# Longshore Sediment Transport

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# Longshore Sediment Transport

## Description

This application provides estimates of the *potential* longshore transport rate under the action of waves. The method used is based on the empirical relationship between the longshore component of wave energy flux entering the surf zone and the immersed weight of sand moved (Galvin, 1979). Three methods are available to the user depending on whether available input data are breaker wave height and direction, deepwater wave height and direction, or using a Wave Information Study (WIS) hindcast data file created by the Coastal Engineering Data Retrieval System (CEDRS). The material presented herein can be found in Chapter 4 of the *Shore Protection Manual* (1984) and in Gravens (1988).

## Introduction

The longshore transport rate, $Q$, is the volumetric rate of movement of sand parallel to the shoreline. Much longshore transport occurs in or near the surf zone and is caused by the approach of waves at an angle to the shoreline. $Q$ is expressed in terms of sand volume per unit time (such as cubic yards per year or cubic meters per year). The method used to calculate the longshore transport in this ACES application is based on the assumption that longshore transport rate $Q$ is dependent upon the longshore component of wave energy flux $P_{ls}$ entering the surf zone (Equation 4-49 of the SPM (1984)).

$$Q = \frac{K}{(\rho_s - \rho) g a'} P_{ls} \qquad \text{(unit volume per sec)} \qquad (1)$$

where

$K$ = dimensionless empirical coefficient (based on field measurements)
= 0.39

$\rho_s$ = density of sand

$\rho$ = density of water

$g$ = acceleration due to gravity

$a'$ = ratio of the volume of solids to total volume, accounting for sand porosity
= 0.6

## General Assumptions and Limitations

General assumptions and limitations used to derive the longshore energy flux in the surf zone are:

- Conservation of energy flux in shoaling waves.
- Linear wave theory.
- Evaluating the energy flux relation at the breaker position.
- Breaker characteristics described by solitary wave theory.
- Straight bathymetric contours parallel to the shoreline.

Judgment is required in using the empirical relationship between longshore transport rate and the energy flux factor (Equation 1). The accuracy of $Q$ found using the energy flux factor can be estimated to be ±50 percent (SPM, 1984).

## General Energy Flux Equation

The energy flux per unit length of wave crest or, equivalently, the rate at which wave energy is transmitted across a plane of unit width perpendicular to the direction of wave advance is

$$P = E C_g \tag{2}$$

where

$E$ = wave energy density

$C_g$ = wave group speed

The wave energy density is calculated by:

$$E = \frac{\rho g H^2}{8} \tag{3}$$

where

$\rho$ = mass density of water

$g$ = acceleration of gravity

$H$ = wave height

If the wave crests make an angle $\alpha$ with the shoreline, the energy flux in the direction of wave advance *per unit length of beach* is

$$P \cos\alpha = \frac{\rho g H^2}{8} C_g \cos\alpha \tag{4}$$

The longshore component of wave energy flux is

$$P_l = P \cos\alpha \sin\alpha = \frac{\rho g H^2}{8} C_g \cos\alpha \sin\alpha \tag{5}$$

By use of the identity

$$\cos\alpha \sin\alpha = \frac{1}{2} \sin 2\alpha \tag{6}$$

the general equation for $P_l$ becomes

$$P_l = \frac{\rho g}{16} H^2 C_g \sin 2\alpha \tag{7}$$

Up to this point, all result have been for small-amplitude linear wave theory. However, the assumed relation between longshore transport and energy flux in the surf zone requires that $P_l$ be evaluated at the breaker line, where small-amplitude theory is less valid. To indicate approximations for waves entering the surf zone, the symbol $P_{ls}$ will be used in place of $P_l$. this approximation is called the energy flux factor $P_{ls}$ in the SPM (1984), and like $P_l$, it is measured in units of energy per second per unit length of shoreline. The next two sections derive approximate formulas for computing the longshore energy flux factor $P_{ls}$ entering the surf zone based on known breaking or deepwater significant wave conditions.

## Energy Flux Equation (Breaking Wave Conditions)

If the breaker values of the wave characteristics ($H_{sb}$ significant breaker height and $\alpha_b$ angle of approach) are applied to Equation 7, the energy flux factor results.

$$P_{ls} = \frac{\rho g}{16} H_{sb}^2 C_{gb} \sin 2\alpha_b \tag{8}$$

From linear wave theory (Equation 2-37 of the SPM (1984))

$$C_{gb} = \sqrt{g d_b} \tag{9}$$

where

$d_b$ = breaking depth

The breaking depth is related to the breaking wave height by Equation 2-91 of the SPM (1984).

$$\frac{d_b}{H_b} = 1.28 \tag{9a}$$

Substituting Equation 9a into Equation 9 and simplifying yields the celerity at breaking (Equation 2-37 of the SPM (1984)).

$$C_{gb} = \sqrt{1.28 g \ H_b} \tag{10}$$

Substituting Equation 10 into Equation 8 and simplifying yields the longshore energy flux factor for breaking wave conditions (Equation 4-44 of the SPM (1984)).

$$P_{ls} = 0.0707 \, \rho \, g^{\frac{3}{2}} H_{sb}^{\frac{5}{2}} \sin 2 \, \alpha_b \tag{11}$$

## Energy Flux Equation (Deepwater Wave Conditions)

Using deepwater values of wave characteristics ($H_{s0}$ significant wave height and $\alpha_0$ angle of approach), Equation 7 becomes:

$$P_{ls} = \frac{\rho \, g}{16} H_{s0}^2 \, C_{gb} \sin 2 \, \alpha_0 \tag{12}$$

Again, from linear wave theory:

$$C_{gb} = n \, C_b \tag{13}$$

where

$$n \approx \frac{1}{2} \text{ (in deep water)}$$

Celerity at breaking (Equation 2-37 of the SPM (1984)).

$$C_b = \sqrt{1.28 \ H_b} \tag{14}$$

Local wave height $H_b$ can be related to deepwater height $H_0$ by refraction and shoaling coefficients, where the coefficients are evaluated at the breaker position.

$$\sqrt{H_b} = \sqrt{K_r K_s H_0} \tag{15}$$

where

$K_r$ = refraction coefficient

$$\sqrt{K_r} = \left( \frac{\cos \alpha_0}{\cos \alpha_b} \right)^{\frac{1}{4}} \tag{16}$$

$K_s$ = shoaling coefficient (approximated by the breaker height index)
= 1.3 (Galvin and Schweppe, 1980)

In Equation 16, $\cos\alpha_b$ equals 1.0 to a good approximation. For example, if $\alpha_b$ has a high value of 20°, then $(\cos 20°)^{1/4} = 0.98$.

Substituting the above variables into Equation 11 for $P_{ls}$ and simplifying yields the longshore energy flux factor entering the surf zone using deepwater wave conditions (Equation 4-45 of the SPM (1984)).

$$P_{ls} = 0.04 \ \rho \, g^{\frac{3}{2}} H_{s0}^{\frac{5}{2}} \left(\cos \alpha_0\right)^{\frac{1}{4}} \sin 2 \, \alpha_0 \tag{17}$$

**Estimating Potential Sand Transport Rates Using WIS-CEDRS Data**

The following paragraphs describe the procedure that is used in this ACES application for calculating potential longshore transport rates using WIS hindcast wave estimates as provided in a specially formatted data file by the CEDRS (see section titled *Coastal Engineering Data Retrieval System).*

The potential longshore sand transport rate using WIS-CEDRS data is calculated using Equations 1 and 11 which require the breaking wave height and incident angle with respect to the shoreline. WIS-CEDRS hindcast estimates, however, are given for water depths greater than or equal to 10 meters (Jensen, 1983). Therefore, a transformation of the WIS-CEDRS hindcast wave estimates to breaking conditions is necessary. Refraction and shoaling of incident waves provided by WIS-CEDRS is accomplished using linear wave theory and numerically solving Snell's law for wave direction and the equation of conservation of wave energy flux for wave height. The governing equations are given below (Coastal Engineering Technical Note-II-19, 1989). The subscripts 0 and b denote values in deep water and at breaking, respectively.

Wave direction is obtained through Snell's law

$$\frac{\sin(\alpha_0)}{L_0} = \frac{\sin(\alpha_b)}{L_b} \tag{18}$$

where

$\alpha_0$ = deepwater wave approach angle (extracted from the CEDRS data file)
$\alpha_b$ = wave breaker angle
$L$ = wavelength

$$= \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right) \tag{19}$$

$T$ = wave period (extracted from the CEDRS data file)

The wave height is obtained by invoking the conservation of wave energy flux directed onshore

$$E_0 C_{g0} \cos(\alpha_0) = E_b C_{gb} \cos(\alpha_b) \tag{20}$$

where

$E_{0/b}$ = wave energy

$$= \frac{1}{8}\rho g H_{0/b}^2$$

$H_0$ = deepwater wave height (extracted from the CEDRS data file)
$H_b$ = breaking wave height
$C_g$ = wave group speed

$$= \frac{L}{2T}\left[1 + \frac{\frac{4\pi d}{L}}{\sinh \frac{4\pi d}{L}}\right]$$

The breaking wave height is linearly related to the depth at breaking as

$$H_b = \gamma d_b \tag{21}$$

where

$\gamma$ = wave breaking index
= 0.78

## Wave Transformation Procedure

The first step in the wave transformation procedure (Gravens, 1988) is to calculate the wavelength (Equation 19) at the location (denoted by subscript 0) where the wave height, incident angle, period, and water depth are known. These parameters are read by this ACES application directly from the *formatted* WIS-CEDRS data file. Equation 19 is solved by using a Newton-Raphson iteration.

The second step is to determine wave height, water depth, and incident angle at breaking (denoted by subscript b in Equations 18 and 20). Equation 18 is first solved for cos $\alpha_b$ and substituted together with Equation 21 into the right-hand side of Equation 20. This yields an expression for the conservation of wave energy flux in terms of the known wave characteristics (left-hand side of Equation 20) and the unknown wave characteristics at breaking (right-hand side of Equation 20). Equation 19 evaluated at breaking gives another expression in terms of the wavelength and water depth at breaking. A Newton-Raphson type solution can be used to iterate for the two unknown variables of wavelength and water depth at breaking.

Having determined the wavelength and depth at breaking, breaking wave height is calculated using Equation 21, and the breaking wave angle is calculated using Equation 18. The potential longshore sand transport rate is then estimated using Equations 1 and 11.

### Coastal Engineering Data Retrieval System

The CEDRS (available only to Corps of Engineers offices) is an interactive microcomputer resident database system, distinct and separate from ACES, which provides both hindcast and measured wind and wave data for use in the field of coastal engineering. The general goal of CEDRS is to assemble, archive, and make available regional databases containing data applicable to requirements of individual coastal Districts of the Corps of Engineers. The CEDRS databases contain both measured data from several sources and computer model generated hindcast data. The CEDRS system resides completely on an auxiliary hard disk furnished for each regional database. For more information regarding the system, forward inquiries to:

Coastal Engineering Research Center
US Army Engineer Waterways Experiment Station
ATTN: CEWES-CR-O
3909 Halls Ferry Road
Vicksburg, MS 39180-6199

The CEDRS data file that is used by this ACES application provides percent occurrence tables of waves in height and period ranges for specified direction bands at numerous stations on all US coasts. Values in the tables represent the percentage of a 20- or 32-year period during which waves occur from specified azimuth ranges for the indicated height and period ranges (see reports in the *References and Bibliography* section dealing with Wave Information Studies of US Coastlines for more information).

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# NUMERICAL SIMULATION OF TIME-DEPENDENT BEACH AND DUNE EROSION

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# Numerical Simulation of Time-Dependent Beach and Dune Erosion

## Description

This application is a numerical beach and dune erosion model that predicts the evolution of an equilibrium beach profile from variations in water level and breaking wave height as occur during a storm. The model is one-dimensional (only onshore-offshore sediment transport is represented). It is based on the theory that an equilibrium profile results from uniform wave energy dissipation per unit volume of water in the surf zone. The general characteristics of the model are based on a model described by Kriebel (1982, 1984a, 1984b, 1986). Because of the complexity of this methodology and the input requirements, familiarization with the above references is strongly recommended.

## Introduction

The model described herein, called XSHORE, is a 1-D model in that only cross-shore sediment transport is represented. This implies that the effect of longshore transport is negligible as, for example, if the gradient of longshore transport were near zero. The model is based on the theory that the equilibrium shape of a beach profile is related to uniform wave energy dissipation per unit volume of water in the surf zone under breaking waves (Dean, 1977).

## General Assumptions and Limitations

General assumptions and limitations adopted in developing the beach and dune erosion numerical model are:

- An equilibrium beach profile can be attained if a beach is exposed to constant wave and water level conditions for a sufficiently long time.
- Longshore sediment transport is neglected, and beach and dune erosion results solely from cross-shore transport.
- The model is restricted to calculating beach and dune erosion; i.e., recovery (accretive) processes are not well represented.
- Cross-shore sediment transport is caused by breaking of short-period waves.
- Wave transformation in the nearshore is approximated using the assumptions of straight and parallel bathymetric contours and linear wave theory.
- Surf zone wave heights are assumed to be proportional to the local water depth.
- Wave runup and setup can be neglected.
- Waves are considered normally incident inside the surf zone.
- Median grain size (sand-size material) is constant along the profile.

## THEORETICAL DEVELOPMENT OF THE MODEL

The development of the model begins by assuming that the nearshore profile can be described by a monotonic function of depth, $h$ , which increases in the seaward direction, $x$ , across the actively modified portion of the profile according to the relationship (Bruun, 1954; Dean, 1977)

$$h = A x^{\frac{2}{3}} \tag{1}$$

where $A$ is a profile shape coefficient that has been shown to be correlated with median grain size (Moore, 1982). The coefficient $A$ governs the steepness of the subaqueous beach profile and has been related to a unique value of the wave energy dissipation per unit volume, $D_{eq}$ , which exists everywhere in the surf zone if the profile is in equilibrium (Dean, 1977).

Based on these concepts and using shallow-water linear wave theory, the cross-shore sediment transport rate, $Q$ , is expressed in terms of the excess energy dissipation per unit volume of water across the surf zone as

$$Q = K(D - D_{eq}) \tag{2}$$

where

$K$ = empirically determined sand transport rate coefficient

$D, D_{eq}$ = actual and equilibrium energy dissipation per unit volume of water under given local wave and water level conditions

Moore (1982) determined a design curve for $K$ by simulating the large-scale laboratory tests of Saville (1957) (see Kraus and Larson, 1988) using a numerical model based on Equation *2*. Profile evolution was best reproduced for the value of

$$K = 1.144 \times 10^{-3}\, ft^4/lb \quad \text{or} \quad 2.2 \times 10^{-6}\, m^4/N$$

In differential form, wave energy dissipation per unit volume of water in the surf zone may be written as

$$D = \frac{1}{h}\frac{\partial P}{\partial x} \tag{3}$$

where

$P$ = wave energy flux

This wave energy flux is given by

$$P = E\, C_g \tag{4}$$

where

$E$ = total wave energy in one wavelength per unit crest width

$C_g$ = wave group speed

The total wave energy is given by

$$E = \frac{1}{8}\rho g H^2 \tag{5}$$

where

$\rho$ = fluid density
$g$ = acceleration of gravity
$H$ = wave height

The quantity $C_g$ is the wave group speed and in shallow water is given by

$$C_g = (gh)^{\frac{1}{2}} \tag{6}$$

Substituting the wave energy $E$ and group speed $C_g$ into Equation *4* produces the following relationship:

$$P = \frac{1}{8}\rho g H^2 (gh)^{\frac{1}{2}} \tag{7}$$

Breaking wave heights are assumed to be proportional to local water depth and given by

$$H = \gamma h \tag{8}$$

where $\gamma$ is a dimensionless breaking wave constant, equal to 0.78 in the model. Wave energy flux in the surf zone can now be written as a function of total water depth.

$$P = \frac{1}{8}\rho g^{\frac{3}{2}}\gamma^2 h^{\frac{5}{2}} \tag{9}$$

By differentiating Equation *9* according to Equation *3*, an expression for energy dissipation per unit volume of water can be derived.

$$D = \frac{5}{16}\rho g^{\frac{3}{2}}\gamma^2 h^{\frac{1}{2}} \frac{\partial h}{\partial x} \tag{10}$$

Assuming a profile shape in equilibrium (which implies $D = D_{eq}$) with uniform energy dissipation per unit volume of water, integrating Equation *10* provides a relationship between water depth, $h$ , and distance offshore, $x$ .

$$h = \left[\frac{24}{5}\left(\frac{D_{eq}}{\rho \gamma^2 g^{\frac{3}{2}}}\right)\right]^{\frac{2}{3}} x^{\frac{2}{3}} \tag{11}$$

Using the equilibrium profile shape given by Equation *1*, Equation *11* can be solved for equilibrium energy dissipation, $D_{eq}$ , within the surf zone.

$$D_{eq} = \frac{5}{24}\rho \gamma^2 g^{\frac{3}{2}} A^{\frac{3}{2}} \tag{12}$$

## Numerical Solution

In the present model, time-dependent profile response is determined by an explicit finite difference solution of the equation for continuity of sand in the onshore-offshore direction.

$$\frac{\partial h}{\partial t} = \frac{\partial Q}{\partial x} \tag{13}$$

The surf zone is represented by a series of cells in which the incremental change in cross-shore distance from a baseline, $\Delta x$ , is uniform, and $h$ is the total water depth. Figure 6-2-1 is a schematic representation of the surf zone showing sediment transport over a horizontal grid of uniform width $\Delta x$.

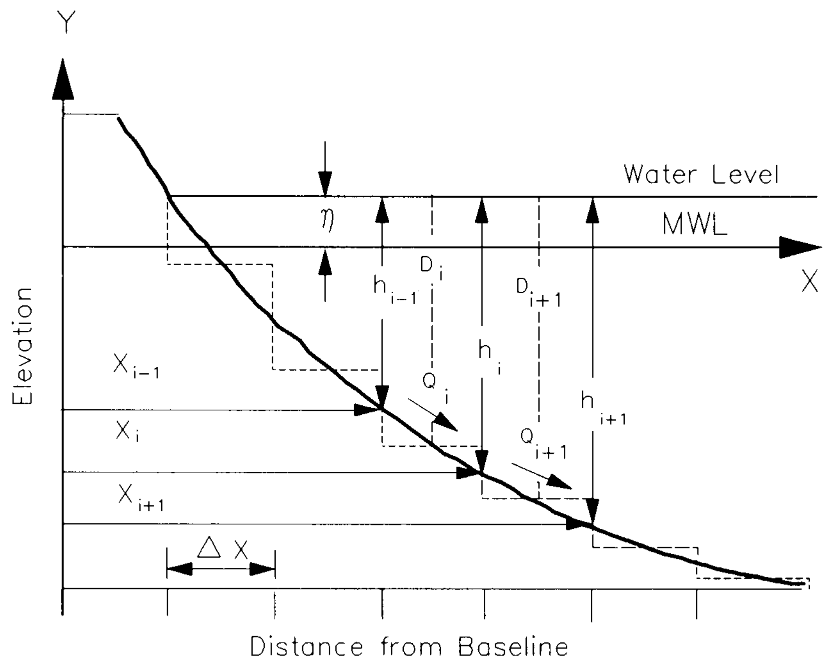


**Figure 6-2-1. Numerical Representation of the Beach Profile**

Since water level can fluctuate with respect to storm surge and tide, then

$$h_i = \eta_i - d_i \tag{14}$$

$\eta_i$ = water surface elevation due to storm surge and tide

$d_i$ = bottom elevation

Both $\eta_i$ and $d_i$ are defined at $h_i$ and are relative to mean water level (MWL). Wave setup in the surf zone and wave runup on the beach face are not represented.

Since $D$ and $Q$ vary with water depth and local bottom slope, the rate of change at each discrete cell differs from adjacent cells. Therefore, Equation *13* can be written as

$$\frac{\Delta h_i}{\Delta t} = \frac{Q_{i+1} - Q_i}{\Delta x} \tag{15}$$

From Equation *2*

$$Q_i = K(D_i - D_{eq}) \tag{16}$$

$$Q_{i+1} = K(D_{i+1} - D_{eq}) \tag{17}$$

where

$D_i$ = energy dissipation per unit volume of water at $i$

$D_{i+1}$ = energy dissipation per unit volume of water at $i+1$

Substituting Equations *16* and *17* into Equation *15*

$$\frac{\Delta h_i}{\Delta t} = \frac{K(D_{i+1} - D_{eq}) - K(D_i - D_{eq})}{\Delta x} \tag{18}$$

Since $\Delta h_i = h_i' - h_i$ at an internal grid point and $D_{eq}$ is constant across the surf zone

$$h_i' = h_i + \frac{K \Delta t}{\Delta x}(D_{i+1} - D_i) \tag{19}$$

where

$h_i'$ = water depth at the succeeding time step

From this expression, if $D_{i+1}$ is greater than $D_i$ over time $\Delta t$, there will be a net increase in water depth at the succeeding time step.

As a wave breaks over a sloping bottom, the energy dissipated in a distance $\Delta x$ may be expressed as the difference between the energy flux $P$ entering and exiting a control volume defined by $\Delta x$ and the water depth, where $P_i$ and $P_{i+1}$ are defined at $h_i$ and $h_{i+1}$. Therefore,

$$D_{i+1} = \frac{P_{i+1} - P_i}{h_{i+\frac{1}{2}} \ \Delta x} \tag{20}$$

Combining Equations *9* and *20*, the energy dissipation per unit volume of water in the cell between grid points $i$ and $i+1$ is

$$D_{i+1} = \alpha \left[ \frac{h_{i+1}^{\frac{5}{2}} - h_i^{\frac{5}{2}}}{0.5(h_{i+1} + h_i)\Delta x} \right] \tag{21}$$

where

$$\alpha = \frac{\rho g^{\frac{3}{2}} \gamma^2}{8} \tag{22}$$

Substituting Equation *21* into the finite difference form of the continuity Equation *19* yields

$$h_i' = h_i + \frac{2\alpha K \Delta t}{(\Delta x)^2} \left[ \frac{(h_{i+1}^{2.5} - h_i^{2.5})}{(h_{i+1} + h_i)} - \frac{(h_i^{2.5} - h_{i-1}^{2.5})}{(h_i + h_{i-1})} \right] \tag{23}$$

where

$h_i'$ = calculated water depth at position $x_i$ for the succeeding time step

## Initial Boundary Conditions

Figure 6-2-2 is a schematic representation of the macro-morphologic features of a beach-dune system, illustrating initial parameters required for constructing an idealized beach profile.

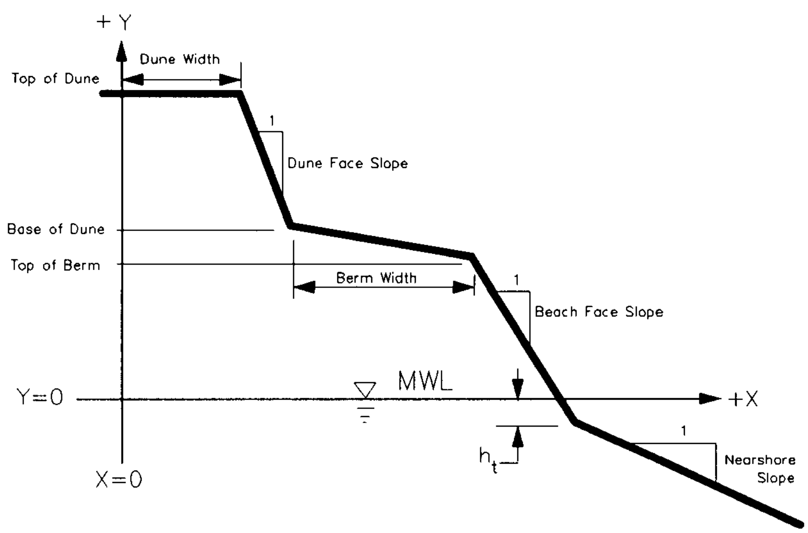


Figure 6-2-2. Idealized Berm, Dune, and Offshore System

A linearly sloping beach face is assumed to intersect an equilibrium profile at a depth, $h_t$, such that there is a continuous decrease in depth in the offshore direction. This parameter is calculated in the model knowing the beach face slope and the equilibrium profile shape factor, $A$, which has been shown to be a function of median grain diameter ($D_{50}$) (Moore, 1982). Once this depth has been defined, erosion of the subaerial beach is assumed to respond geometrically, as determined solely from continuity, from $h_t$ to the top of the berm or top of the dune, depending on profile geometry and water level elevation. When actual profile data are supplied, the user defines the initial landward boundary condition, and all other geometrical parameters are calculated internally.

The offshore boundary condition for the profile response portion of the model is defined at the depth of wave breaking, shoreward of which refraction is assumed to be negligible. The region between the point of wave breaking and $h_t$ is governed by energy dissipation and may be thought of as the dynamic zone. Beyond the breaking depth, energy dissipation is assumed negligible, and sediment transport is defined as zero. Seaward of wave breaking, wave refraction and shoaling are calculated from data input supplied by the user (significant wave height, peak period, and wave angle relative to the shoreline).

## MODEL CAPABILITIES

The preceding discussion summarizes the basic equations and conditions used for simulating beach and dune erosion caused by varying water levels and wave heights, as occur during a storm. The present model differs from that of Kriebel (1982) in two significant ways:

- A horizontal grid is used ($\Delta x$ constant) rather than a vertical grid ($\Delta y$ constant).
- An explicit finite difference calculation scheme is used rather than an implicit scheme.

Model characteristics that have been implemented in XSHORE, include:

- Capability for generating an idealized beach profile (see schematization shown in Figure 6-2-2) according to criteria provided by the user (e.g. dune width, dune height, dune slope, height at base of dune, berm width, berm height, beach slope, nearshore slope) or use of actual profile data to be entered as x,y pairs.
- Use of time-dependent water level data (tide and/or storm surge) as recorded by tide gages or from predicted tidal variations.
- Use of a time series of wave height, peak period, and wave angle data representing the primary forcing parameters used to operate the model.
- A limitation on the maximum run time of 5 days, since most storm events (periods of assumed dominant erosion through cross-shore sand transport) have a duration less than this period of time.

The smallest time step for data entry and output is 1 hr. Summary statistics include change in shoreline position at the 0, +5, +10, +15 ft contours and associated adjustments in sand volume ($yd^3/ft$) above mean water level for integer multiples of 1 hr. Simulated profile change data are also written to an ASCII file at the chosen time interval for the user to view with an available graphics software package.

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# CALCULATION OF COMPOSITE GRAIN-SIZE DISTRIBUTIONS

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# Calculation of Composite Grain-Size Distributions

## Description

The major concern in the design of a sediment sampling plan for beach-fill purposes is determining the composite grain-size characteristics of both the native beach and the potential borrow site. This application calculates a composite grain-size distribution that reflects textural variability of the samples collected at the native beach or the potential borrow area.

## Introduction

Sediment of a similar grain size (particle diameter) to that of the project, or native beach, is used to construct a beach fill. Particle diameters smaller than that of the native beach will eventually be transported (winnowed) and lost from the project site. Sediment containing finer particle diameters can still be used for construction by placing an overage amount with the assumption that, after the winnowing process is complete, the volume of material remaining is equal to the design volume. This overage is known as the overfill ratio or fill factor.

The fill factor is computed by comparing the grain-size distributions (mean and standard deviation) of the native and borrow sediments (see the ACES application, **Beach Nourishment Overfill Ratio and Volume**). Since only one grain-size distribution (GSD) is used to characterize each population, several (5 to 100) individual sediment samples are mathematically combined into one to form a composite sample. Obviously, the number of samples and the spatial and temporal distribution are critical in determining the composite so that each population is properly represented.

## Sediment Analysis

Both native beach and borrow sediments should be analyzed using standard sieving techniques. The sieving process mechanically separates a sediment sample containing an infinite number of particle diameters into finite particle size classes. Table A-4 in Appendix A is a list of standard size classes and corresponding sieve mesh sizes. Statistical parameters such as the median, mean, standard deviation, kurtosis, and skewness can be mathematically computed using data from the GSD.

In this application the individual sediment weight retained on each sieve has been normalized so that the *sum* of the individual sediment weights retained on all sieves equals 100. Stated mathematically, normalized sediment weight for each $\phi$ size is:

$$w_{\phi\ normalized} = w_{\phi} \frac{100}{\sum w} \tag{1}$$

where

$w_{\phi\ normalized}$ = normalized sediment weight for a specific $\phi$

$w_{\phi}$ = sediment weight on each sieve

$\sum w$ = total weight of sample

**Composite Surface Samples**

The sediment characteristics of the native beach can vary considerably.

- Across the beach profile through varied energy zones.
- Along the beach.
- At depths within the active profile.
- Between seasons.

The native beach composite must reflect these components. A typical plan would call for surficial sediment samples to be taken at regular vertical elevation intervals across the beach profile such as +12, +9, +6, +3, 0, -3, etc., offshore to the point of profile closure. Depending upon the longshore variability, this plan would be repeated along 1 to 4 transects for every mile of the project. To capture the seasonal variation, the beach would be sampled 2 to 3 times a year during the winter storm erosion and summer accretion periods. As an alternative, shallow cores could be taken along profile transects that would represent the spatial and temporal variations captured by the surface samples.

Surface samples are combined into one composite *average* GSD by summing the weights retained on each sieve interval and then dividing by the number of samples. Stated mathematically, the composite sediment weight for a given size is

$$w_{\phi\ composite} = \frac{w_{\phi s_1} + w_{\phi s_2} + w_{\phi s_3} + \ldots + w_{\phi s_n}}{n} \tag{2}$$

where

$w_{\phi\ composite}$ = composite weight for a specific sieve

$w_{\phi s_n}$ = sediment weight retained on a specific sieve for each sample

$n$ = number of samples

### Composite Core Samples

Cores are usually used to sample potential borrow sites and in some instances the native beach. Cores are brought back to the laboratory to be split, photographed, subsampled, and sieved using standard sieving techniques. Cores are typically subsampled based upon unique lithologic units. A 20-ft core may contain five or more unique units. Some units may be only inches in length, whereas others may be several feet in length.

The composite GSD of each lithologic unit is mathematically weighted proportionally based on its length relative to the core length. For example, if a unit is 1-ft in length from a 10-ft core, each sieve weight for the entire GSD is multiplied by 0.1 (1/10). This proportional weighting is done for all samples in the core resulting in a composite GSD for each core. This scheme is repeated for all cores that are to be included in the core composite. After all individual core composites are created, the entire GSD (consisting of individual sieve weights) for each core is summed and divided by the number of cores. This final step is identical to the procedure in which surface samples are combined into one composite.

A slight variation of the previously described procedure occurs when the user specifies the core composite by an upper and lower elevation limit. For example, a core is a total of 10 ft in length and contains two lithologic units, one 2 ft in length and the other 8 ft in length. The 2-ft unit represents a lithologic unit from an elevation of +11 to +9 ft, and the 8-ft unit represents a lithologic unit from an elevation of +9 to +1 ft. If for example, the user specifies a composite from an elevation of +10 to 0 ft, 1 ft of the 2-ft unit is above the requested upper elevation limit (+10 ft) and 1 ft below the limit. Proportionally, this units represents 10 percent (1/10) of the core within the requested elevation limits and is thereby weighted accordingly; i.e. the GSD is multiplied by 0.1. The 8-ft unit represents 80 percent (8/10) of the core within the elevation limits; thereby its GSD is multiplied by 0.8. Since no sample was present for the +1 to 0 elevation, the weighting scheme accounts for only 90 (10 + 80) percent of the entire core. This procedure is done for all cores located within the elevation limits.

## Soil Classification

A grade scale commonly used for classifying sediments is the phi ( $\phi$ ) scale devised by Krumbein (1934, 1938). The phi scale is a geometric scale to the power of 2 with individual size classes defined as "twice as large or half as large" as some other class (Table A-4, Appendix A). The phi transformation is given by:

$$\phi = -\log_2 d \qquad \text{or} \qquad 2^{-(\phi)} = d \tag{3}$$

where

$d$ = grain-size particle diameter in millimeters

The advantages of the phi scale are

- The distribution of particle sizes can be plotted on arithmetic graph paper, obviating the necessity of logarithmic graph paper; thus the graphical method of computing sediment statistics is simplified.

- Particle diameters for each size class become whole numbers instead of fractions of millimeters.

The disadvantages of the phi scale are

- Frequent unfamiliarity on the part of users.
- No intuitive relation to numerical results of sieve analysis.
- Progression of numerical scales is counter-intuitive (larger value means smaller grain size).

Sediments are classified as GRAVEL, COARSE SAND, MEDIUM SAND, FINE SAND, SILT, and CLAY to indicate the dominant particle size diameter of the sample. The two size schemes used today by coastal engineers are the Unified and Wentworth Soils classifications. These classifications assign similar but different size ranges to each category (Table A-4, Appendix A).

## Statistical Analysis of Sediment Data

There are two basic methods of obtaining statistical parameters of a GSD. The first method is known as the **Folk Graphic Method**, which was developed before the advent of advanced computers. The Folk method involves plotting a cumulative curve of the sample and determining the particle diameter that corresponds with certain cumulative percentages, i.e. 5, 16, 25, 50, 75, 84, and 95. The second method of obtaining statistical parameters is called the **Method of Moments**. It is a computational (not graphic) method using every size class, and it provides a more accurate measure than the graphic method, which relies on only a few selected percentages.

### Folk's Graphical Method

*Median ($M_d$)*

The graphic median is the size class in which half of the particles by weight are coarser than the median and half are finer.

*Mean ($\mu$)*

The mean is probably the best statistical parameter for determining the overall particle size and is given by:

$$\mu = \frac{(\phi_{16} + \phi_{50} + \phi_{84})}{3} \qquad (4)$$

*Standard Deviation (σ)*

The inclusive graphic standard deviation is a measure of sorting and is given by:

$$\sigma = \frac{\phi_{84} - \phi_{16}}{4} + \frac{\phi_{95} - \phi_{5}}{6.6} \qquad (5)$$

Most beach sands have a standard deviation ranging from 0.5 to 2.0.

*Skewness ($S_k$)*

The inclusive graphic skewness is a measure of asymmetry and is given by:

$$S_k = \frac{\phi_{16} + \phi_{84} - 2(\phi_{50})}{2(\phi_{84} - \phi_{16})} + \frac{\phi_{5} + \phi_{95} - 2(\phi_{50})}{2(\phi_{95} - \phi_{5})} \qquad (6)$$

A positive value indicates the sediment has an excess amount of fines, whereas a negative value indicates an excess amount of coarse material. The mathematical limits of skewness are -1.0 to +1.0.

*Kurtosis (K)*

The graphic kurtosis is a measure of the ratio between the sorting of the *tails* (ends) of the curve and the sorting of the central portion. When the central portion is better sorted (values greater than 1.5), the curve is said to be peaked or leptokurtic. If the tails are better sorted than the central portion, the curve is said to be flat-peaked or platykurtic (values between 0.0 and 1.1). Kurtosis is given by:

$$K = \frac{\phi_{95} - \phi_{5}}{2.44(\phi_{75} - \phi_{25})} \qquad (7)$$

## Method of Moments

*First Moment*

By definition, the first moment equals the mean $\overline{X}$ and is expressed as:

$$\overline{X} = \frac{\sum f m_{\phi}}{100} \tag{8}$$

where

$f$ = frequency in percentage for each size class

$m_{\phi}$ = midpoint of each $\phi$ size class

*Second Moment*

The second moment is a measure of the dispersion about the mean and is expressed as:

$$\frac{\sum f (m_{\phi} - \overline{X})^2}{100} \tag{9}$$

The second moment represents the numerical value of the standard deviation; therefore, the standard deviation is expressed as:

$$\sigma = \sqrt{\frac{\sum f (m_{\phi} - \overline{X})^2}{100}} \tag{10}$$

*Third Moment*

The third moment (known as the *mean cubed deviation*) is the measure of symmetry about the mean and is expressed as:

$$\frac{\sum f (m_{\phi} - \overline{X})^3}{100} \tag{11}$$

Skewness is computed by dividing the *mean cubed deviation* by the cube of the standard deviation and is expressed as:

$$S_k = \frac{\sum f(m_\phi - \overline{X})^3}{100 \ \ \sigma^3} \tag{12}$$

*Fourth Moment*

The fourth moment is the distribution about the mean and is expressed as:

$$\frac{\sum f(m_\phi - \overline{X})^4}{100} \tag{13}$$

Kurtosis is derived by dividing the fourth moment by the standard deviation raised to the fourth power:

$$S_k = \frac{\sum f(m_\phi - \overline{X})^4}{100 \ \ \sigma^4} \tag{14}$$

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# Beach Nourishment Overfill Ratio and Volume

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# Beach Nourishment Overfill Ratio and Volume

## Description

The methodologies represented in this ACES application provide two approaches to the planning and design of nourishment projects. The first approach is the calculation of the *overfill ratio*, which is defined as the volume of actual borrow material required to produce a unit volume of usable fill. The second approach is the calculation of a *renourishment factor*, which is germane to the long-term maintenance of a project, and addresses the basic question of how often renourishment will be required if a particular borrow source is selected that is texturally different from the native beach sand.

## Introduction

Beaches can effectively dissipate wave energy and are classified as shore protection structures of adjacent uplands when maintained at proper dimensions. Existing beaches are part of the natural coastal system, and their wave dissipation usually occurs without creating adverse environmental effects. Since most beach erosion problems occur when there is a deficiency in the natural supply of sand, the placement of borrow material on the shore should be considered as one shore stabilization measure.

An important question arises in these situations with respect to the volume of material taken from the borrow zone that will be effectively lost from the project during and following placement due to sorting processes. The design engineer is required to estimate the proportion of a proposed borrow material that will serve a useful function in a specific project requiring beach fill.

Quantitative methods for evaluating the suitability of borrow material as beach fill are those that give overfill ratios or factors or renourishment factors. Overfill ratios apply when the beach in the project area is expected to be stable if composed of sand of certain characteristics, or will be stabilized with engineering structures. Renourishment factors apply where the beach is undergoing erosion and the project requires periodic nourishment for beach stabilization. The methods described can be found in James (1975) and the SPM (1984).

## General Assumptions and Limitations

The overfill ratio and renourishment factor described herein are not physically related. Each value results from unique models of predicted beach-fill behavior that are dissimilar, although both use the comparison of native and borrow sand texture as input. The overfill ratio is calculated on the assumption that some portion of the borrow material is absolutely stable and hence a finite proportion of the original material will remain on the beach indefinitely. The renourishment factor is calculated on the opposing assumption that no material is absolutely stable, but that a finer material is less stable than coarse material, and hence a coarse beach fill will require renourishment less frequently than a fine one. The models address the different problems in determining nourishment requirements when fill that is dissimilar to native sediments is to be used (overfill ratio) and in predicting how quickly a particular fill will erode (renourishment factor). For design purposes, the overfill ratio, $R_A$, should be applied to adjust both initial and renourishment volumes. The renourishment factor, $R_J$ , should be considered an independent evaluation of when renourishment will be required.

It is recommended that selection of borrow material be based on all available historical information on the project area. Computations such as those performed by this ACES application provide useful supplemental inputs for planning and designing, but should not be regarded as accurate predictors. Engineering judgment and experience must accompany design application.

The procedures described herein require enough core borings and samples in the borrow zone, on the beach, and in the nearshore zones to adequately describe the size distribution of borrow and beach material. Size analyses of the borings and samples are used to compute composite size distributions for the two types of materials. These composite distributions are compared to determine the suitability of the borrow material. The concept of *composite* material properties is discussed in Krumbein (1957) and Hobson (1977).

## Soil Classification

A grade scale most commonly used for classifying sediments is the phi ( $\phi$ ) scale devised by Krumbein (1934, 1938). The phi scale is a geometric scale to the power of 2 with individual size classes defined as "twice as large or half as large" as some other class (Table A-4, Appendix A). The phi transformation is given by:

$$\phi = -\log_2 d \qquad \text{or} \qquad 2^{-(\phi)} = d \tag{1}$$

where

$d$ = grain-size particle diameter in millimeters

The advantages of the phi scale are

- The distribution of particle sizes can be plotted on arithmetic graph paper, obviating the necessity of logarithmic graph paper; thus the graphical method of computing sediment statistics is simplified.
- Particle diameters for each size class become whole numbers instead of fractions of millimeters.

The disadvantages of the phi scale are

- Frequent unfamiliarity on the part of users.
- No intuitive relation to numerical results of sieve analysis.
- Progression of numerical scales is counter-intuitive (larger value means smaller grain size).

Sediments are classified as GRAVEL, COARSE SAND, MEDIUM SAND, FINE SAND, SILT, and CLAY to indicate the dominant particle size diameter of the sample. The two size schemes used today by coastal engineers are the Unified and Wentworth Soils classifications. These classifications assign similar but different size ranges to each category (Table A-4, Appendix A).

## Grain-Size Distributions

The first stage in specifying beach-fill material is analysis of the sampling material from the existing beach to determine grain-size characteristics (*grain-size distribution*) that will be used as a basis for specification of the fill.

There are two basic methods of obtaining statistical parameters of a *grain-size distribution*. The first is known as the Folk Graphic Method, which was developed before the advent of advanced computers. The Folk method involves plotting a cumulative curve of the sample and determining the particle diameter that corresponds with certain cumulative percentages, i.e. 5, 16, 25, 50, 75, 84, and 95. The second method of obtaining statistical parameters is called the Method of Moments. It is a computational (not graphic) method using every size class, and it provides a more accurate measure than the graphic method, which relies on only a few selected percentages. These two methods are described in more detail in the ACES application titled **Calculation of Composite Grain-Size Distributions.**

### Phi Mean and Phi Sorting

Grain-size distributions are usually characterized by two parameters.

- The phi mean, $M_\phi$, which is a measure of the location of the central tendency of the *grain-size distribution.*
- The phi sorting or phi standard deviation, $\sigma_\phi$, which is a measure of the gradation or scale of the spread of the grain size about the phi mean. A low value ($< 0.5\ \phi$) indicates that the *grain-size distribution* contains only a narrow range of sizes; it is *well sorted* or *poorly graded*. Conversely, a high value of phi sorting ($> 1.0\ \phi$) indicates the presence of a wide range of grain sizes. Material of this type is *poorly sorted* or *well graded*.

### Comparison Parameters

It is convenient to define two statistical parameters used to compare the *grain-size distributions* of native and borrow materials. The first parameter, $\delta$, is the phi mean difference and is defined as a scaled difference between borrow and native phi means:

$$\delta = \frac{M_{\phi b} - M_{\phi n}}{\sigma_{\phi n}} \tag{2}$$

where

$-_b$ = subscript $b$ refers to borrow material

$-_n$ = subscript $n$ refers to natural sand on beach

$M_\phi$ = the phi mean

$$= \frac{(\phi_{84} + \phi_{16})}{2} \tag{3}$$

$\phi_{84}$ = 84th percentile in phi units

$\phi_{16}$ = 16th percentile in phi units

$\sigma_\phi$ = standard deviation

$$= \frac{(\phi_{84} - \phi_{16})}{2} \tag{4}$$

The phi mean difference has positive values where borrow materials are, on the average, finer than native materials, and negative values where borrow materials have a phi mean coarser than that of the native materials.

The second comparison parameter, $\sigma$, is the phi sorting ratio (phi standard deviation) and is defined as phi sorting of the borrow material over phi sorting of the native material:

$$\sigma = \frac{\sigma_{\phi b}}{\sigma_{\phi n}} \tag{5}$$

Borrow materials more poorly sorted than native materials have values of $\sigma$ greater than unity; where borrow materials are well sorted in comparison with native materials, $\sigma$ is less than unity.

## Beach-Fill Models

Two basic types of mathematical models have been proposed to handle beach-fill problems. The first model (Overfill Ratio) enables calculation of a fill factor that is an estimate of the volume of a specific fill material needed to create a unit volume of *stable* native beach material. In most cases, fill factors exceed one, indicating that the particular borrow material is less than ideal and that winnowing processes will selectively remove unsuitable parts from the fill until it becomes compatible with existing beach sediments. The second model (Renourishment Factor) enables calculation of a factor that is used to estimate how often placement of a particular fill will be required to maintain specific beach dimensions. A more detailed and formalized discussion of the two methods can be found in James (1975) and the SPM (1984).

## Overfill Ratio

There are four possible combinations that result from a comparison of the composite grain-size distribution statistical parameters (mean and standard deviation) of native material and borrow material. These are listed in Table 6-4-1.

Table 6-4-1. Relationships of Phi Means and Phi Standard Deviations

| Category | Phi Means | Phi Standard Deviations |
|---|---|---|
| I | $M_{\phi b} > M_{\phi n}$ Borrow is finer than native material | $\sigma_{\phi b} > \sigma_{\phi n}$ Borrow material is more poorly sorted than native material |
| II | $M_{\phi b} < M_{\phi n}$ Borrow is coarser than native material | |
| III | $M_{\phi b} < M_{\phi n}$ Borrow is coarser than native material | $\sigma_{\phi b} < \sigma_{\phi n}$ Borrow material is better sorted than native material |
| IV | $M_{\phi b} > M_{\phi n}$ Borrow is finer than native material | |

The overfill ratio is given by:

$$\frac{1}{R_A} = 1 - F\left(\frac{\theta_2 - \delta}{\sigma}\right) + F\left(\frac{\theta_1 - \delta}{\sigma}\right) + \left[\frac{F(\theta_2) - F(\theta_1)}{\sigma}\right] \exp\left\{\frac{1}{2}\left[\theta_1^2 - \left(\frac{\theta_1 - \delta}{\sigma}\right)^2\right]\right\} \tag{6}$$

where

$F$ = integral of the standard normal curve

| Case | $\theta_1$ | $\theta_2$ |
|---|---|---|
| I and II | $\mathrm{Max}\left\{-1\ ,\ \frac{-\delta}{\sigma^2 - 1}\right\}$ | $\infty$ |
| III and IV | $-1$ | $\mathrm{Max}\left\{-1\ ,\ 1 + \frac{2\delta}{1 - \sigma^2}\right\}$ |

### Renourishment Factor

The renourishment factor ($R_J$) is a dynamic approach to answering how beach processes can be expected to modify specific fill sediments. It is an estimate of how often renourishment will be needed and helps evaluate the long-term performance of different fill materials with regard to suitability, maintenance, and expense. The conceptual approach is that the active beach system can be viewed as a compartment which receives sediments through longshore transport and from gradual erosion of the inactive *reservoir* of sediments that form the backshore. The compartment loses sediments by longshore and offshore transport beyond its boundaries. In this scheme, a fill is viewed essentially as an increase to the backshore reservoir. Sediment particle residence time in the compartment is longer for coarse-grained sediments than for fine; thus, a comparison between *composite size distributions* of native and borrow sediments can be used to predict the lifetime of a fill. The scheme thus becomes a "bookkeeping" problem of monitoring material going in and out of the system by using mass-balance equations that are similar to the more familiar sediment budget calculations.

To determine periodic renourishment requirements, James (1975) defines a renourishment factor, $R_J$, which is the ratio of the rate at which borrow material will erode to the rate at which natural beach material is eroding. The renourishment factor is given as:

$$R_J = \exp\left[\Delta\left(\frac{M_{\phi b} - M_{\phi n}}{\sigma_{\phi n}}\right) - \frac{\Delta^2}{2}\left(\frac{\sigma^2_{\phi b}}{\sigma^2_{\phi n}} - 1\right)\right] \tag{7}$$

where

$\Delta$ = winnowing function = 1.0 (recommended value)

A renourishment factor of one-third implies that the borrow material is three times as stable as the native, or that renourishment with this borrow material would be required one-third as often as renourishment with nativelike sediments. However, an $R_J$ of 3 indicates the borrow is one-third as stable, and if used as beach fill, will require renourishment three times as often as the nativelike sediments.

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