IRREGULAR WAVE RUNUP ON BEACHES

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IRREGULAR WAVE RUNUP ON BEACHES

DESCRIPTION

This application provides an approach to calculate runup statistical parameters for wave runup on smooth slope linear beaches. To account for permeable and rough slope natural beaches, the present approach needs to be modified by multiplying the results for the smooth slope linear beaches by a reduction factor. However, there is no guidance for such a reduction due to the sparsity of good field data on wave runup. The approach used in this ACES application is based on existing laboratory data on irregular wave runup (Mase and Iwagaki, 1984; Mase, 1989).

GENERAL ASSUMPTIONS AND LIMITATIONS

At present there are no theoretical approaches to calculate either monochromatic or irregular wave runup on beaches. The lack of a theoretical approach to solve the problem is due to the numerous difficulties inherent in the runup prediction problem such as:

- Nonlinear transformation of wave energy in the breaking wave zone.
- Wave reflection effects.
- Three-dimensional effects such as standing or progressive edge waves.
- Permeability.
- Porosity.
- Roughness.
- Ground-water table level.

Present approaches to calculating monochromatic wave runup on smooth steep slope coastal structures have been limited to empirical expressions of a Hunt (1959) equation form with limiting runup as determined via analytical breaking wave steepness limiting expressions (Walton and Ahrens, 1989; Walton, et al., 1989). Additionally, empirical nonlinear power law expressions exist for predicting irregular wave runup on smooth linear slopes (Mase, 1989) for the following runup statistics in a stationary wave train:

R_{max} = maximum wave runup

R_2 = runup value exceeded by 2 percent of the runups

$R_{1/10}$ = average of the highest one-tenth of the wave runups

$R_{1/3}$ = significant or average of the highest third of the runups

$\overline{R}$ = average wave runup

This ACES application calculates these runup statistics and quantiles based on the coefficients provided by Mase (1989).

Wave Runup Equation

The methodology is based on a fitting of the relative wave runup, R , via an equation of the form:

$$R_p = H_{SO} \quad a_p \quad I^{(b_p)} \tag{1}$$

where

$-_p$ = quantile or statistic value desired $\left(\max, 2\%, \frac{1}{3}, \frac{1}{10}, average\right)$

a_p, b_p = constants based on the statistic or quantile value of desired runup

I = Iribarren number

$$= \frac{\tan\theta}{\left(\frac{H_{SO}}{L_o}\right)^{\frac{1}{2}}} \tag{2}$$

$\tan\theta$ = tangent of the beach slope

H_{SO} = deepwater significant wave height

L_o = deepwater wavelength

Until further research is conducted, it is suggested that the beach foreshore slope be used as the required beach slope in Equation *(1)*.

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WAVE RUNUP AND OVERTOPPING ON IMPERMEABLE STRUCTURES

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Wave Runup and Overtopping on Impermeable Structures

Description

This application provides estimates of wave runup and overtopping on rough and smooth slope structures that are assumed to be impermeable. Run-up heights and overtopping rates are estimated independently or jointly for monochromatic or irregular waves specified at the toe of the structure. The empirical equations suggested by Ahrens and McCartney (1975), Ahrens and Titus (1985), and Ahrens and Burke (unpublished report) are used to predict runup, and Weggel (1976) to predict overtopping. Irregular waves are represented by a significant wave height and are assumed to conform to a Rayleigh distribution (Ahrens, 1977). The overtopping rate is estimated by summing the overtopping contributions from individual runups in the distribution. Portions of the material presented herein can also be found in Chapter 7 of the SPM (1984).

Introduction

As waves encounter certain types of coastal structures, the water rushes up and sometimes over the structure. These closely related phenomena, wave runup and wave overtopping, often strongly influence the design and the cost of coastal projects. Wave runup is defined as the vertical height above still-water level to which a wave will rise on the structure (of assumed infinite height). Overtopping is the flow rate of water over the top of the finite height structure as a result of wave runup. Waves are assumed to be normally incident to the structure.

General Assumptions and Limitations

The various relationships for runup and overtopping employed in this application are empirically derived from physical model studies originally conducted for specific structures and wave climates. General assumptions applicable to the various expressions can be summarized as:

- Waves are normally incident to the structure and are unbroken in the vicinity of the structure toe.
- Waves are considered to be monochromatic. Irregular wave conditions are characterized by significant wave height H_s.
- Waves are specified at the structure location. Linear wave theory is applied to determine unrefracted deepwater wave height where necessary.
- The crest of the structure must be above still-water level.
- For run-up estimates, structures are considered to be impermeable and to have infinite height and simple plane slopes.
- For overtopping estimates, the actual finite structure height is employed. Results for additional structure configurations (such as curved and recurved walls) can be obtained if runup is known.

As reported in the references, the expressions for runup were primarily determined by empirical curve fitting procedures and consequently do not formally represent "best fit" curves derived by statistical procedures. The exception is the expression for smooth slope runup for nonbreaking wave conditions that was developed using regression analysis.

Similarly, the expression for overtopping rate was originally derived by a graphical curve fitting procedure (Weggel, 1976). In practice, the empirical coefficients required for the overtopping rate equation are often difficult to obtain. While a representative value of α is easy to estimate as a function of the structure slope, no satisfactory functional approximations for Q_0^* are available. An estimate is usually made by interpolation or extrapolation of the values presented in the SPM (1984), which are tabulations derived from the original data set and curve fitting procedure.

While expressions empirically derived from model data represent a useful and valid technology, engineering judgment should always be applied to the results, particularly when applying the formulas in situations much different from the bounds and character of the original data from which they were derived.

Wave Runup

Numerous laboratory tests have been conducted over the years resulting in data for wave runup. Figure 5-2-1 shows parameters involved in discussing wave runup, and the next two sections present equations used in ACES for rough and smooth slopes.

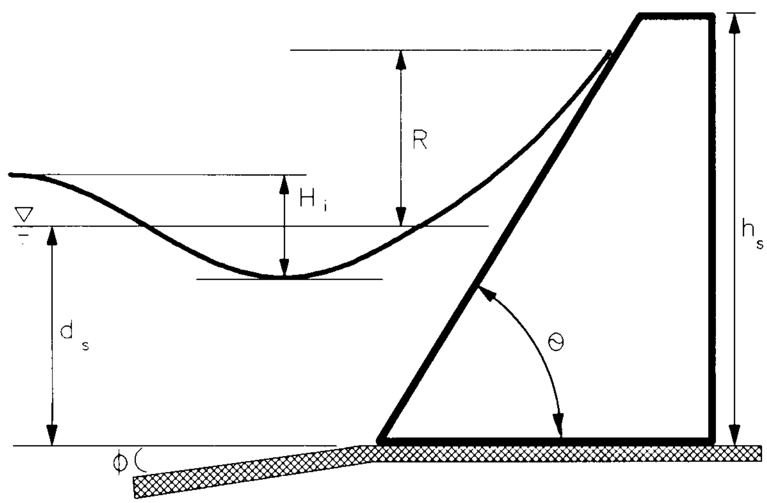

Figure 5-2-1. Wave Runup and Overtopping

Rough Slope Runup

Ahrens and McCartney (1975) present an empirical method for estimating the runup on structures protected by various types of primary armor faces. In their method, the runup is predicted as a nonlinear function of the surf parameter, ξ .

$$R = H_i \frac{a\xi}{1 + b\xi} \qquad (1)$$

where

R = runup

H_i = incident wave height

a, b = empirical coefficients associated with corresponding types of armor unit (see Table A-3 of Appendix A)

$$\xi = \frac{\tan\theta}{\sqrt{\frac{H_i}{L_0}}} \tag{2}$$

θ = angle between structure seaward face and horizontal (a measure of structure slope)

L_0 = deepwater wavelength

Smooth Slope Runup

Ahrens and Titus (1985) recommend the following general equation for runup on smooth slopes:

$$R = C H_i \tag{3}$$

The coefficient C is characterized by the surf parameter ξ according to the following three wave-structure regimes:

- $(\xi \le 2)$ waves plunging directly on the run-up slope.
- $(\xi \ge 3.5)$ wave conditions that are nonbreaking and are regarded as standing or surging waves.
- $(2 < \xi < 3.5)$ transition conditions where breaking characteristics are difficult to define.

Recommended expressions for coefficient C corresponding to these regimes are then:

- *Plunging wave conditions* $(\xi \le 2)$

$$C_p = 1.002\xi \tag{4}$$

° *Nonbreaking wave conditions* ($\xi \geq 3.5$)

$$C_{nb} = 1.181\left(\frac{\pi}{2\theta}\right)^{0.375} \exp\left[3.187\left(\frac{\eta_c}{H_i} - 0.5\right)^2\right] \qquad (5)$$

where

η_c = crest height of the wave above the still-water level calculated using the Stream Function Wave Theory (Dean, 1974)

° *Transitional wave conditions* ($2 < \xi < 3.5$)

$$C_t = \left(\frac{3.5 - \xi}{1.5}\right) C_p + \left(\frac{\xi - 2}{1.5}\right) C_{nb} \qquad (6)$$

This ACES application uses a more convenient but less accurate expression for the coefficient under nonbreaking conditions derived by Ahrens and Burke (unpublished report):

$$C_{nb} = 1.087\sqrt{\frac{\pi}{2\theta}} + 0.775\Pi \qquad (7)$$

where

Π = Goda's (1983) nonlinearity parameter

$$= \frac{\frac{H_i}{L}}{\tanh^3\left(\frac{2\pi d_s}{L}\right)} \qquad (8)$$

L = incident wavelength

Overtopping Rate

Several consequences of overtopping are important to engineers designing coastal structures. For structures along the shoreline (seawalls, bulkheads, or revetments), the volume of water that flows over the structure significantly impacts backside flooding. For breakwaters, wave transmission to the leeward side is an important criterion in harbor design. Also for breakwaters, the stability of armor material on the backslope of the structure is an important consideration. This ACES methodology estimates the overtopping flow rate for simple structures.

Monochromatic Wave Overtopping

The method implemented within this ACES application was developed by Weggel (1976) using data reported by Saville (1955) and by Saville and Caldwell (1953). It consists of an empirical expression for the monochromatic-wave overtopping rate:

$$Q = C_w \sqrt{g Q_0^* H_0^3} \left(\frac{R + F}{R - F} \right)^{\frac{-0.1085}{\alpha}} \qquad (9)$$

where

Q = overtopping rate/unit length of structure

C_w = wind correction factor

g = gravitational acceleration

Q_0^*, α = empirical coefficients (see SPM (1984) figures)

Note: An average value for $\bar{\alpha}$ as a function of structure slope may be approximated by:

$$\bar{\alpha} = 0.06 - 0.0143 \ln(\sin\theta)$$

This option is available in the application.

H_0 = unrefracted deepwater wave height

R = runup

$F = h_s - d_s$ = freeboard

h_s = height of structure

d_s = water depth at structure

Wind Effects

Onshore winds can increase the overtopping rate at a barrier. The effect is dependent upon wind velocity, direction with respect to the axis of the structure, and structure characteristics. This increased overtopping rate is approximated by adjusting the above value for Q with a wind correction factor C_w (SPM, 1984):

$$C_w = 1 + W_f \left(\frac{F}{R} + 0.1 \right) \sin\theta \qquad (10)$$

where

$$W_f = \frac{U^2}{1800} \qquad (11)$$

U = onshore wind speed (mph)

Irregular Wave Overtopping

Douglass (1986) presents a summary of methods available for estimating overtopping rates from irregular waves. The method summarized below is that of Ahrens (1977) and embodies the following assumptions:

- Run-up values caused by an irregular sea will follow a Rayleigh distribution.
- Significant deepwater wave, $H_{1/3}$, causes the significant runup, $R_{1/3}$.
- α, Q_0^*, and H_0 in Weggel's overtopping equation remain constant for all members of the distribution.

Ahrens estimates the overtopping rate by summing the overtopping contributions from the individual members of the run-up distribution:

$$Q = \frac{1}{199} \sum_{i=1}^{199} Q_i \tag{12}$$

where

Q = volume rate of overtopping caused by irregular waves

Q_i = volume rate of overtopping caused by one runup on the run-up distribution

$$= C_w \sqrt{g Q_0^* (H_{so})^3} \left(\frac{R_i + F}{R_i - F} \right)^{\frac{-0.1085}{\alpha}} \tag{13}$$

H_{so} = deepwater significant wave height

R_i = run-up value having exceedance probability p

$$= \sqrt{\frac{\ln \frac{1}{p}}{2}} R_s \tag{14}$$

$p = 0.005 * i, i = 1, 2, 3, \ldots, 199$

R_s = runup with a given deepwater significant wave height and period

These equations *modify* Weggel's monochromatic expressions to account for the effect of irregular waves when the freeboard, F, is less than the runup of the significant wave, R_s. When the freeboard is greater than the runup, Weggel's equations yield no overtopping while larger runups in the distribution may still overtop the structure. For these relatively high freeboards, the run-up distribution is broken into 999 elements, instead of 199, to better account for the effect of the higher runups. The overtopping equation for this larger distribution becomes:

$$Q = \frac{1}{999} \sum_{i=1}^{999} Q_i \tag{15}$$

where

$p = 0.001 * i, i = 1, 2, 3, \ldots, 999$

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WAVE TRANSMISSION ON IMPERMEABLE STRUCTURES

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Wave Transmission on Impermeable Structures

Description

This application provides estimates of wave runup and transmission on rough and smooth slope structures. It also addresses wave transmission over impermeable vertical walls and composite structures. In all cases, monochromatic waves are specified at the toe of a structure that is assumed to be impermeable. For sloped structures, a method suggested by Ahrens and Titus (1985) and Ahrens and Burke (1987) is used to predict runup, while the method of Cross and Sollitt (1971) as modified by Seelig (1980) is used to predict overtopping. For vertical wall and composite structures, a method proposed by Goda, Takeda, and Moriya (1967) and Goda (1969) is used to predict wave transmission.

Introduction

The transmission of wave energy beyond protective structures involves a number of complex processes. Some incident wave energy may be reflected by the structure, some wave energy may be dissipated by turbulent interaction with primary armor units (if present), some may be dissipated internally by the finer materials beneath the armor layers of an impermeable structure, and some may be transmitted through or over the structure with resultant wave regeneration. Important factors identifiable in the process include the shape and material composition of the structure, the incident wave environment, as well as the degree of immergence or submergence of the structure.

General Assumptions and Limitations

The various relationships for runup and transmission employed in this application are empirically derived from physical model studies originally conducted for specific structures and wave climates. For sloped structures, the run-up methodology is described in the section entitled **Wave Runup and Overtopping on Impermeable Structures** of this *ACES Technical Reference*. For convenience, the pertinent assumptions and limitations are restated below. General assumptions applicable to the various methods can be summarized as:

- Waves are monochromatic, normally incident to the structure, and unbroken in the vicinity of the structure toe.
- Waves are specified at the structure location.
- All structure types are considered to be impermeable.
- For sloped structures the crest of the structure must be above still-water level.
- For vertical and composite structures, partial and complete submersion of the structure is considered.
- Run-up estimates on sloped structures require the assumption of infinite structure height and a simple plane slope.
- The expressions for transmission by overtopping use the actual finite structure height.

As reported in the references, the expressions for runup were primarily determined by empirical curve fitting procedures and consequently do not formally represent "best fit" curves derived by statistical procedures. The exception is the expression for smooth slope runup for nonbreaking wave conditions that was developed using regression analysis.

The methodology for wave transmission was also empirically derived. The transmission predicted by the expression for sloped structures with freeboard was tested over the range of $(0 \le B/h_s \le 0.86)$ for smooth impermeable structures, and $(0.88 \le B/h_s \le 3.2)$ for rough impermeable breakwaters (Seelig, 1980). Seelig also recommended that the expression be applied in the range $(0.006 \le d_s/gT^2 \le 0.03)$. For transmission over vertical or composite structures, the empirical coefficients α and β were determined from laboratory experiments for three breakwater types and wave conditions in the range $(0.14 \le d_s/L \le 0.5)$ (Goda, 1969, and Seelig, 1976).

While expressions empirically derived from model data represent a useful and valid technology, engineering judgment should always be applied to the results, particularly when applying the formulas in situations much different from the bounds and character of the original data from which they were derived. Familiarity with the history, techniques, and data bounds of original experimental results should complement the use of sound engineering judgment when applying such procedures.

Wave Transmission by Overtopping

In general, wave transmission at structures is characterized by the following expression:

$$H_T = K_{TO} H_i \tag{1}$$

where

H_T = transmitted wave height

K_{TO} = wave transmission coefficient (overtopping)

H_i = incident wave height

The next two sections discuss the wave transmission coefficient for the simple, idealized impermeable structures.

Transmission Coefficient for Sloped Structures with Freeboard

Wave transmission over a sloped breakwater occurs when runup exceeds the freeboard. Some of the pertinent parameters for the following discussion are shown in Figure 5-3-1 below.

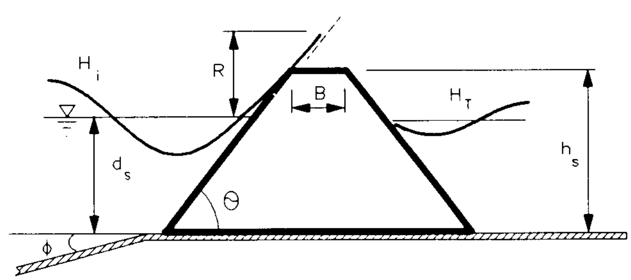


Figure 5-3-1. Transmission over Sloped Structures

The transmission coefficient for sloped structures subject to wave overtopping is estimated by an empirical equation based on the work of Cross and Sollitt (1971) and on 2-D laboratory tests conducted by Seelig (1980).

$$K_{TO} = C\left(1 - \frac{F}{R}\right) \tag{2}$$

where

C = empirical coefficient

$$= 0.51 - 0.11\frac{B}{h_s} \tag{3}$$

B = crest width of structure

h_s = structure height

$F = h_s - d_s$ = freeboard

d_s = water depth at structure

R = runup

As stated previously, runup is calculated as outlined in the section entitled **Wave Runup and Overtopping on Impermeable Structures** of this *ACES Technical Reference.*

Transmission Coefficient for Vertical or Composite Structures

The transmission coefficient for impermeable vertical-faced structures is estimated by an empirical equation based on the work of Goda, Takeda, and Moriya (1967) and Goda (1969). The equation is presented in Seelig (1976) as:

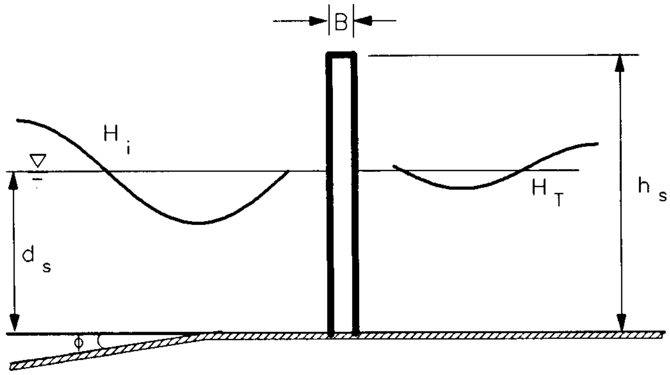
$$K_{TO} = 0.5\left\{1 - \sin\left[\frac{\pi}{2\alpha}\left(\frac{F}{H_i} + \beta\right)\right]\right\} \tag{4}$$

The empirical coefficients, α and β, were determined from laboratory experiments for three breakwater types for water depth to wavelength ratios of

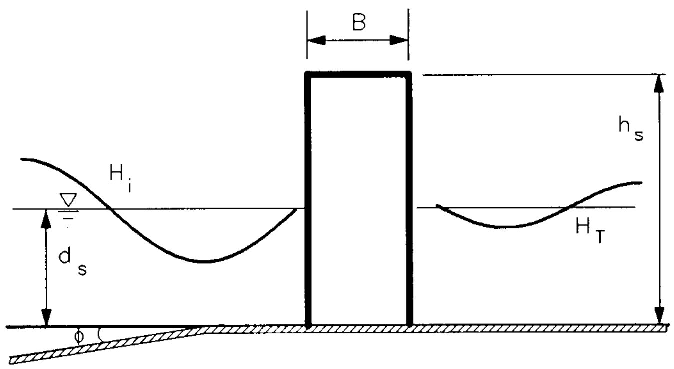
$$0.14 \le \frac{d_s}{L} \le 0.5 \tag{5}$$

The breakwater types and definition of terms and symbols are shown in Figure 5-3-2, which is taken from Seelig (1976).

Vertical Thin-Wall Breakwater
$B \approx 0$
$\alpha = 1.8 \qquad \beta = 0.1$



Vertical Wall Breakwater
$B \approx d_s$
$\alpha = 2.2 \qquad \beta = 0.4$



Composite Breakwater
$B \approx d_s$

$\frac{d_l}{d_s} = 0.3 \qquad \alpha = 2.2 \qquad \beta = 0.10$

$\frac{d_l}{d_s} = 0.5 \qquad \alpha = 2.2 \qquad \beta = 0.25$

$\frac{d_l}{d_s} = 0.7 \qquad \alpha = 2.2 \qquad \beta = 0.35$

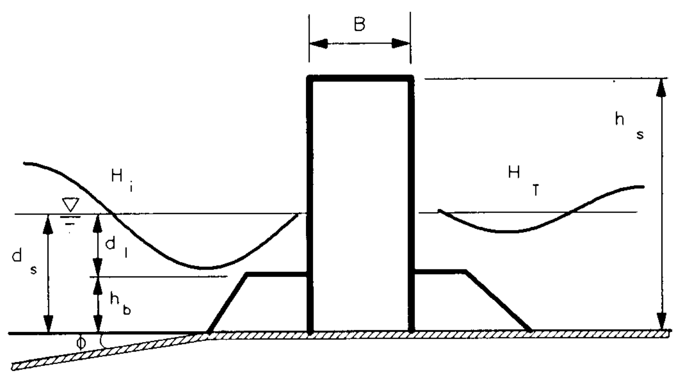


Figure 5-3-2. Transmission over Vertical/Composite Breakwaters (Seelig, 1976)

The range of applicability of K_{TO} is given in Table 5-3-1.

Table 5-3-1 K_{TO} for Vertical and Composite Breakwaters	
K_{TO}	Domain
$K_{TO} = 1.0$	$\frac{F}{H_i} \le -(\alpha + \beta)$
$K_{TO} = 0.5\left\{1 - \sin\left[\frac{\pi}{2\alpha}\left(\frac{F}{H_i} + \beta\right)\right]\right\}$	$-(\alpha + \beta) < \frac{F}{H_i} < (\alpha - \beta)$
$K_{TO} = 0.0$	$\frac{F}{H_i} \ge (\alpha - \beta)$

β , is defined as

$$\beta = C_1 \beta_1 + C_2 \beta_2 \tag{6}$$

where

$$C_1 = \max\left(0, 1 - \frac{B}{d_s}\right) \quad \text{and} \quad C_2 = \min\left(1, \frac{B}{d_s}\right) \tag{7}$$

Table 5-3-2 presents equations for α and β_1 , and β_2 used to calculate the transmission coefficient for the various breakwater types shown in Figure 5-3-2. These equations were established from analysis of the data in Figure 5-3-2.

Table 5-3-2 α and β_1, β_2 for Vertical and Composite Breakwaters				
Coefficient	Domain - Vertical Breakwaters		Domain - Composite Breakwaters	
	$0 \le \frac{B}{d_s} < 1.0$	$\frac{B}{d_s} \ge 1.0$	$\frac{d_l}{d_s} \le 0.3$	$0.3 < \frac{d_l}{d_s} \le 1.0$
α	$1.8 + 0.4\left(\frac{B}{d_s}\right)$	2.2	2.2	2.2
β_1	$0.1 + 0.3\left(\frac{B}{d_s}\right)$	0.4	N/A	N/A
β_2	N/A	N/A	0.1	$0.527 - \frac{0.130}{\left(\frac{d_l}{d_s}\right)}$
d_l = water depth above berm or toe (see Figure 5-3-2)				

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Wave Transmission Through Permeable Structures

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WAVE TRANSMISSION THROUGH PERMEABLE STRUCTURES

DESCRIPTION

Porous rubble-mound structures consisting of quarry stones of various sizes often offer an attractive solution to the problem of protecting a harbor against wave action. It is important to assess the effectiveness of a given breakwater design by predicting the amount of wave energy transmitted by the structure. This application determines wave transmission coefficients and transmitted wave heights for permeable breakwaters with crest elevations at or above the still-water level. This application can be used with breakwaters armored with stone or artificial armor units. The application uses a method developed for predicting wave transmission by overtopping coefficients using the ratio of breakwater freeboard to wave runup (suggested by Cross and Sollitt, 1971). The wave transmission by overtopping prediction method is then combined with the model of wave reflection and wave transmission through permeable structures of Madsen and White (1976). Seelig (1979,1980) had developed a similar version for mainframe processors.

The material presented here is intended as a brief summary of the methodology. More detailed discussion is presented in Madsen and White (1976) and Seelig (1980).

INTRODUCTION

The transmission of wave energy beyond protective structures involves a number of complex processes. Some incident wave energy may be reflected by the structure, some wave energy may be dissipated by turbulent interaction with primary armor units (if present), some may be dissipated internally by the finer materials beneath the armor layers of a permeable structure, and some may be transmitted through or over the structure with resultant wave regeneration. Important factors identifiable in the process include the shape and material composition of the structure, the incident wave environments, and the degree of immergence or submergence of the structure.

The methodology summarized in the following sections provides an attempt to account for the various processes of wave transmission at an unsubmerged rubble-mound structure subjected to relatively long-period waves. Figure 5-4-1 presents general symbology for the following discussions.

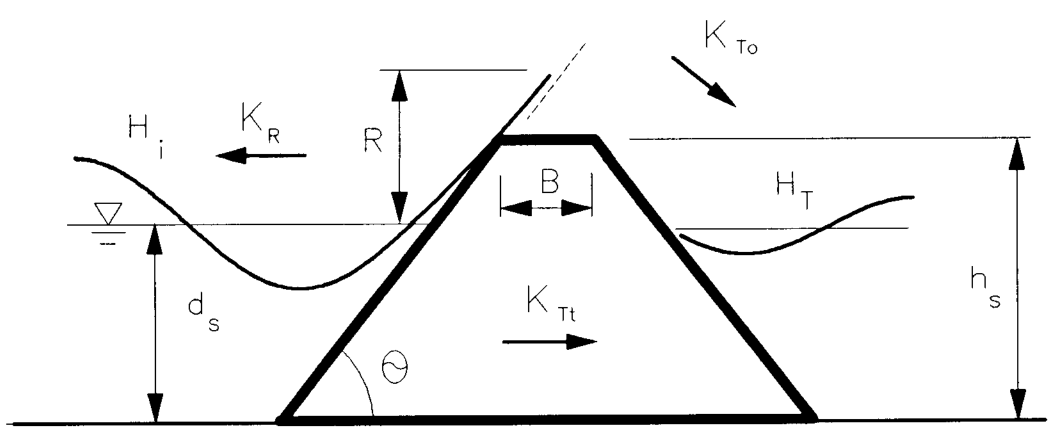

Figure 5-4-1. Wave Transmission over and Through Permeable Breakwaters

General Assumptions and Limitations

General assumptions and limitations for the model are:

- Incident waves are periodic, relatively long, and normally incident.
- Fluid motion is adequately described by the linearized governing equations.
- The model can be used only for breakwaters with crests above the still-water line.
- The model can be used for unbroken waves.
- Tests of breakwaters armored with dolos units suggest that the model can be used for artificial armor units.
- Laboratory data showed that the model gives best predictions for shallow-water waves.
- Predictions of transmission coefficients tend to be conservative for transitional or deepwater waves.

Total Wave Transmission

A common measure of breakwater performance is that of wave transmission, expressed as a transmission coefficient generally defined as the ratio of the transmitted wave height to the incident wave height.

$$K_T = \frac{H_T}{H_i} \tag{1}$$

where

H_T = transmitted wave height

H_i = incident wave height

There are two basic types of wave transmission considered in this ACES application:

- Wave regeneration caused by overtopping of the structure crest.
- Wave energy transmitted through the permeable materials of the structure.

A total wave transmission coefficient, K_t, is given by

$$K_T = \sqrt{(K_{To})^2 + (K_{Tt})^2} \tag{2}$$

where

K_{To} = overtopping coefficient

K_{Tt} = transmission coefficient

The following sections discuss the formulations for both the overtopping and transmission coefficients.

Overtopping Coefficient

The overtopping transmission coefficient is estimated by an empirical equation based on 2-D laboratory tests (Seelig, 1980).

$$K_{To} = C\left(1 - \frac{F}{R}\right) \tag{3}$$

where

C = empirical coefficient = $0.51 - 0.11(B/h_s)$

$F = h_s - d_s$ = freeboard

h_s = structure height

d_s = water depth

R = wave runup

Wave runup is estimated using the following formula (Ahrens and McCartney, 1975).

$$R = H_I\left(\frac{a\zeta}{1 + b\zeta}\right) \tag{4}$$

where

a = 0.692

ζ = surf parameter

$$= \frac{\tan\theta}{\sqrt{\frac{H_I}{L_o}}} \tag{5}$$

θ = angle of seaward face of breakwater

L_0 = deepwater wavelength (linear wave theory)

b = 0.504

Transmission Coefficient

The coefficient of wave transmission through permeable breakwaters, K_{Tt}, is estimated using the analytical model of Madsen and White (1976). In this model the transmission coefficient is related to a complex function of the following parameters:

- Size, porosity, and placement of materials in the breakwater
- Breakwater geometry
- Seaward slope of the structure
- Water depth
- Wave height and period
- Kinematic viscosity of the water

The Madsen and White model combines an analytical treatment with empirical relationships for the hydraulic characteristics of the porous material and for the friction factor representing energy dissipation on the seaward face of the breakwater. Important assumptions of the model include the following:

- Incident waves are periodic, relatively long, unbroken, and normally incident.
- Fluid motion is adequately described by the linear long-wave equations.

Madsen and White base their analytical solution on the fundamental argument that the problem of reflection from and transmission through a structure may be regarded as one of determining the partition of incident wave energy among reflected, transmitted, and dissipated energy. The analytical model is divided into four analyses.

- Internal energy dissipation - Idealized by considering the problem of the interaction of waves with a homogeneous porous structure of rectangular cross section that is "hydraulically" equivalent to the trapezoidal, multilayered breakwater.
- External energy dissipation - Based upon the associated problem of energy dissipation on a rough impermeable slope.
- Synthesis of the two analyses - Combines the two analyses into a rational procedure for the estimation of reflection and transmission coefficients of trapezoidal, multilayered breakwaters.
- Equivalent breakwater analysis - A simple method to determine characteristics of an idealized homogeneous rectangular breakwater that is hydraulically equivalent to a trapezoidal, multilayered breakwater.

Each of these analyses is briefly discussed in the next four sections.

Internal Energy Dissipation

This section presents a discussion of the treatment of internal energy dissipation within the porous media of the structure. The actual geometry of this portion of the breakwater is replaced by an idealized rectangular crib-style structure of homogeneous material of known properties. The theory also embodies the following additional assumptions:

- The idealized structure is subject to relatively long, normally incident unbroken waves described by linear wave theory.
- The flow resistance within the porous structure is a linear function of velocity.

The theoretical consideration for internal energy dissipation is based upon an analytic solution to simplified long wave equations. The problem domain is depicted in Figure 5-4-2.

Internal energy dissipation is represented by a friction term in the momentum equation only for the subdomain that involves the porous rectangular structure. Within the subdomain representing the structure, a flow resistance of the Dupuit-Forchheimer type (Bear, et al. 1968) is assumed, and an empirical relationship relating flow resistance to stone size, porosity, and fluid viscosity is used to provide a representation of experimentally observed hydraulic properties of porous media. Adopting this empirical formulation of the flow resistance for a porous medium in conjunction with Lorentz' principle of equivalent work leads to a determination of a linearized flow resistance factor in terms of the characteristics of the porous material and the incident wave characteristics.

The resulting analytic solutions to the long-wave equations are manipulated to provide reflection and transmission coefficients for the idealized crib-style breakwater. These coefficients will be used in the synthesis of the separate analyses for energy dissipation and transmission.

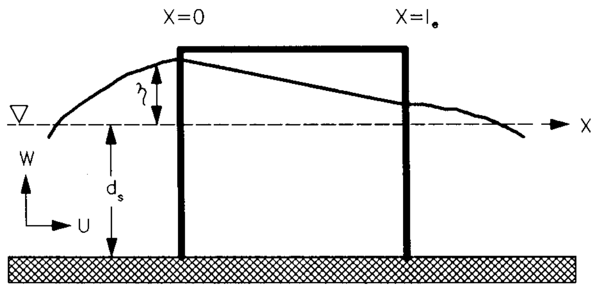

Figure 5-4-2. Wave Transmission Through a Rectangular Porous Breakwater

The equations governing the motion outside and within the structure are

Outside of structure		Within porous structure	
$\frac{\partial \eta}{\partial t} + d_s\left(\frac{\partial U}{\partial x}\right) = 0$	continuity	$n\frac{\partial \eta}{\partial t} + d_s\left(\frac{\partial U}{\partial x}\right) = 0$	(6)
$\frac{\partial U}{\partial t} + g\left(\frac{\partial \eta}{\partial x}\right) = 0$	conservation of momentum	$\frac{S}{\eta}\left(\frac{\partial U}{\partial t}\right) + g\left(\frac{\partial \eta}{\partial x}\right) + f\frac{\omega}{n}U = 0$	(7)

where

η = free surface elevation

d_s = water depth

U = horizontal water particle velocity

g = acceleration due to gravity

S = factor for the effect of unsteady motion (taken as 1)

f = nondimensional friction factor

$\omega = 2\pi/T$ = angular frequency

n = porosity of the porous medium

Since the equations are linear, complex variables may be used. Requiring a periodic solution in terms of radian frequency ω, the following are used:

$$\eta = \zeta(x)e^{i\omega t} \tag{8}$$

$$U = u(x)e^{i\omega t} \tag{9}$$

In these equations $i = \sqrt{-1}$, and the amplitude functions ζ and u are complex functions of x only. Only the real part of the complex solutions for ζ and u constitutes the physical solutions.

General solutions for the governing Partial Differential Equations by region are:

$$\zeta = a_i e^{-ik_o x} + a_r e^{ik_o x} \tag{10}$$

$$(x \leq 0)$$

$$u = \sqrt{\frac{g}{d_s}}\left(a_i e^{-ik_o x} - a_r e^{ik_o x}\right) \tag{11}$$

$$\zeta = a_t e^{-ik_o(x - l_e)} \tag{12}$$

$$(x \geq l_e)$$

$$u = \sqrt{\frac{g}{d_s}}\left[a_t e^{-ik_o(x - l_e)}\right] \tag{13}$$

$$\zeta = a_+ e^{-ikx} + a_- e^{ik(x - l_e)} \tag{14}$$

$$(0 \leq x \leq l_e)$$

$$u = \sqrt{\frac{g}{d_s}}\frac{n}{\sqrt{s - if}}\left[a_+ e^{-ikx} - a_- e^{ik(x - l_e)}\right] \tag{15}$$

where

a_i = complex incident wave amplitude

k_o = wave number

$$= \frac{\omega}{\sqrt{g d_s}} \tag{16}$$

a_r = complex reflected wave amplitude

a_t = complex transmitted wave amplitude

a_+ = complex amplitude of wave propagating in the positive x-direction within the structure

k = complex wave number $= n k_o \sqrt{S - if}$ (17)

a_- = complex amplitude of wave propagating in the negative x-direction within the structure

The general solutions for the motions in the three regions, given Equations *10* through *15*, show the problem to involve four unknown quantities. These unknowns are the complex wave amplitudes a_r, a_t, a_+, and a_-. They may be determined by matching solutions at the common boundaries of the regions and further manipulated to eliminate a_+ and a_- and provide expressions for the complex amplitudes of the transmitted and reflected waves:

$$\frac{a_t}{a_i} = \frac{4\epsilon}{(1+\epsilon)^2 e^{ikl_e} - (1-\epsilon)^2 e^{-ikl_e}} \tag{18}$$

$$\frac{a_r}{a_i} = \frac{(1-\epsilon^2)(e^{ikl_e} - e^{-ikl_e})}{(1+\epsilon)^2 e^{ikl_e} - (1-\epsilon)^2 e^{-ikl_e}} \tag{19}$$

where

$$\epsilon = \frac{\frac{n}{\sqrt{s}}}{\sqrt{\left(1 - \frac{if}{S}\right)}} \tag{20}$$

$$f = \frac{n_r}{k_o l_e}\left\{\left[1 + \left(1 + \frac{170}{R_d}\right)\frac{16\beta_r}{3\pi} a_i \frac{l_e}{d_s}\right]^{\frac{1}{2}} - 1\right\} \tag{21}$$

n_r = porosity of reference material = 0.435

l_e = width of idealized breakwater

R_d = particle Reynolds number

$$= \frac{|u_s| d_r}{\nu} \tag{22}$$

$|u_s|$ = horizontal velocity within structure

$$= a_i \sqrt{\frac{g}{d_s}}\left(\frac{1}{1+\lambda}\right) \tag{23}$$

$$\lambda = \frac{k_o l_e f}{2n_r} \tag{24}$$

d_r = 1/2 mean diameter of reference material

ν = kinematic viscosity = 0.0000141

β_r = hydrodynamic characteristic of reference material

$$= 2.7\left(\frac{1-n_r}{n_r^3}\right)\frac{1}{d_r} \tag{25}$$

General solutions for the transmission and reflection coefficients for this idealized structure follow directly:

$$T_l = \frac{|a_t|}{a_i} \tag{26}$$

$$R_l = \frac{|a_r|}{a_i} \tag{27}$$

Numerically, T_l, R_l are solved iteratively by first assuming a value for λ and solving for u_s, R_a, f, then solving for a new value of λ, and repeating the procedure until convergence is achieved.

External Energy Dissipation

In the previous section, an analytical solution for the idealized problem of wave transmission through and reflection from rectangular breakwaters was obtained. Since most breakwaters are of trapezoidal, rather than rectangular cross section, a considerable amount of energy may be dissipated on the seaward slope of the breakwater. This external dissipation of energy is not accounted for in the analysis of porous crib-style breakwaters. To account for the external dissipation of energy on the seaward slope of a trapezoidal breakwater, theoretical and empirical relationships are presented for the problem of energy dissipation on a rough, impermeable slope. A theoretical analysis of this problem is based on the following assumptions:

- Relatively long, normally incident unbroken waves described by linear wave theory.
- Energy dissipation on the rough impermeable slope may be represented as bottom friction.

The problem to be considered is illustrated in Figure 5-4-3.

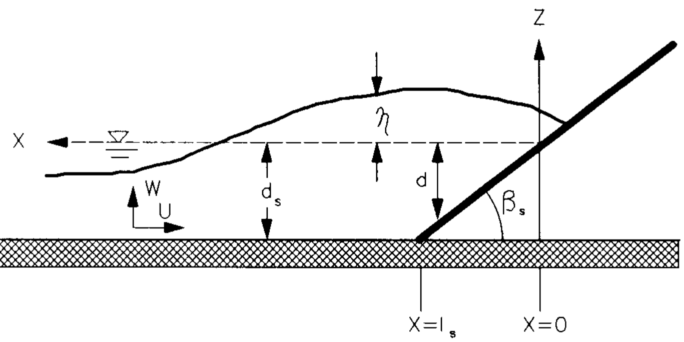

Figure 5-4-3. Wave Runup and Energy Dissipation on Impermeable Slope

The linear long-wave equations for the problem subdomain bounded by the structure slope ($x < l_s$) are given below. These equations also describe the subdomain approaching the structure $x \geq l_s$ with the omission of the friction term in the momentum equation. The orientation of the x-axis has reversed from the previous notation.

continuity	conservation of momentum	
$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(dU) = 0$	$\frac{\partial U}{\partial t} + g\left(\frac{\partial \eta}{\partial x}\right) + f_b \omega U = 0$	(28)

where

η = surface elevation relative to still water

d = water depth along sloping breakwater face

U = horizontal velocity component

g = acceleration due to gravity

f_b = linearized bottom friction factor

$$= \frac{\frac{1}{2} f_w |U|}{\omega d} \tag{29}$$

f_w = wave friction factor relating bottom shear stress, τ_b , fluid density, ρ , and velocity

$$\tau_b = \frac{1}{2} \rho f_w |U| U \tag{30}$$

$\omega = \frac{2\pi}{T}$ = radian frequency

Like the previous procedure, the equations are solved by assuming a periodic solution of radian frequency, ω, and introducing complex variables:

$$\eta = \zeta(x)e^{i\omega t} \tag{31}$$

$$U = u(x)e^{i\omega t} \tag{32}$$

General solutions for the governing Partial Differential Equations for the two subdomains are

$$\left.\begin{aligned} \zeta &= a_i e^{ik_o x} + a_r e^{-ik_o x} && (33) \\ u &= -\sqrt{\frac{g}{d_s}}\left(a_i e^{ik_o x} - a_r e^{-ik_o x}\right) && (34) \end{aligned}\right| (x \ge l_s)$$

$$\left.\begin{aligned} \zeta &= A J_o 2\left[\frac{\omega^2(1 - i f_b)x}{g \tan\beta_s}\right]^{\frac{1}{2}} && (35) \\ u &= -iA\left[\frac{g}{(1 - i f_b)x \tan\beta_s}\right]^{\frac{1}{2}} J_1 2\left[\frac{\omega^2(1 - i f_b)x}{g \tan\beta_s}\right]^{\frac{1}{2}} && (36) \end{aligned}\right| (0 \le x \le l_s)$$

where

a_i = amplitude of incident wave

$$k_o = \frac{\omega}{\sqrt{g d_s}} \tag{37}$$

d_s = constant water depth seaward of breakwater

a_r = complex amplitude of reflected wave

l_s = submerged horizontal length of the impermeable slope

A = arbitrary constant that is the complex vertical amplitude of the wave motion at the intersection of the still-water level and the slope. The modulus (or magnitude) $|A|$ is treated as an approximate value of the runup on the slope.

J_0 = Bessel function of the first kind of order zero

β_s = angle of impermeable slope

J_1 = Bessel function of the first kind of order one

Simultaneous solution at their common boundary ($x = l_s$) yields:

$$a_i e^{ik_o l_s} + a_r e^{-ik_o l_s} = A J_o 2k_o l_s \sqrt{1 - i f_b} \tag{38}$$

$$a_i e^{ik_o l_s} - a_r e^{ik_o l_s} = A \frac{i}{\sqrt{1 - i f_b}} J_1 2k_o l_s \sqrt{1 - i f_b} \tag{39}$$

The above complex solutions may be manipulated to produce expressions for reflection and runup on rough impermeable slopes:

$$\frac{a_r}{a_i} = \left(\frac{J_0 2k_o l_s \sqrt{1 - i f_b} - \frac{i}{\sqrt{1 - i f_b}} 2 J_1 k_o l_s \sqrt{1 - i f_b}}{J_0 2k_o l_s \sqrt{1 - i f_b} + \frac{i}{\sqrt{1 - i f_b}} 2 J_1 k_o l_s \sqrt{1 - i f_b}} \right) e^{i2k_o l_s} \tag{40}$$

$$\frac{A}{2a_i} = \frac{e^{ik_o l_s}}{J_0 2k_o l_s \sqrt{1 - i f_b} + \frac{i}{\sqrt{1 - i f_b}} 2 J_1 k_o l_s \sqrt{1 - i f_b}} \tag{41}$$

Expressions for a reflection coefficient and nondimensional run-up amplitude follow directly:

$$R_{II} = \frac{|a_r|}{a_i} \tag{42}$$

$$R_u = \frac{|A|}{2a_i} \tag{43}$$

The important parameters in determining the reflected wave amplitude and the run-up amplitude are the submerged horizontal length of the slope relative to incident wavelength, l_s/L and the linearized friction factor, f_b. Since the linearized friction factor appears in the form $\sqrt{1 - i f_b}$, it is expedient to introduce the friction angle ϕ defined by

$$\tan 2\phi = f_b \tag{44}$$

since

$$\sqrt{1 - i f_b} = (1 + \tan^2 2\phi)^{\frac{1}{4}} e^{-i\phi} \tag{45}$$

The expression for the friction angle ϕ, is

$$\tan 2\phi = f_w \frac{|A|}{d_s} \frac{1}{\tan \beta_s} F_s \tag{46}$$

where

f_w = wave friction factor (empirically determined)

$$= 0.29\left(\frac{d}{d_s}\right)^{-0.5}\left(\frac{d\tan\beta_s}{|A|}\right)^{0.7} \qquad (47)$$

d = average stone diameter

F_s = slope friction constant

$$= \frac{4}{3\pi}\frac{\int_0^1\left(\frac{J_1 2\psi\sqrt{y}}{\psi\sqrt{y}}\right)^3 dy}{\int_0^1 y\left(\frac{J_1 2\psi\sqrt{y}}{\psi\sqrt{y}}\right)^2 dy} \qquad (48)$$

$$\psi = k_o l_s \sqrt{1 - i\tan 2\phi} \qquad (49)$$

$$y = \frac{x}{l_s} \qquad (50)$$

These equations are solved iteratively by first assuming a value of ϕ, next evaluating R_u and F_s , then calculating a new ϕ, and repeating the procedure until convergence is achieved.

Synthesis of the Two Preceding Analyses

This section discusses the synthesis of the results of the two preceding analyses. The procedure provides approximate values for wave reflection and transmission for trapezoidal multilayered rubble-mound breakwaters.

For most trapezoidal, multilayered breakwaters, the stone size in the layer under the armor layer of the seaward slope is small relative to the material of the armor layer. As a first approximation, the structure may therefore be treated as having an impermeable rough slope. Thus, with incident wave characteristics, rubble-mound armor, and seaward structure slope, the procedure developed in the section entitled **External Energy Dissipation** may be used to approximately account for the energy dissipation on the seaward slope. The remaining wave energy may be expressed as the energy associated with a progressive wave of the following amplitude:

$$a_I = R_{II} a_i \qquad (51)$$

where

a_I = wave amplitude representing remaining energy after dissipation on the seaward slope of the breakwater

R_{II} = reflection coefficient of a rough impermeable sloped structure (from external energy dissipation analysis)

a_i = amplitude of incident wave

This remaining energy (represented by wave amplitude a_I) is partitioned among the reflected, transmitted, and internally dissipated energy of a hydraulically equivalent homogeneous rectangular breakwater for which a reflection coefficient R_I and transmission coefficient T_I have been determined as shown in the section entitled **Internal Energy Dissipation**. A rational method for obtaining a homogeneous rectangular breakwater that is hydraulically equivalent to a trapezoidal, multilayered breakwater is developed in the next section entitled **Hydraulically Equivalent Rectangular Breakwater**.

Having now accounted for the external as well as the internal energy dissipation, the amplitude of the reflected wave is found to be

$$|a_r| = R_I a_I = R_I R_{II} a_i \tag{52}$$

The transmitted wave amplitude is

$$|a_t| = T_I a_I = T_I R_{II} a_i \tag{53}$$

Therefore, the approximate values of the reflection R and transmission coefficients K_{Tt} of a trapezoidal, multilayered breakwater are

$$R = \frac{|a_r|}{a_i} = R_I R_{II} \tag{54}$$

$$K_{Tt} = \frac{|a_t|}{a_i} = T_I R_{II} \tag{55}$$

Hydraulically Equivalent Rectangular Breakwater

This section will present a method for determining an idealized homogeneous rectangular breakwater that is hydraulically equivalent to a trapezoidal, multilayered breakwater.

A hydraulically equivalent breakwater is taken as a homogeneous rectangular breakwater that yields the same discharge as would an actual trapezoidal, multilayered breakwater *with its top layer of stones on the seaward slope removed*. This definition of the equivalent breakwater is illustrated schematically in Figure 5-4-4. Typical realistic breakwaters consist of several different porous materials that are identified by their stone size, d_n, and their hydraulic characteristics, β_n. The idealized homogeneous rectangular breakwater consists of a reference material of stone size, d_r, and hydraulic characteristics, β_r. The reference material is considered representative of the porous materials of the multilayered breakwater.

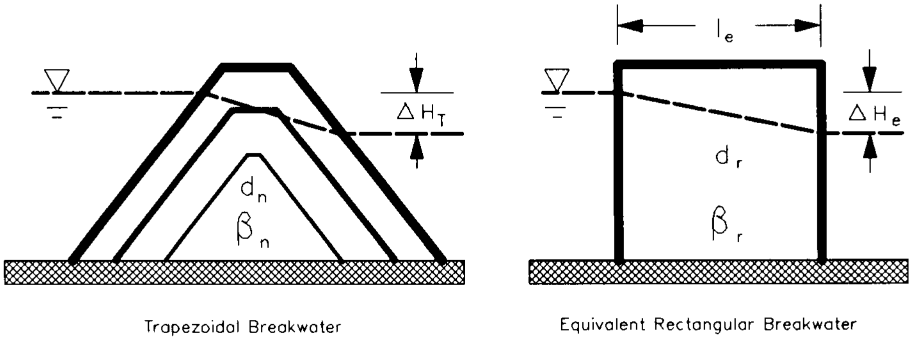

Figure 5-4-4. A Multilayered Trapezoidal Breakwater and Its Idealized Homogeneous Rectangular Equivalent

The flow through the structure is assumed to be one-dimensional (1-D), and the discharge per unit length of the equivalent rectangular breakwater is

$$Q_{\frac{equivalent}{structure}} = \sqrt{\left(\frac{g \Delta H_e}{\beta_r}\right)} \frac{d_s}{\sqrt{l_e}} \tag{56}$$

where

g = acceleration due to gravity

ΔH_e = head difference

β_r = hydrodynamic characteristic of reference breakwater

d_s = water depth

l_e = width of the equivalent breakwater

To evaluate the discharge per unit length of the multilayered trapezoidal breakwater, the structure is segmented into horizontal slices. The slices may be selected arbitrarily; however, it is expeditious to place them at material boundaries or at elevations where slopes change. A typical horizontal slice of height, Δh_j , is shown in Figure 5-4-5. Each slice consists of segments of different porous materials with individual hydraulic characteristics, β_n , and lengths, l_n .

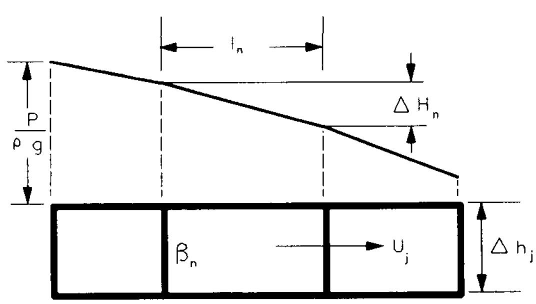

Figure 5-4-5. Illustrative Slice of a Multilayered Trapezoidal Breakwater Section

By summing contributions from all horizontal slices of the trapezoidal breakwater, the total discharge is

$$Q_{\frac{trapezoidal}{breakwater}} = \sqrt{\left(\frac{g\Delta H_T}{\beta_r}\right)} d_s \sum_j \left[\frac{1}{\sqrt{\sum_n \left(\frac{\beta_n}{\beta_r} l_n\right)}} \left(\frac{\Delta h_j}{d_s}\right) \right] \tag{57}$$

It is required that the discharges per unit length for the rectangular and trapezoidal breakwaters be identical:

$$Q_{\frac{equivalent}{structure}} = Q_{\frac{trapezoidal}{breakwater}} \tag{58}$$

The above relationship is reordered to solve for l_e, the equivalent rectangular breakwater.

$$l_e = \left\{ \sum_j \left[\frac{1}{\sqrt{\sum_n \left(\frac{\beta_n}{\beta_r} l_n\right)}} \left(\frac{\Delta h_j}{d_s}\right) \right] \right\}^{-2} \left(\frac{\Delta H_e}{\Delta H_T}\right) \tag{59}$$

where

$\sum_j$ = summation over number of layers in the breakwater

$\sum_n$ = summation over number of materials in the breakwater

$$\beta_n = \beta_o \left(\frac{1 - n_n}{n_n^3}\right) \frac{1}{d_n} \tag{60}$$

β_o = 2.7

d_n = mean diameter of individual material

$$\beta_r = \beta_o \left(\frac{1 - n_r}{n_r^3}\right) \frac{1}{d_r} \tag{61}$$

n_r = 0.435

d_r = mean diameter of reference material

The equation for l_e shows that the width of the equivalent breakwater may be determined from knowledge of the configuration of the trapezoidal, multilayered breakwater and the corresponding head differences, ΔH_e and ΔH_T.

Runup on the seaward face of both the equivalent and trapezoidal breakwater is taken as a representative value of the head difference.

$$\Delta H_e = (1 + R_I) a_I = (1 + R_I) R_{II} a_i \tag{62}$$

$$\Delta H_T = R_u H_i = 2 R_u a_i \tag{63}$$

where

$H_i = 2a_i$ = incident wave height

$$\frac{\Delta H_e}{\Delta H_T} = \frac{(1 + R_I) R_{II}}{2 R_u} \tag{64}$$

The head difference ratio is a function of the reflection coefficient R_I of the equivalent breakwater, which cannot be determined till the width of the equivalent breakwater l_e is known (i.e., an iterative procedure).

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