac{H_i}{L}\right)\coth\left(\frac{2\pi d}{L}\right)\left[(1+\chi)^2\Theta_1 + (1-\chi)^2\Theta_2\right] \tag{13}$$

In the Miche-Rundgren formulation, the height of the orbit center above the SWL is represented by the second term in Equations *12* and *13*, i.e.,

$$\frac{h_0}{H_i} = \frac{\pi}{4}\left(\frac{H_i}{L}\right)\coth\left(\frac{2\pi d}{L}\right)\left[(1+\chi)^2\Theta_1 + (1-\chi)^2\Theta_2\right] \tag{14}$$

The similarity between the Miche-Rundgren equations and the Sainflou equations is easily recognized. In the case of complete reflection when $\chi = 1.0$, the only difference between the corresponding equations is the Θ_1-term present in the Miche-Rundgren equations.

Miche-Rundgren Pressure

The pressure at any vertical elevation under a standing wave when the crest or trough is at the vertical wall was given by Rundgren (1958) as:

Crest

$$\frac{p_{cr}}{\gamma} = -y_0 - \frac{H_i}{2}(1+\chi)\frac{\sinh\left(\frac{2\pi y_0}{L}\right)}{\sinh\left(\frac{2\pi d}{L}\right)\cosh\left(\frac{2\pi d}{L}\right)} - \frac{\pi H_i}{4}\left(\frac{H_i}{L}\right)\frac{\sinh\left(\frac{2\pi y_0}{L}\right)}{\sinh^2\left(\frac{2\pi d}{L}\right)}\left[(1+\chi)^2\Theta_3 + (1-\chi)^2\Theta_4\right] \tag{15}$$

Trough

$$\frac{p_{tr}}{\gamma} = -y_0 + \frac{H_i}{2}(1+\chi)\frac{\sinh\left(\frac{2\pi y_0}{L}\right)}{\sinh\left(\frac{2\pi d}{L}\right)\cosh\left(\frac{2\pi d}{L}\right)} - \frac{\pi H_i}{4}\left(\frac{H_i}{L}\right)\frac{\sinh\left(\frac{2\pi y_0}{L}\right)}{\sinh^2\left(\frac{2\pi d}{L}\right)}\left[(1+\chi)^2\Theta_3 + (1-\chi)^2\Theta_4\right] \tag{16}$$

where

$$\Theta_3 = \left[1 - \frac{1}{4\cosh^2\left(\frac{2\pi d}{L}\right)}\right]\cosh\left[\frac{2\pi}{L}(2d+y_0)\right] - 2\tanh\left(\frac{2\pi d}{L}\right)\sinh\left[\frac{2\pi}{L}(2d+y_0)\right] + \frac{3}{4}\left[\frac{\cosh\left(\frac{2\pi y_0}{L}\right)}{\sinh^2\left(\frac{2\pi d}{L}\right)} - \frac{2\cosh\left[\frac{2\pi}{L}(d+y_0)\right]}{\cosh\left(\frac{2\pi d}{L}\right)}\right] \tag{17}$$

$$\Theta_4 = \frac{\cosh\left[\frac{2\pi}{L}(2d+y_0)\right]}{4\cosh^2\left(\frac{2\pi d}{L}\right)} - 2\tanh\left(\frac{2\pi d}{L}\right)\sinh\left[\frac{2\pi}{L}(2d+y_0)\right] + \frac{3}{4}\left[\frac{\cosh\left(\frac{2\pi y_0}{L}\right)}{\sinh^2\left(\frac{2\pi d}{L}\right)} - \frac{2\cosh\left[\frac{2\pi}{L}(d+y_0)\right]}{\cosh\left(\frac{2\pi d}{L}\right)}\right] \tag{18}$$

IMPLEMENTATION

The determination of wave forces and moments acting on a vertical wall (when the crest or trough of a standing wave is at the wall) is accomplished in this ACES application by numerically integrating the equations given in the preceding sections.

Numerical Implementation

After performing initial bookkeeping chores, the numerical code first determines the sea surface elevation for when the crest and when the trough are at the wall (corresponds to $y_0 = 0$). This calculation is performed using Equations *12* and *13* from the Miche-Rundgren method. The resulting values for crest and trough elevation are taken as valid for both the Miche-Rundgren solution and for the Sainflou solution. The corresponding Sainflou equations (Equations *3* and *4*) would give similar results, but the increased accuracy produced by the fully second-order Miche-Rundgren equations, and the fact that reduced reflection could be accommodated, led to the decision of using only the Miche-Rundgren result. The nomogram method in the SPM also follows this convention.

Next, the water depth is divided into 90 equal increments for calculation of wave pressure. Each incremental depth represents an at-rest value of y_0 for use in the Lagrangian equations for vertical elevation. During each program loop at an incremental depth, the following values are calculated and stored:

Sainflou Method

- Vertical elevation of particle when crest is at wall (Equation *1*).
- Vertical elevation of particle when trough is at wall (Equation *2*).
- Pressure when crest is at wall (Equation *6*).
- Pressure when trough is at wall (Equation *7*).
- Incremental component to overturning moment about the bottom of the wall, given as $p_{cr}(y_{cr} + d)$ and $p_{tr}(y_{tr} + d)$ for the crest and trough, respectively.

Miche-Rundgren Method

- Vertical elevation of particle when crest is at wall (Equation *8*).
- Vertical elevation of particle when trough is at wall (Equation *9*).
- Pressure when crest is at wall (Equation *15*).
- Pressure when trough is at wall (Equation *16*).
- Incremental component to overturning moment about the bottom of the wall, given as $p_{cr}(y_{cr} + d)$ and $p_{tr}(y_{tr} + d)$ for the crest and trough, respectively.

After all loops have been completed, the total force per unit horizontal width of the wall is found for both methods by integrating the pressure over the entire water column (summing the incremental pressure values calculated at each depth). Corresponding overturning moments are found by integrating the incremental moment components.

Implementation Notes

Forces and moments per unit length of wall calculated by this program are those caused by the wave action and hydrostatic water pressure. The designer must add any other external force loads due to soil pressure from backfill, etc.

As mentioned previously, the Miche-Rundgren method overpredicts when the waves are very long (low steepness), whereas the Sainflou method provides a more realistic answer for this condition. Conversely, the Miche-Rundgren method is better for steeper wave conditions. This application reports answers from both methods, with the recommendation of using the smaller values for force and moment.

The Sainflou method *does not* include provision for reflection coefficients less than one. Therefore, it gives conservative results if reflection is less than complete. The SPM (1984) gives the following guidance on choosing the reflection coefficient:

> It should be assumed that smooth vertical walls completely reflect incident waves and $\chi = 1.0$. Where wales, tiebacks, or other structural elements increase the surface roughness of the wall by retarding vertical motion of the water, a lower value of χ may be used. A lower value of χ also may be assumed when the wall is built on a rubble base or when rubble has been placed seaward of the structure toe. Any value of χ less than 0.9 should not be used for design purposes.

This application should provide results that closely match the design curves given in the SPM. An exception may occur in determination of crest and trough elevation at very low values of H_i/gT^2 using SPM Figures 7-90 or 7-93. The SPM method first determines h_0 and then calculates η_{cr} and η_{tr} as a distance $H_i(1+\chi)/2$ above or below h_0. As H_i/gT^2 approaches zero, the curves for h_0 in the SPM were forced to a value of $h_0/H_i = 1.0$, representing the limit of a solitary wave. On the other hand, this ACES application calculates η_{cr} and η_{tr} directly, and the values of h_0 obtained from Equations *5* and *14* may be different from the SPM at very small values of wave steepness.

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Rubble-Mound Revetment Design

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Rubble-Mound Revetment Design

Description

Quarrystone is the most commonly used material for protecting earth embankments from wave attack because high-quality stone, where available, provides a stable and unusually durable revetment armor material at relatively low cost. This ACES application provides estimates for revetment armor and bedding layer stone sizes, thicknesses, and gradation characteristics. Also calculated are two values of runup on the revetment, an expected extreme and a conservative runup value.

Introduction

Structures are often needed along either bluff or beach shorelines to provide protection from wave action or to retain in situ soil or fill. Vertical structures are classified as either seawalls or bulkheads, according to their function, and protective materials laid on slopes are called revetments.

Revetments are generally constructed of durable stone or other material that will provide sufficient armoring for protected slopes. They consist of an armor layer, filter layer, and toe protection. The filter layer assures drainage and retention of the underlaying soil. Toe protection is needed to provide stability against undermining at the bottom of the structure.

General Assumptions and Limitations

Empirical formulas that were developed for the design of rubble-mound structures are generally expressed in terms of the stone weight required to withstand design wave conditions. These formulas have been largely derived from physical model studies. They are guides and must be used with experience and engineering judgment. Physical modeling is often a cost-effective measure to determine the final cross-section design for most rubble-mound structures. A definition sketch for some of the terms used in this section is shown in Figure 4-4-1.

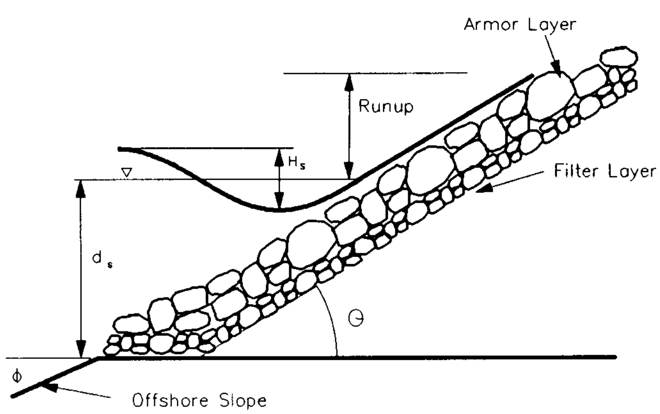

Figure 4-4-1. Definition Sketch for Riprap Revetments

Riprap Armor Stability Formula

For irregular wave conditions, a stability formula is defined that is similar to the one developed by Hudson (1958) for monochromatic waves:

$$W_{50} = w_r \left[\frac{H_s}{N_s \left(\frac{w_r}{w_w} - 1 \right)} \right]^3 \qquad (1)$$

where

W_{50} = median weight of the armor stone

w_r = unit weight of the armor stone

H_s = significant wave height

N_s = stability number

w_w = unit weight of water

The following sections present two stability numbers for use in Equation *1*. These stability numbers are based on riprap stability studies conducted at CERC and by the Dutch. This ACES application uses the larger of the two in Equation *1* to compute the required stone weight for the revetment.

CERC Stability Number

Based on findings of Broderick (1983), the *zero-damage* riprap stability number given in Ahrens (1981) as:

$$N_{s-zero} = 1.45 (\cot\theta)^{\frac{1}{6}} \qquad (2)$$

should be changed to:

$$N_{s-zero} = \frac{1.45}{1.27} (\cot\theta)^{\frac{1}{6}} \qquad (3)$$

where

$\cot\theta$ = cotangent of structure slope

The factor, 1.27, is the ratio between the average of the highest 10 percent of the waves and the significant wave height in a Rayleigh distribution. By changing the coefficient in the stability number equation, the significant wave height can continue to be used in the stability formula, and at the same time the essential findings of Broderick can be addressed. Broderick found that a wave height greater than the significant gave better correlation to damage observed in laboratory tests of riprap stability. Broderick suggested that the average of the highest 10 percent of the waves was the appropriate wave height to use in a riprap stability formula.

Dutch Stability Number

Based on findings of Van der Meer and Pilarczyk (1987) and Van der Meer (1988a, 1988b) two formulas for riprap stability numbers were derived, one for plunging waves and one for surging (nonbreaking) waves. Basic assumptions for the formulas are:

- A rubble-mound structure with an armor layer consisting of rock.
- Little or no overtopping (less than 10 to 15 percent of the waves).
- The slope of the structure should be generally uniform.

The formulas are:

Plunging Waves

$$N_s = 6.2\,P^{0.18}\left(\frac{S}{\sqrt{N}}\right)^{0.2}(\zeta_z)^{-0.5} \qquad (4)$$

Surging (Nonbreaking) Waves

$$N_s = 1.0\,P^{-0.13}\left(\frac{S}{\sqrt{N}}\right)^{0.2}\sqrt{\cot\theta}\;\zeta_z^{P} \qquad (5)$$

where

P = permeability coefficient (Figure 4-4-2)
S = damage level (Table 4-4-1)
N = number of waves
NOTE: The equations are valid in the range $1,000 < N < 7,000$, so $N = 7,000$ represents a logical limiting value that is used in this ACES application and should be conservative.
ζ_z = surf similarity parameter (see Equation *6*)

The permeability coefficient P was introduced to describe the influence of the permeability of the structure on its stability. Van der Meer investigated three structures. The lower boundary value of P is that given by an impermeable core (clay or sand) (see Figure 4-4-2(a)). The ratio of armor/filter stone diameter was 4.5. With this impermeable core, a value of $P = 0.1$ was assumed. The upper boundary value of P is that given by a homogeneous structure, consisting only of armor stones (see Figure 4-4-2(d)). For this structure, a value of $P = 0.6$ was assumed. The third structure consisted of a two-diameter-thick armor layer on a permeable.core. The ratio of armor/core stone diameter was 3.2 (see Figure 4-4-2(c)). For this structure, a value of $P = 0.5$ was assumed. The value of P for other structures with, for example, more than one layer of stones (Figure 4-4-2(b) or a thicker armor layer must be estimated from the values established for the three specific structures. The design engineer's experience is obviously important when selecting a value of P.

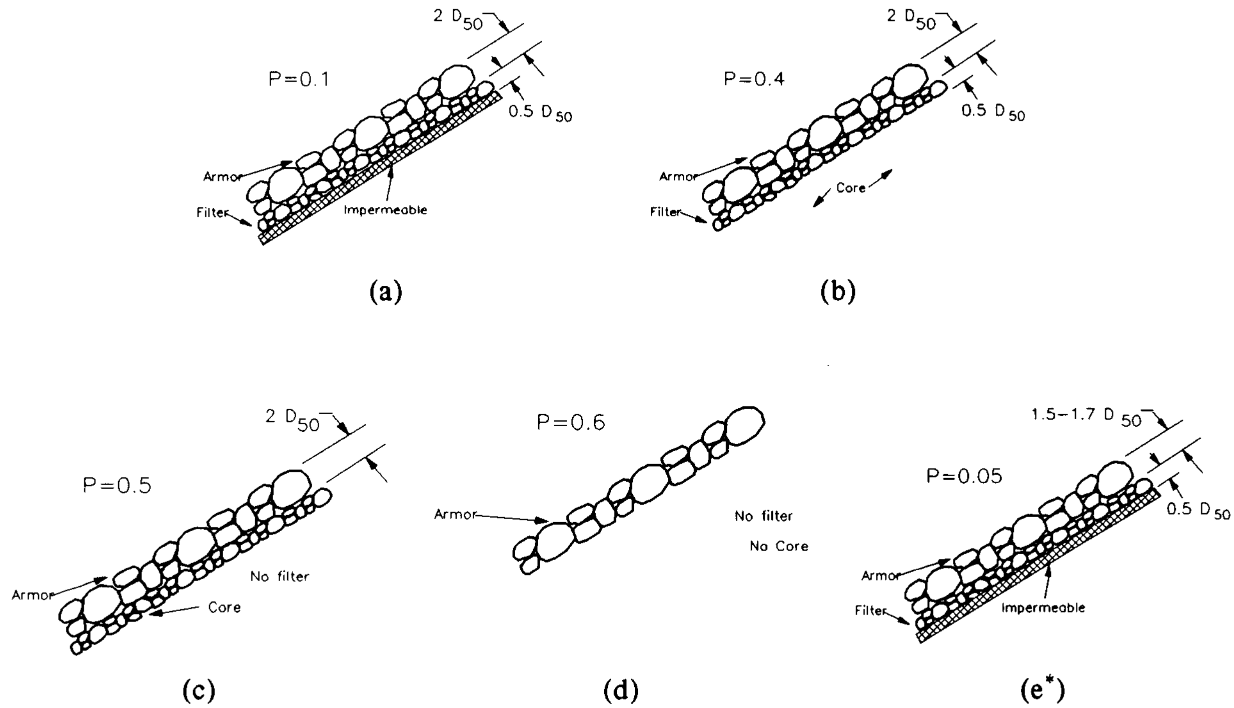

Figure 4-4-2. Permeability Coefficient (Van der Meer, 1988a;Bradbury, Allsop, and Latham, 1990*)

Table 4-4-1
Damage Levels for Two Diameter Thick Rock Slopes
(Van der Meer, 1988a)

cot θ	Damage Level S	
	Start of Damage	Failure (Filter Layer Visible)
2.0	2	8
3.0	2	12
4.0	3	17
6.0	3	17

Equations *4* and *5* were developed for deepwater wave conditions, and Van der Meer (1988a) recommends a correction for shallow-water conditions. This ACES application uses a factor of 1.2 for the correction.

The next section describes the surf similarity parameter and how it is used in this ACES application.

Surf Similarity Parameter

The surf similarity parameter has been found to be a very useful variable to characterize the breaker conditions on coastal structures or beaches. For irregular waves the surf similarity parameter can be defined as (Battjes, 1974):

$$\zeta_z = \frac{\tan\theta}{\left(\frac{2\pi H_s}{g T_z^2}\right)^{1/2}} \tag{6}$$

where

T_z = average wave period

$= T_s \left(\frac{0.67}{0.80}\right)$

NOTE: This ratio, (0.67/0.80), is based on laboratory data collected by Ahrens (1987).

The surf similarity parameter is used to determine which of the Van der Meer/Pilarczyk formulas for stability number (Equations *4* or *5*) should be compared with the CERC stability number and finally used in the stability formula (Equation *1*). Van der Meer (1988a) derives the following formula for the surf similarity parameter, ζ_{ztp}, at the transition point from plunging to surging waves:

$$\zeta_{ztp} = \left(6.2\, P^{0.31} \sqrt{\tan\theta} \right)^{\frac{1}{(P+0.5)}} \tag{7}$$

The following criteria are used to determine which of the Van der Meer/Pilarczyk stability numbers (Equations *4* or *5*) is to be compared with the CERC stability number (Equation *3*):

$\zeta_z \le \zeta_{ztp}$ -> use Equation *4*

$\zeta_z > \zeta_{ztp}$ -> use Equation *5*

REVETMENT DESIGN

The following sections describe the formulas for median weight and other percentiles of stone, armor and filter layer thicknesses, and stone dimensions used in this ACES application for design of rubble-mound revetments.

Weight of Armor Unit

The median weight of the armor unit is computed using Equation *1*.

$$W_{50} = w_r \left[\frac{H_s}{N_s \left(\frac{w_r}{w_w} - 1 \right)} \right]^3$$

The stability number, N_s , used in Equation *1* is the larger of the CERC stability number (Equation *3*) or the Dutch stability number (Equation *4* or *5*).

Armor Layer Thickness

The minimum armor layer thickness is given as:

$$r_{armor} = 2 \left(\frac{W_{50}}{w_r} \right)^{\frac{1}{3}} \qquad (8)$$

Filter Layer Thickness

The filter layer thickness is given as the maximum of:

$$r_{filter} = \frac{r_{armor}}{4} \quad \text{or} \quad 1 \text{ foot} \qquad (9)$$

The total horizontal thickness of the armor layer and first underlayer, l , must satisfy the following relation:

$$l \geq 2H_s \qquad (10)$$

where

$$l = r_t \sqrt{1 + \cot^2 \theta} \qquad (11)$$

$$r_t = r_{armor} + r_{filter} \qquad (12)$$

The purpose of Equation *10* is to ensure there is sufficient stone between the violent wave attack on the surface of the riprap and the geotextile filter cloth to dissipate a considerable portion of the wave energy. A geotextile filter cloth is relatively permeable to ground-water seepage but not to the short duration loads and high velocity impacts of breaking waves.

Stone Sizes (Gradation)

Armor Layer

Gradation is based on guidance given in EM 1110-2-2300 (1971), which specifies that the maximum and minimum weight of the riprap stone is given by:

$$W_{max} = 4W_{50} \qquad (13)$$

$$W_{min} = \frac{1}{8}W_{50} \qquad (14)$$

where W_{max} and W_{min} are the weight of the largest and smallest stone respectively in the gradation.

In addition, laboratory tests (Ahrens, 1975) provide the following approximate relations:

$$W_{85} = 1.96W_{50} \qquad (15)$$

$$W_{15} = 0.4W_{50} \qquad (16)$$

where the subscript indicates the percentage of the total weight of gradation contributed by stones of lesser weight.

Stone dimensions, D, are computed by the following relationship:

$$D_x = \left(\frac{W_x}{w_r}\right)^{\frac{1}{3}} \qquad (17)$$

where the subscript x indicates the percentage of the weight of the total gradation contributed by stones of lesser weight.

Filter Layer

The ratio of the filter layer stone size to armor stone size is given by Ahrens (1981) as:

$$\frac{D_{15(armor)}}{D_{85(filter)}} = 4.0 \qquad (18)$$

Knowing $D_{85(filter)}$ the following relationship is used to calculate the median stone dimension, $D_{50(filter)}$, of the filter layer:

$$\frac{D_x}{D_{50}} = e^{(0.01157x - 0.5785)} \tag{19}$$

where

$x = 85$

Knowing $D_{50(filter)}$ Equation *19* is used to determine stone dimensions for the 0 (minimum), 15, and 100 (maximum) percentile size of the filter layer. Equation *17* is then used to determine corresponding stone weights for the filter layer.

Irregular Wave Runup on Riprap

Recent research by Ahrens and Heimbaugh (1988) provides an improved method to estimate the maximum runup caused by irregular waves on riprap revetments. An unusual advantage of this method is that it works well for both shallow and deep water at the toe of the revetment. The approach is based on the surf parameter discussed earlier. In this instance the surf parameter is calculated using the energy-based variables H_{mo} and T_p. The energy-based surf parameter, ζ, is defined as:

$$\zeta = \frac{\tan\theta}{\left(\frac{2\pi H_{mo}}{gT_p^2}\right)^{1/2}} \tag{20}$$

where

T_p = period of peak energy density of the wave spectrum

$$= \frac{T_s}{0.80} \tag{21}$$

H_{mo} = energy-based zero-moment wave height

T_s = average period of the highest one-third of the waves

In this ACES application, the energy-based zero-moment wave height, H_{mo}, is computed by two methods, and the smaller value is then selected for use in the runup equation. The two methods for calculating H_{mo} are:

$$H_{mo} = 0.10\, L_p \tanh\left(\frac{2\pi d_s}{L_p}\right) \qquad (22)$$

or

$$H_{mo} = \frac{H_s}{\exp\left[C_o\left(\frac{d}{gT_p^2}\right)^{-C_1}\right]} \qquad (23)$$

where

C_o = 0.00089

C_1 = 0.834

The expected maximum runup is calculated using the following equation:

$$R_{max} = H_{mo}\frac{a\zeta}{1 + b\zeta} \qquad (24)$$

where
a and b = dimensionless runup coefficients

In research for improved estimates of runup, Ahrens and Heimbaugh (1988) conducted investigations of the systematic error in predicting the maximum runup. They found approximately 25 percent of their tests had a percent error greater than ± 10 percent. Because of this, they suggest that it may be useful in some critical situations to use a conservative value of runup. Therefore, two sets of runup coefficients are provided in this ACES application, one for the maximum runup and the other for a conservative runup:

Expected Maximum Runup	Conservative Runup
a = 1.022 ; b = 0.247	a = 1.286 ; b = 0.247

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