WINDSPEED ADJUSTMENT AND WAVE GROWTH

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Windspeed Adjustment and Wave Growth

Description

The methodologies represented in this ACES application provide quick and simple estimates for wave growth over open-water and restricted fetches in deep and shallow water. Also, improved methods (over those given in the *Shore Protection Manual* (SPM), 1984) are included for adjusting the observed winds to those required by wave growth formulas.

Introduction

Wind-generated wave growth is a complex process of considerable practical interest. Although the process is only partially understood, substantial demand remains for quick estimates required for design and analysis procedures. The most accurate estimates available are those provided by sophisticated numerical models such as those presented in Cardone et al. (1976), Hasselmann et al. (1976), Resio (1981), and Resio (1987). Yet many studies, especially at the preliminary level, attempt to describe wind-generated wave growth without the benefit of intensive large-scale modeling efforts. The prediction methods that follow present a first-order estimate for the process, but their simplification of the more complex physics should always be considered.

Methods are included for adjusting observed winds of varying character and location to the conditions required by wave growth formulas. A model depicting an idealized atmospheric boundary layer over the water surface is employed to estimate the low-level winds above the water surface. Stability effects (air-sea temperature gradient) are included, but barotropic effects (horizontal temperature gradient) are ignored. The numerical descriptions of the planetary boundary layer model are based upon similitude theory. Additional corrections are provided for the observed bias of ship-based wind observations as well as short fetches. Formulas for estimating winds of alternate durations are also included. The methodology for this portion of the application is largely taken from Resio, Vincent, and Corson (1982).

The simplified wave growth formulas predict deepwater wave growth according to fetch- and duration-limited criteria and are bounded (at the upper limit) by the estimates for a fully developed spectrum. The shallow-water formulations are based partly upon the fetch-limited deepwater forms and do not encompass duration effects. The methods described are essentially those in Vincent (1984), the SPM (1984), and Smith (1991).

Unless otherwise annotated, metric units are assumed for the following discussion.

General Assumptions and Limitations

The deep- and shallow-water wave growth curves are based on limited field data that have been generalized and extended on the basis of dimensionless analysis. The wind estimation procedures are based on a combination of boundary layer theory and limited field data largely from the Great Lakes. Wind transformation from land to water tends to be highly site and condition specific. The derivation of an individual site from these generalized conditions can create significant errors. Collection of site-specific field data to calibrate the techniques is suggested.

WIND ADJUSTMENT

The methodology for preparing wind observations for use in the wave growth formulas is based upon an idealized model of the planetary boundary layer depicted in Figure 1-1-1. For typical mid-latitude conditions, this planetary boundary layer exists in the lowest kilometer of the atmosphere and contains about 10 percent of the atmospheric mass (Holton, 1979).

Low-level winds directly over the water surface are considered to exist in a region characterized as having relatively constant stress at the air-sea interface. This surface layer will be designated the constant stress region for the remainder of this discussion.

Above the constant stress region is the Ekman layer, where the additional forces of Coriolis force, pressure gradient force, viscous stress, and convectively driven mixing are considered important.

Finally, above the Ekman region, geostrophic winds are considered to exist which result from considering the balance between pressure gradient forces and Coriolis force for synoptic scale systems.

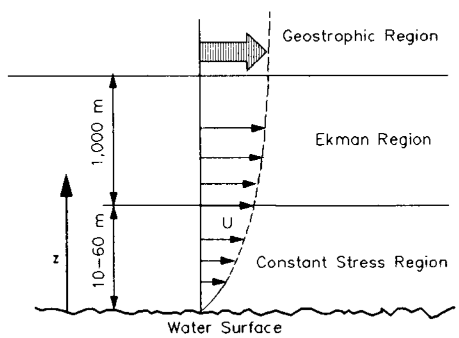

Figure 1-1-1. Idealized Atmospheric Boundary Layer over Water

Observed winds for use in the wave growth equations are considered to be characterized by six categories summarized in Table 1-1-1.

Table 1-1-1
Character and Action for Wind Observations

Observation type	Initial Action	Solution Domain
Over water (non-ship obs)	-------	Constant stress layer
Over water (ship obs)	Adjusted	Constant stress layer
At shoreline (onshore winds)	-------	Constant stress layer
At shoreline (offshore winds)	Geostrophic wind estimated	Full PBL* model
Over land	Geostrophic wind estimated	Full PBL model
Geostrophic wind	-------	Full PBL model

* PBL = Planetary Boundary Layer

Although the above six wind observation categories are presented for user convenience, only two separate cases are ultimately considered by the methodology: low-level winds observed within the constant stress region and known or estimated geostrophic winds. In the ACES application, adjustments for ship-based observations are made before proceeding with a solution in the constant stress region, and geostrophic winds are estimated for cases where low-level observed winds are predominantly over land masses. The case of observed winds blowing onshore and measured at the shoreline is considered to be effectively identical to the case of winds observed over water. Similarly, winds observed at the shoreline but blowing from the land mass in an offshore direction are considered effectively equivalent to winds observed at a more inland location. Complex wind patterns caused by local frictional characteristics or topography are obviously not considered by these simplifications.

Initial Adjustments and Estimates

Wind observations over water are typically the most desirable choice of available data sources for wave prediction. Observers on ships at sea frequently record such data and make qualitative estimates. Cardone (1969) reviewed the bias of ship-based observations and suggested the following adjustment:

$$U = 1.864\,U_{obs}^{\frac{7}{9}} \qquad (mps) \tag{1}$$

where

U = adjusted ship-based wind speed

U_{obs} = ship-based observations

For cases where the observed winds are predominantly over land surfaces, similar models of the boundary layer are sometimes employed for other prediction purposes. However, in this application, the following simple estimate for geostrophic winds is made from low-level wind observations (cgs units):

$$V_g = \frac{U_*}{\sqrt{C_{D\,land}}} \tag{2}$$

where

U_* = friction velocity

$$= \frac{k\,U_{obs}}{\ln\left(\frac{z_{obs}}{z_0}\right)} \tag{3}$$

k = von Karman constant ($k \sim 0.4$)

z_{obs} = elevation of wind observation

z_0 = surface roughness length (assumed = 30 cm)

$C_{D\,land}$ = drag coefficient over land

$$C_{D\,land} \sim 0.00255\,z_0^{0.1639} \tag{4}$$

Constant Stress Region

The major features of the constant stress region can be summarized as follows:

- The constant stress region is confined to the lowest few meters of the boundary layer.
- Wind flow is assumed parallel to the water surface.
- The wind velocity is adjusted so that the horizontal frictional stress is nearly independent of height.
- The stress remains constant within the layer and is characterized by the friction velocity U_* .

Stability (air-sea temperature gradient) has an important effect on wave growth. The wind profile within this region is described by the following modified logarithmic form:

$$U_z = \frac{U_*}{k}\left[\ln\left(\frac{z}{z_0}\right) - \Psi\left(\frac{z}{L'}\right)\right] \tag{5}$$

where

U_z = wind velocity at elevation z

z_0 = surface roughness length

$$= \frac{C_1}{U_*} + C_2 U_*^2 + C_3 \tag{6}$$

$$\left(C_1 = 0.1525 \quad , \quad C_2 = \frac{0.019}{980} \quad , \quad C_3 = -0.00371\right) \tag{7}$$

Ψ = universal similarity function
KEYPS formula (Lumley and Panofsky, 1964)

L' = Obukov stability length

$$= 1.79\frac{U_*^2}{\Delta T}\left[\ln\left(\frac{z}{z_0}\right) - \Psi\left(\frac{z}{L'}\right)\right] \tag{8}$$

ΔT = air-sea temperature gradient

$$\left.\begin{array}{ll} \Psi = 0 & |\ \Delta T = 0 \\ \Psi = C\dfrac{z}{L'} & |\ \dfrac{z}{L'} > 0 \\ \Psi = 1 - \phi_u - 3\ln\phi_u + 2\ln\left(\dfrac{1+\phi_u}{2}\right) + 2\tan^{-1}\phi_u - \dfrac{\pi}{2} + \ln\left(\dfrac{1+\phi_u^2}{2}\right) & |\ \dfrac{z}{L'} \le 0 \end{array}\right\} \tag{9}$$

$$\phi_u = \frac{1}{1 - 18R_i^{1/4}} \tag{10}$$

$$R_i = \frac{z}{L'}(1 - 18R_i)^{1/4} \tag{11}$$

The solution of the above equations is an iterative process that converges very rapidly. The convergence criterion (ϵ) for U_* and L' are given below:

$$\epsilon_{U_*} \rightarrow 0.1(cm/\sec) \qquad \text{and} \qquad \epsilon_{L'} \rightarrow 1(cm) \tag{12}$$

The wave growth equations discussed later require the equivalent wind speed at a 10-m elevation under conditions of neutral stability ($\Delta T = 0$). Having solved the equations in the constant stress region for U_*, the required equivalent neutral wind speed U_e may be easily obtained from Equation 5 using (U_*, $z = 10m$, $\Delta T = 0$):

$$U_{e_{1000}} = \frac{U_*}{k}\left[\ln\left(\frac{1000}{z_0}\right) - 0\right] \tag{13}$$

Full Boundary Layer

For cases where the geostrophic winds are known or have been estimated, the similitude equations describing the entire planetary boundary layer are solved. In addition to the relations described above for the constant stress region, the following relationships describe the model from water surface level to the geostrophic level:

$$\ln\frac{|\vec{V}_g|}{f z_0} = A - \ln\frac{U_*}{|\vec{V}_g|} + \sqrt{\frac{k^2|\vec{V}_g|^2}{U_*^2} - B^2} \tag{14}$$

$$\sin\theta = \frac{BU_*}{k|\vec{V}_g|} \tag{15}$$

where

$\vec{V}_g$ = geostrophic wind

f = Coriolis acceleration

A, B = nondimensional functions of stability

$$\left.\begin{aligned} A &= A_0[1 - e^{(0.015\mu)}] \\ B &= B_0 - B_1[1 - e^{(0.03\mu)}] \end{aligned}\right| \quad \mu \le 0 \tag{16}$$

$$\left.\begin{aligned} A &= A_0 - 0.96\sqrt{\mu} + \ln(\mu + 1) \\ B &= B_0 + 0.7\sqrt{\mu} \end{aligned}\right| \quad \mu > 0 \tag{17}$$

μ = dimensionless stability parameter

$$= \frac{kU_*}{fL'} \tag{18}$$

A_0, B_0, B_1 = constants

θ = angle between $\vec{V}_g$ and the surface stress

Equations *14-18* are solved simultaneously together with Equations *5-11* until the convergence of U_*, L', and A is obtained. A slightly different value of ($C_2 = 0.0144/980$) in Equation *7* is used (Dr. C. Linwood Vincent, CERC, personal communication, September 1989). The convergence criteria for the iterative solution to the equations are as follows:

$$\epsilon_{U_*} \rightarrow 0.1(cm/\sec) \quad \text{and} \quad \epsilon_{L'} \rightarrow 1(cm) \quad \text{and} \quad \epsilon_A \rightarrow 0.1 \tag{19}$$

The solution procedure converges very rapidly. As before, Equation *13* is then used to determine the equivalent neutral wind speed at the 10-m elevation using (U_*, $z = 10$ m, $\Delta t = 0$).

Final Adjustments

An additional adjustment is made for situations having relatively short fetch lengths before application of the wave growth equations. For fetch lengths shorter than 16 km, the following reduction is applied:

$$U_e = 0.9 U_e \tag{20}$$

Finally, it is necessary to evaluate the effects of winds of varying duration, t_i, on the wave growth equations. The following expressions are used to adjust the wind speed to a duration of interest:

$$\frac{U_i}{U_{3600}} = 1.277 + 0.296 \tanh\left(0.9 \log \frac{45}{t_i}\right) \quad \Big| \quad (1 < t_i < 3600 \sec) \tag{21}$$

$$\frac{U_i}{U_{3600}} = -0.15 \log t_i + 1.5334 \quad \Big| \quad (3600 < t_i < 36000 \sec) \tag{22}$$

The 1-hr wind speed U_{3600} is first determined (using $t_i = t_{obs}$). The wind speed U_i at the desired duration of interest is then determined by selecting the desired t_i and using the appropriate equation.

WAVE GROWTH

Having estimated the winds above the water surface at a duration of interest, the objective is to provide an estimate of the wave growth caused by the winds. The simple wave growth formulas that follow provide quick estimates for wind-wave growth in deep and shallow water. The open-water expressions correspond to those listed in the SPM (1984) and Vincent (1984). The restricted fetch deepwater expressions can be found in Smith (1991). It should be noted that the drag law (Garratt, 1977) employed differs from that in the SPM. The major assumptions regarding the use of the simplified wave growth expressions include:

- Energy from the presence of other existing wave trains is neglected.
- Relatively short fetch geometries ($F \le 75\,\mathrm{mi}$).
- Relatively constant wind speed ($\Delta U \le 5\,\mathrm{kts}$) and direction ($\Delta\alpha \le 15°$).
- Winds prescribed at the *10-m* elevation ($z = 10\,\mathrm{m}$).
- Neutral stability conditions.
- Fixed value of drag coefficient ($C_D = 0.001$).

The wind adjustment methodology described earlier in this report adjusts the observed wind, U_{obs}, to the 10-m elevation under neutrally stable conditions U_e. Vincent (1984) maintains the wind speed should be adjusted to consider the nonlinear effect on the wind stress creating the waves. The drag law reported by Garratt (1977) is used:

$$\tau = \rho_a C_D U^2 \tag{23}$$

where

ρ = air density

$$C_D = 0.001\,(0.75 + 0.067\,U) \tag{24}$$

The equivalent neutral wind speed, then, is adjusted (or linearized) to a constant drag coefficient ($C_D = 0.001$) before application in the wave growth formulas:

$$U_a = U_e\sqrt{\frac{C_D}{0.001}} \tag{25}$$

Fetch Considerations

The wave growth formulations which follow are segregated into four categories: deep and shallow-water forms for both simple open-water fetches and for more complex, limiting geometries (designated "restricted fetch"). A brief discussion of fetch delineation is useful.

Open-Water Fetches

In open water, wave generation is limited by the dimensions of the subject meteorological event, and fetch widths are of the same order of magnitude as the fetch length. The simplified estimates for wave growth in open water attribute significance to the fetch length (but not width or shape). The wave growth is assumed to occur along the fetch in the direction of the wind.

Restricted Fetches

The more limiting or complex geometries of water bodies such as lakes, rivers, bays, and reservoirs have an impact on wind-wave generation. This restricted fetch methodology applies the concept of wave development in off-wind directions and considers the shape of the basin. The details of the method are reported by Smith (1991), and are based upon a concept reported by Donelan (1980) whereby the wave period (as a function of fetch lengths at off-wind directions) is maximized. For this approach, the radial fetch lengths (as measured from various points along the shoreline of the basin to the point of interest) are used to describe the geometry of the basin. In addition, the wind direction must be specified. Figure 1-1-2 illustrates the relevant geometric data required for the restricted fetch approach.

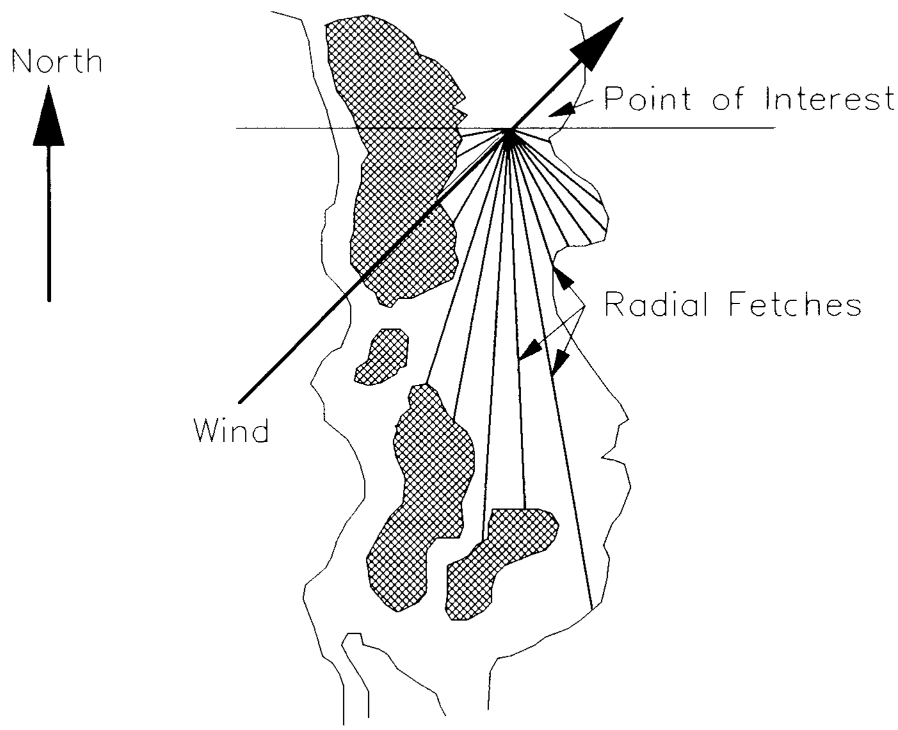

Figure 1-1-2. Restricted Fetch Geometry Data

The conventions used for specifying wind direction and fetch geometry are illustrated in Figure 1-1-3. The approach wind direction (α) as well as the radial fetch angles (β_1), and ($\Delta\beta$) should be specified in a clockwise direction from north from the point of interest where wave growth prediction is required.

From the specified radial fetch data, intermediate values are interpolated at 1-deg increments around the entire 360-deg compass. These interpolated fetches are subsequently averaged over 15-deg arcs centered at each whole 1-deg value.

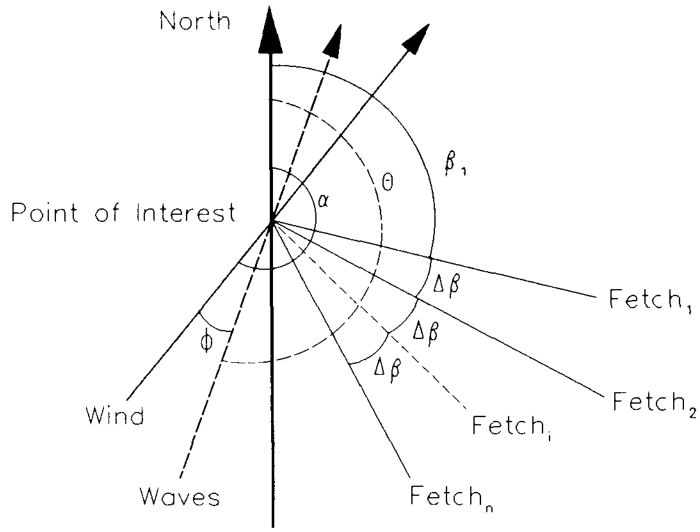

Figure 1-1-3. Restricted Fetch Conventions

The direction of wave development (θ) is solved by maximizing the product

$$F_{\phi}^{0.28} \cdot (\cos\phi)^{0.44} \tag{26}$$

This procedure maximizes the relevant terms in the expression for wave period (T_p) (Equation *36*). The angle (ϕ) is defined as the off-wind direction angle associated with the interpolated averaged fetch length value (F_{ϕ}). Product results (Equation *26*) are evaluated from ($\phi = 0 \pm 90°$) at 1-deg increments. When the product (Equation *26*) is maximized, (ϕ) represents the angle between the wind and waves, and (θ) represents the compass direction from which wave development occurs along (F_{ϕ}). For a specified wind direction, there will be a corresponding wave development direction where (T_p) is maximized by Equation *26*.

Deepwater Wave Growth

The formulas for wave growth in deep water encompass the effects of fetch and duration. The open-water formulas for fetch- and duration-limited wave growth are taken from Vincent (1984) and are based upon the spectrally based results given in Hasselmann et al. (1973, 1976). The fetch-limited and fully developed forms are also tabulated in the SPM (1984). The expressions for restricted fetch wave growth in deep water are from Smith (1991). In all cases, the wave growth estimates are bounded by the expressions for a fully developed equilibrium spectrum. The procedure is outlined as follows:

- Determine the minimum duration, t_{fetch}, required for a wave field to become fetch-limited:

Open Water

$$t_{fetch} = 68.8 \frac{F^{2/3}}{g^{1/3} U_a^{1/3}} \qquad (27)$$

Restricted Fetch

$$t_{fetch} = 51.09 \frac{F^{0.72}}{g^{0.28} \hat{U}_a^{0.44}} \qquad (28)$$

- Determine the character of the wave growth (duration-limited or fetch-limited):

Open Water		Restricted Fetch
$H = 0.0000851 \left(\frac{U_a^2}{g}\right)\left(\frac{g t_i}{U_a}\right)^{5/7}$ (29)	Duration Limited	$H = 0.000103 \left(\frac{\hat{U}_a^2}{g}\right)\left(\frac{g t_i}{\hat{U}_a}\right)^{0.69}$ (30)
$T = 0.0702 \left(\frac{U_a}{g}\right)\left(\frac{g t_i}{U_a}\right)^{0.411}$ (31)	$(t_i < t_{fetch})$	$T = 0.082 \left(\frac{\hat{U}_a}{g}\right)\left(\frac{g t_i}{\hat{U}_a}\right)^{0.39}$ (32)

--- or ---

$H = 0.0016 \left(\frac{U_a^2}{g}\right)\left(\frac{g F}{U_a^2}\right)^{1/2}$ (33)	Fetch Limited	$H = 0.0015 \left(\frac{\hat{U}_a^2}{g}\right)\left(\frac{g F}{\hat{U}_a^2}\right)^{1/2}$ (34)
$T = 0.2857 \left(\frac{U_a}{g}\right)\left(\frac{g F}{U_a^2}\right)^{1/3}$ (35)	$(t_i \geq t_{fetch})$	$T = 0.3704 \left(\frac{\hat{U}_a}{g}\right)\left(\frac{g F}{\hat{U}_a^2}\right)^{0.28}$ (36)

- Determine the "fully developed" condition:

Open Water		Restricted Fetch
$H_{fd} = 0.2433 \left(\frac{U_e^2}{g}\right)$ (37)	Fully Developed	$H_{fd} = 0.2433 \left(\frac{\hat{U}_e^2}{g}\right)$ (38)
$T_{fd} = 8.134 \left(\frac{U_e}{g}\right)$ (39)		$T_{fd} = 8.134 \left(\frac{\hat{U}_e}{g}\right)$ (40)

- Ensure that the "fully developed" condition is not exceeded:

$$H_{m0} = \min(H, H_{fd}) \qquad (41)$$

$$T_p = \min(T, T_{fd}) \qquad (42)$$

where

g = acceleration due to gravity

t_i = wind duration used in duration-limited expressions

F = fetch length used in fetch-limited expressions

$\hat{U}_a = U_a\cos(\phi)$ = fetch-parallel component of U_a for restricted fetch approach

$\hat{U}_e = U_e\cos(\phi)$ = fetch-parallel component of U_e for restricted fetch approach

H = wave height determined by duration-limited or fetch-limited expressions

T = wave period determined by duration-limited or fetch-limited expressions

H_{fd} = wave height limited by fully developed spectrum criteria

T_{fd} = wave period limited by fully developed spectrum criteria

H_{m0} = final wave height determined from spectrally based methods

T_p = final wave period determined from spectrally based methods

Shallow-Water Wave Growth

Estimates for wave growth in shallow water are based upon the fetch-limited deepwater formulas, but modified to include the effects of bottom friction and percolation (Bretschneider and Reid, 1954). Water depth is assumed to be constant over the fetch. Duration-limited effects are not embodied by these formulas. The relationships have not been verified and may (or may not) be appropriate for the conditions and assumptions of the original Bretschneider-Reid work. **The expressions represent an interim method pending results of further research.** The open-water forms are also presented in the SPM (1984).

Open-Water Forms:

$$H_{m0} = \frac{U_a^2}{g} 0.283 \tanh\left[0.530\left(\frac{gd}{U_a^2}\right)^{0.75}\right] \tanh\left\{\frac{\frac{0.0016}{0.283}\left(\frac{gF}{U_a^2}\right)^{0.5}}{\tanh\left[0.530\left(\frac{gd}{U_a^2}\right)^{0.75}\right]}\right\} \qquad (43)$$

$$T_p = \frac{U_a}{g} 7.54 \tanh\left[0.833\left(\frac{gd}{\hat{U}_a^2}\right)^{0.375}\right] \tanh\left\{\frac{\frac{0.2857}{7.54}\left(\frac{gF}{\hat{U}_a^2}\right)^{0.333}}{\tanh\left[0.833\left(\frac{gd}{\hat{U}_a^2}\right)^{0.375}\right]}\right\} \qquad (44)$$

Restricted Fetch Forms:

$$H_{m0} = \frac{U_a^2}{g} 0.283 \tanh\left[0.530\left(\frac{gd}{\hat{U}_a^2}\right)^{0.75}\right] \tanh\left\{\frac{\frac{0.0015}{0.283}\left(\frac{gF}{\hat{U}_a^2}\right)^{0.5}}{\tanh\left[0.530\left(\frac{gd}{\hat{U}_a^2}\right)^{0.75}\right]}\right\} \tag{45}$$

$$T_p = \frac{U_a}{g} 7.54 \tanh\left[0.833\left(\frac{gd}{\hat{U}_a^2}\right)^{0.375}\right] \tanh\left\{\frac{\frac{0.3704}{7.54}\left(\frac{gF}{\hat{U}_a^2}\right)^{0.28}}{\tanh\left[0.833\left(\frac{gd}{\hat{U}_a^2}\right)^{0.375}\right]}\right\} \tag{46}$$

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BETA-RAYLEIGH DISTRIBUTION

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BETA-RAYLEIGH DISTRIBUTION

DESCRIPTION

This application provides a statistical representation for a shallow-water wave height distribution. The Beta-Rayleigh distribution is expressed in familiar wave parameters: H_{mo} (energy-based wave height), T_p (peak spectral wave period), and d (water depth). After constructing the distribution, other statistically based wave height estimates such as H_{rms}, H_{mean}, $H_{1/10}$ can be easily computed. The Beta-Rayleigh distribution features a finite upper bound corresponding to the breaking wave height, and the expression collapses to the Rayleigh distribution in the deepwater limit. The methodology for this portion of the application is taken exclusively from Hughes and Borgman (1987).

INTRODUCTION

Economic coastal engineering design requires accurate specification of the characteristics of the irregular wave field in nearshore waters. In the absence of a fully deterministic understanding of irregular water waves, coastal engineers often describe the sea surface in terms of meaningful wave field parameters which appear to vary in a consistent and predictable manner.

An important and useful statistical descriptor of irregular waves is the wave height distribution. It provides knowledge of the range of wave heights under a given sea condition, as well as the probability of occurrence of a particular wave height within the range. This knowledge can be used to better represent the effects of irregular waves in coastal engineering calculations.

The Rayleigh distribution has proven to be a reliable measure of the wave height distribution for waves in deep water. It is theoretically valid for a wave field composed of a very large number of superimposed deepwater, small-amplitude sinusoidal waves with random phasing and with frequencies very narrowly spread about a single value. For deepwater waves, only the tail in the region of the highest waves exhibits any noticeable deviation from the theory (SPM, 1984, Figure 3-4), and this tendency occurs primarily in the most energetic conditions such as hurricanes (Earle, 1975; Forristall, 1978). Nevertheless, application of the Rayleigh distribution to field wave data has shown repeatedly that the distribution does a remarkable job of predicting the wave height distribution in cases well outside its strict theoretical limits.

GENERAL ASSUMPTIONS AND LIMITATIONS

All wave parameters derived from the Beta-Rayleigh distribution are limited by the data used to derive this parametric formulation. This derivation was based on preliminary examination of surf zone wave height distributions which indicated that the Beta-Rayleigh distribution can provide a reasonable fit to the data. If the input information H_{mo}, T_p, and d do not fall within the range of data used to derive the parametric formulation, inaccuracies may result. Also, the assumption that the maximum individual wave condition can equal the water depth (upper limit of the Beta-Rayleigh distribution) was suggested by Hughes and Borgman (1987), but they concluded it was only a recommendation until further research is conducted. Because the Beta-Rayleigh distribution reverts to the Rayleigh distribution as depth increases, parameters given by this application when $d/gT^2 \geq 0.01$ are calculated from the Rayleigh distribution.

RAYLEIGH DISTRIBUTION

One of the most significant contributions to the parameterization of ocean waves was the demonstration that the wave height distribution of a narrowly banded Gaussian sea state is described by the Rayleigh distribution (Longuet-Higgins, 1952) as

$$p(H) = \frac{2H}{H_{rms}^2} \exp\left[-\left(\frac{H}{H_{rms}} \right)^2 \right] \qquad (1)$$

where:

$p(H)$ = probability density function of wave heights

H = wave height (vertical distance between wave trough and peak)

H_{rms} = root-mean-squared wave height

$$= \left[\frac{1}{N} \sum_{i=1}^{N} H_i^2 \right]^{\frac{1}{2}}$$

The Rayleigh probability distribution also can be written as

$$p(H) = \frac{2H}{8\sigma^2} \exp\left[\frac{-H^2}{8\sigma^2} \right] \qquad (2)$$

where:

σ^2 = variance of the sea surface elevations

Equating Equations *1* and *2* gives

$$H_{rms} = 2\sqrt{2}\,\sigma \qquad (3)$$

The frequently used definition of significant wave height is given by

$$H_{mo} = 4\sigma \qquad (4)$$

The Rayleigh distribution also has been used to describe wave heights in finite depth water with reasonable success if the assumption of a Gaussian sea state is not violated to a great extent. However, as waves approach depth-limited breaking, they become highly nonsinusoidal in shape, they interact with other nonlinear waves (Scheffner, 1986), and the larger waves in the sea state start breaking. Hence, the process becomes very non-Gaussian, and the wave height distributions deviate from the Rayleigh theory. This deviation can become significant during high-energy wave conditions as evidenced in the Atlantic Ocean Remote Sensing Land-Ocean Experiment (ARSLOE) data set (Ochi, Malakar, and Wang, 1982), in data collected during the monsoonal season in India (Dattatri, 1973), and in data from the US coastline (Thompson, 1974). The Rayleigh distribution of wave heights shows a definite weighting toward the higher end of the distribution. For example, Figure 1-2-1 presents two wave height distribution functions measured in shallow water during the DUCK 85 experiment (Ebersole and Hughes, 1987).

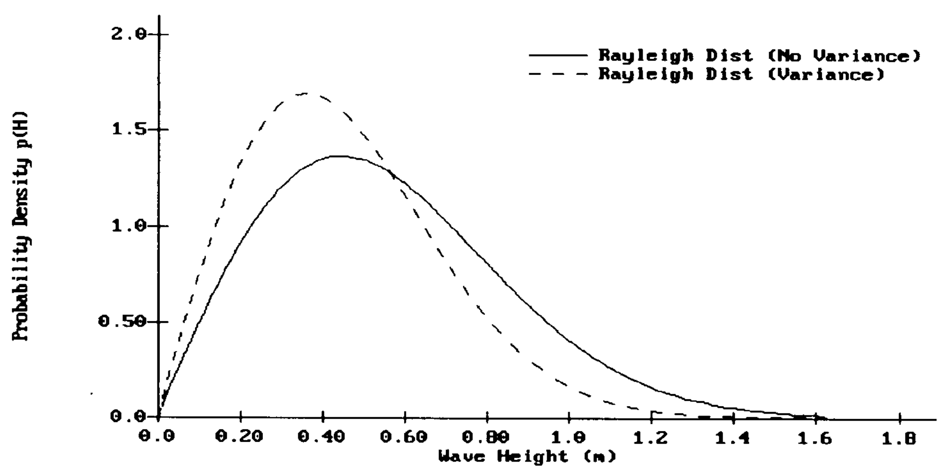

Figure 1-2-1. Rayleigh Distribution Comparison (H_{mo} = 0.72m; T_p = 10.9sec; d= 1.63m)

The solid curve in Figure 1-2-1 is the Rayleigh prediction using the measured H_{rms} (root mean square) and the dashed curve is the Rayleigh distribution using the variance as the governing parameter. Theoretically, the two curves should coincide, but in shallow water the nonlinear shape of the waves causes the statistical measure of the wave heights H_{rms} to be larger than that predicted by Equation *3*. This difference becomes pronounced as the waves approach breaking. From Figure 1-2-1, it is seen that the shallow-water wave height distribution shows a tendency to skew toward the high-wave side of the Rayleigh distribution; that is, there are more of the higher waves than predicted by the Rayleigh theory. Also, using the statistical parameter, H_{rms}, in the Rayleigh distribution provides a better (though still not good) estimation of the observed wave height distributions.

Shallow-Water Distributions

Previous investigators have suggested improved wave height distributions for shallow water and for the surf zone that are either modified Rayleigh distributions or mathematical forms that have the Rayleigh distribution as a deepwater asymptote. Collins (1970) and Battjes (1972) assumed that the Rayleigh distribution was valid up until the point that depth-limited breaking begins to occur. They then truncated the Rayleigh distribution at the breaking wave height and assumed that all broken waves have a height equal to the local breaking wave height. Hence, the truncated probability density is represented by a delta function located at the breaking wave height. This modification becomes pronounced within the surf zone. Kuo and Kuo (1974) truncated the Rayleigh distribution at the breaking wave height and redistributed the truncated probability density over the remaining wave heights. Goda (1975) assumed wave breaking occurs over a small range of wave heights together with a redistribution of the truncated probability density over the smaller wave heights.

The three examples of modified Rayleigh distributions discussed above result in probability density functions (*pdf*) which have a maximum in about the same location as the original Rayleigh distribution; thus, these distributions do not adequately model the characteristic skewing of the shallow-water wave height distribution toward the higher waves as observed in energetic sea conditions.

Beta-Rayleigh Distribution

To better characterize the wave height distribution for shallow-water waves, a *pdf* with the following attributes is desirable:

- The *pdf* should be bounded by a maximum wave height.
- The *pdf* should have the capability to skew toward higher wave heights than predicted by the Rayleigh distribution, as the sea becomes increasingly non-Gaussian.
- The *pdf* should transform into the Rayleigh distribution as the maximum wave height approaches infinity (i.e., deep water).
- The *pdf* should be mathematically tractable and not too complicated.
- The *pdf* should have some physical justification, either from a deterministic or stochastic viewpoint.

In Hughes and Borgman (1987), a plausible candidate for a shallow-water wave height *pdf* that satisfies these five criteria is derived. The resulting shallow-water *pdf* is referred to as the *Beta-Rayleigh distribution*, and it is expressed as a variation of the beta distribution as

$$p_{BR}(H) = \frac{2\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \; \frac{H^{2\alpha-1}}{H_b^{2\alpha}} \left(1 - \frac{H^2}{H_b^2}\right)^{\beta-1} \tag{5}$$

valid in the range $0 < H < H_b$. In Equation 5, H_b denotes the maximum (or breaking) wave height, and α and β are related to the rms wave height by the expression

$$\overline{H^2} = H_{rms}^2 = \int_0^{H_b} H^2 \, p_{BR}(H)\, dH = \frac{\alpha H_b^2}{\alpha + \beta} \tag{6}$$

or

$$\beta = \alpha \left(\frac{H_b^2}{H_{rms}^2} - 1 \right) \tag{7}$$

Other moments of the *pdf* given in Equation *5* also yield relationships between α and β. The first and third moments are complicated by gamma functions, but the fourth moment provides a simple relationship given by

$$\overline{H^4} = H_{rmq}^2 = \int_0^{H_b} H^4 \, p_{BR}(H)\, dh = \frac{\alpha(\alpha + 1) H_b^4}{(\alpha + \beta)(\alpha + \beta + 1)} \tag{8}$$

where:

H_{rmq} = *root-mean-quad* wave height

$$= \left[\frac{1}{N} \sum_{i+1}^{N} H_i^4 \right]^{\frac{1}{2}} \tag{9}$$

Solving Equations *6* and *8* provides expressions for α and β in terms of physical parameters of the wave height distribution.

$$\alpha = \frac{K_1 (K_2 - K_1)}{K_1^2 - K_2} \tag{10}$$

$$\beta = \frac{(1 - K_1)(K_2 - K_1)}{K_1^2 - K_2} \tag{11}$$

where:

$$K_1 = \frac{H_{rms}^2}{H_b^2}$$

$$K_2 = \frac{H_{rmq}^2}{H_b^4}$$

Thus, the *pdf* of Equation *5* can be expressed in terms of three parameters, H_b, H_{rmq}, and H_{rms}. Since this is a three-parameter distribution, Equation *5* will provide a better realization than the Rayleigh *pdf* for shallow-water wave heights, provided the three parameters can be correlated to the sea state.

The transition of Equation *5* to the Rayleigh distribution can be seen by taking the limit, as H_b approaches infinity, of the *pdf* given by Equations *5*, *10*, and *11*. This procedure yields (Hughes and Borgman, 1987) a *generalized Rayleigh distribution* given by

$$p_{GR}(H) = \frac{2H^{2\alpha_0 - 1}}{b_0^{\alpha_0}\ \Gamma(\alpha_0)} \exp\left(\frac{-H^2}{b_0}\right) \tag{12}$$

where:

$$\alpha_0 = \frac{H_{rms}^4}{H_{rmq}^2 - H_{rms}^4} \tag{13}$$

$$b_0 = \frac{H_{rmq}^2 - H_{rms}^4}{H_{rms}^2} \tag{14}$$

By noting that $H_{rmq} = \sqrt{2}\, H_{rms}^2$ for the Rayleigh distribution, Equation *12* reverts to the Rayleigh form given by Equation *1*.

PARAMETERIZATION

The most often used parameters for describing sea states in shallow water are the energy-based significant wave height H_{mo}, the peak spectral period T_p, and the water depth d. Less often used parameters include the width of the spectral peak, some indicators of wave groupiness, and the skewness and kurtosis of sea surface elevations.

Root-Mean-Square Wave Height

Following Thompson and Vincent (1985), the deviation of the ratio H_{rms}/H_{mo} as a function of relative depth, d/gT_p^2, was examined. For the deepwater Rayleigh case, Equations *3* and *4* can be combined to give

$$\frac{H_{rms}}{H_{mo}} = \frac{1}{\sqrt{2}} = 0.707 \tag{15}$$

However, data obtained from the field (Hughes and Ebersole, 1987) and from laboratory tests indicate that this ratio increases as relative depth decreases. A fit to the data is suggested by the following equation:

$$\frac{H_{rms}}{H_{mo}} = \frac{1}{\sqrt{2}} \exp\left[a \left(\frac{d}{g T_p^2} \right)^{-b} \right] \tag{16}$$

where:

a, b = fitting parameters

g = gravitational acceleration

Linear regression of Equation *16* to laboratory and field data yields values of:

$$a = 0.00089 \qquad \text{and} \qquad b = 0.834$$

with a correlation coefficient of $r = 0.848$. Applying linear regression to Equation *16* weights the lower values of H_{rms}/H_{mo} more than the higher values. This procedure tends to force the fit through the data at the larger values of relative depth, which serves as a good control in view of the observed scatter at smaller values of relative depth. A reasonable upper envelope to the data is given by Equation *16* when $a = 0.00136$ and b remains as before.

Root-Mean-Quad Wave Height

As previously mentioned, the Rayleigh distribution has a theoretical value of

$$H_{rmq} = \sqrt{2}\, H_{rms}^2 \tag{17}$$

Combining Equations *15* and *17* provides a deepwater Rayleigh limit for the ratio of H_{rmq} to H_{rms}^2,

$$\frac{H_{rmq}}{H_{mo}^2} = \frac{1}{\sqrt{2}} = 0.707 \tag{18}$$

Plotting this ratio based on laboratory and field data showed a strong deviation from its theoretical value of 0.707 as relative depth decreased. This deviation was most likely caused by the increasing nonlinearity of the waves as they move into shallow water. As the wave crests peak up and the troughs flatten out, individual wave heights increase without a corresponding increase in sea surface variance, as represented by H_{mo}.

The quantity H_{rmq} is sensitive to the accurate determination of wave height because it is calculated from wave heights raised to the fourth power. This sensitivity might account for some of the scatter found in the data; however, it is entirely possible that H_{rmq} depends on more than the three parameters investigated here. Surprisingly, the scatter was not reduced by including skewness of sea surface elevations in the parameterization.

Linear regression of the curve

$$\frac{H_{rmq}}{H_{mo}^2} = \frac{1}{\sqrt{2}} \exp\left[a \left(\frac{d}{gT_p^2} \right)^{-b} \right] \qquad (19)$$

resulted in a fit to the data with

$$a = 0.000098 \qquad \text{and} \qquad b = 1.208$$

and a correlation coefficient of $r = 0.7863$. Here, the weighting has resulted in a fit that will tend to underpredict H_{rmq} at lower values of relative depth. A reasonable upper envelope to the data is given by Equation *19* when $a = 0.00023$ and b remains as before.

Breaking Wave Height

There are numerous methods for determining a breaking wave height for use in the Beta-Rayleigh distribution. Because the standard predictive expression of

$$H_b = 0.78d \qquad (20)$$

appears to underestimate the breaking wave height, as observed in the photopole data (Ebersole and Hughes, 1987), a representative breaking wave height is suggested of simply

$$H_b = 0.9d \qquad (21)$$

To investigate the effect of the choice for breaking wave height on the Beta-Rayleigh distribution, comparisons were made graphically (Hughes and Borgman, 1987), using Equations *20* and *21*. The differences between Equations *20* and *21* were minimal at the lower values of H_{mo}/d, but a distinct deviation occurred at the higher values of H_{mo}/d. Until further research is conducted, it is recommended that H_b be taken as equal to the depth, based on the field data obtained from Ebersole and Hughes (1987) and as expressed by Equation *21*.

APPLICATION

Application of the Beta-Rayleigh distribution requires specification of the water depth, d; the energy-based significant wave height, H_{mo}; and the peak spectral period, T_p. These parameters are used in Equations *16* and *19* to determine H_{rms} and H_{rmq}, respectively. Next, α and β can be computed by Equations *10* and *11* using a value of H_b equal to the depth (Equation *21*). Finally, the Beta-Rayleigh *pdf* is determined from Equation *5*.

The relative improvement of this method over the Rayleigh distribution is illustrated in Figure 1-2-2, where a prebreaking wave height distribution histogram from the photopole experiment is plotted together with the Rayleigh prediction based on H_{mo} (dashed line) and the Beta-Rayleigh prediction (solid line). In this example (typical of the other cases studied), the Beta-Rayleigh *pdf* provides a better estimate, as somewhat expected since these data are part of the set used in the parameterization. The Rayleigh prediction to the data based on H_{rms} has already been presented in Figure 1-2-1.

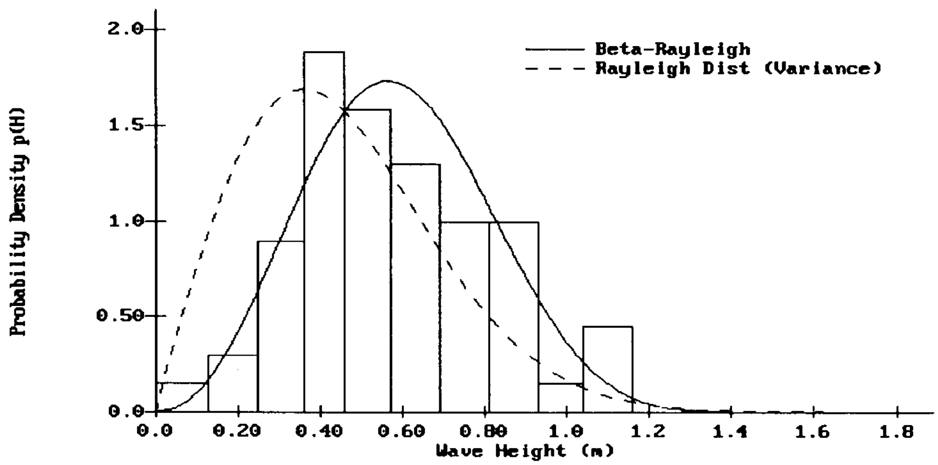

Figure 1-2-2. Beta-Rayleigh Predictions (H_{mo} = 0.72m; T_p = 10.9sec; d= 1.63m)

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EXTREMAL SIGNIFICANT WAVE HEIGHT ANALYSIS

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EXTREMAL SIGNIFICANT WAVE HEIGHT ANALYSIS

DESCRIPTION

This application provides significant wave height estimates for various return periods. Confidence intervals are also provided. The approach developed by Goda (1988) is used to fit five candidate probability distributions to an input array of extreme significant wave heights. Candidate distribution functions are Fisher-Tippett Type I and Weibull with exponents ranging from 0.75 to 2.0. Goodness-of-fit information is provided for identifying the distributions which best match the input data.

GENERAL ASSUMPTIONS AND LIMITATIONS

General assumptions used in this approach are:

- All extreme wave heights come from a single statistical population of storm events. For example, the heights might represent only extratropical storms in the Northern Hemisphere at a site.
- Wave height properties for an event are reasonably represented by the significant height.
- Extreme wave heights are not limited by any physical factors such as shallow-water depth.

DATA FOR EXTREMAL ANALYSIS

The input array of significant wave heights for extremal analysis is extracted from a long-term data source of measurements, hindcasts, or observations. The reliability of predicted extremes is directly related to the accuracy of available data and the number of years of record. Therefore, the longest high-quality data source available should be used. As a general rule-of-thumb, heights can be extrapolated to return periods up to 3 times the length of record, K. Thus, 20 years of data can be used to estimate significant heights up to a 60-year return period. If return periods longer than $3K$ are requested, a warning message is generated.

Each significant height typically represents the maximum from a storm event. The array of significant heights is referred to as a partial duration series. Often only the more severe storms, with significant height above some minimum value, are considered. The total number, N_T, of events from the population during the length of record must be estimated. The parameter N_T can only be approximated, but results are fairly insensitive to the precise value. It is advisable to consider an average of at least one event per year in the partial duration series. If the average number of events per year, N_T/K, is less than one, a warning message is generated.

Another approach is to use only the highest significant height from each year to form an annual maximum series. The partial duration series is usually preferable for waves. Most sources cover relatively short time periods. The additional information available from a partial duration series can be helpful in increasing confidence in the extremal estimates and extrapolating to rare events.

EXTREMAL DISTRIBUTIONS

Probability Distribution Functions

There is no strong physical, theoretical, or empirical evidence for selecting a particular *pdf* for extreme wave heights. The approach commonly used is to try several candidate distributions with each data set and select the one that fits best. The candidate distributions suggested by Goda (1988) and used in this application are as follows:

- Fisher-Tippett Type I (FT-I) Distribution:

$$F(H_s \le \hat{H}_s) = e^{-e^{-\left(\frac{\hat{H}_s - B}{A}\right)}} \tag{1}$$

- Weibull Distribution:

$$F(H_s \le \hat{H}_s) = 1 - e^{-\left(\frac{\hat{H}_s - B}{A}\right)^k} \tag{2}$$

where:

$F(H_s \le \hat{H}_s)$ = probability of $\hat{H}_s$ not being exceeded

H_s = significant wave height

$\hat{H}_s$ = particular value of significant wave height

B = location parameter

A = scale parameter

k = shape parameter

The input data are ranked in descending order of significant height. A probability, or *plotting position*, is assigned to each height as follows (Goda 1988 based on previous work by Gringorten, 1963; Muir and El-Shaarawi, 1986; and Petrauskas and Aagaard, 1970):

$$F(H_s \le H_{sm}) = 1 - \frac{m - 0.44}{N_T + 0.12} \quad ; \quad FT-I$$

$$F(H_s \le H_{sm}) = 1 - \frac{m - 0.20 - \frac{0.27}{\sqrt{k}}}{N_T + 0.20 + \frac{0.23}{\sqrt{k}}} \quad ; \quad Weibull \tag{3}$$

where:

$F(H_s \le H_{sm})$ = probability of the mth significant height not being exceeded

H_{sm} = mth value in the ranked significant heights

m = rank of a significant height value

= 1,2,...,N

N_T = total number of events during the length of record (which may exceed the number of input significant heights)

The parameters A and B in Equations *1* and *2* are estimated by computing a least squares fit of the five candidate distribution functions to the data. Computations are based on linear regression analysis of the relationship

$$H_{sm} = \hat{A} y_m + \hat{B} \quad , \quad m = 1, 2, \ldots, N \tag{4}$$

where:

$$y_m = -\ln[-\ln F(H_s \le H_{sm})] \quad , \quad FT-I$$

$$y_m = \{-\ln[1 - F(H_s \le H_{sm})]\}^{\frac{1}{k}} \quad , \quad Weibull \tag{5}$$

$\hat{A}$ and $\hat{B}$ = estimates of the scale and location parameters from linear regression analysis

Return Period

Return period is defined as the average time interval between successive events of an extreme significant wave height being equalled or exceeded. For example, the 25-year significant height can be expected to be equaled or exceeded an average of once every 25 years. Significant heights for various return periods are calculated from the probability distribution functions by the following equations:

$$H_{sr} = \hat{A} y_r + \hat{B} \tag{6}$$

where:

H_{sr} = significant wave height with return period T_r

$$y_r = -\ln\left[-\ln\left(1 - \frac{1}{\lambda T_r}\right)\right] \quad , \quad FT-I$$

$$y_r = [\ln(\lambda T_r)]^{\frac{1}{k}} \quad , \quad Weibull \tag{7}$$

λ = average number of events per year

$$= \frac{N_T}{K}$$

T_r = return period (years)

K = length of record (years)

Confidence Intervals

Estimation of confidence intervals is an essential part of extremal wave analysis. Typically the period of record is short, and the level of uncertainty in extremal estimates with long return periods is high. Confidence intervals give a quantitative indicator of the level of uncertainty in estimated extremal wave heights. The approach of Gumbel (1958) and Goda (1988) for estimating standard deviation of return value when the true distribution is known is used. The normalized standard deviation is calculated by

$$\sigma_{nr} = \frac{1}{\sqrt{N}}\left[\ 1.0 + a(y_r - c + \epsilon \ \ \ln \nu)^2\ \right]^{\frac{1}{2}} \tag{8}$$

where:

σ_{nr} = normalized standard deviation of significant wave height with return period r

N = number of input significant heights

$$a = a_1 e^{a_2 N^{-1.3} + \kappa\sqrt{-\ln \nu}} \tag{9}$$

$a_1, a_2, c, \epsilon, \kappa$ = empirical coefficients (Table 1-3-1)

ν = censoring parameter

$$= \frac{N}{N_T}$$

Table 1-3-1 Coefficients of Empirical Standard Deviation Formula for Extreme Significant Height (Goda, 1988)					
Distribution	a_1	a_2	κ	c	ϵ
FT-I	0.64	9.0	0.93	0.0	1.33
Weibull (k=0.75)	1.65	11.4	-0.63	0.0	1.15
Weibull (k=1.0)	1.92	11.4	0.00	0.3	0.90
Weibull (k=1.4)	2.05	11.4	0.69	0.4	0.72
Weibull (k=2.0)	2.24	11.4	1.34	0.5	0.54

The absolute magnitude of the standard deviation of significant wave height is calculated by

$$\sigma_r = \sigma_{nr} \, \sigma_{H_s} \tag{10}$$

where:

σ_r = standard error of significant wave height with return period r

σ_{H_s} = standard deviation of input significant heights

Confidence intervals are calculated by assuming that significant height estimates at any particular return period are normally distributed about the assumed distribution function. Factors by which to multiply the standard error (Equation *10*) to get bounds with various levels of confidence are given in Table 1-3-2. It is important to note that the width of confidence intervals depends on the distribution function, N, and ν; but it is not related to how well the data fit the distribution function.

Table 1-3-2 Confidence Interval Bounds For Extreme Significant Height		
Confidence Level (%)	Confidence Interval Bounds Around H_{sr}	Probability of Exceeding Upper Bound (%)
80	$\pm 1.28\sigma_r$	10.0
85	$\pm 1.44\sigma_r$	7.5
90	$\pm 1.65\sigma_r$	5.0
95	$\pm 1.96\sigma_r$	2.5
99	$\pm 2.58\sigma_r$	0.5

Probability of Wave Occurrence

The probability of wave occurrence is defined as the probability that a height with given return period will be equaled or exceeded during some time period. An example is the probability that the 25-year wave height will occur during a 10-year period. The probability of wave occurrence, expressed as a percent chance of occurrence, is calculated as (Headquarters, Department of the Army, 1989)

$$P_e = 100\left[1 - \left(1 - \frac{1}{T_r}\right)^L\right] \tag{11}$$

where:

P_e = percent chance of occurrence

L = time period of concern (years)

Selection of a Distribution Function

The five distribution functions considered in this analysis are sufficiently different that only one or two can be expected to provide a good fit to any particular data set. Two statistics are provided to assist in selecting the best fit distribution function. The correlation between variables in the linear Equation *4* is the primary selection criterion. The distribution function that gives the highest correlation should be selected. The sum of the squares of residuals,

$$\sum_{m=1}^{N} [H_{sm} - (\hat{A} y_m + \hat{B})]^2 \tag{12}$$

is also provided. The sum is usually smallest for the distribution function with the highest correlation. Plots are also available to help judge the fit between data and distribution functions and the width of confidence intervals. If a second distribution function fits nearly as well as the best fit (i.e., the correlation is nearly as high and the sum of the squares of residuals is comparable), then it would be appropriate to consider extremal heights from both distributions. Extreme heights from the two distribution functions could be averaged together. Alternatively, the higher of the two could be used if a conservative estimate is desired.

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CONSTITUENT TIDE RECORD GENERATION

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Constituent Tide Record Generation

Description

This ACES application predicts a tide elevation record at a specific time and locale using known amplitudes and epochs for individual harmonic constituents.

Introduction

Tides are periodic in nature and are caused by the gravitational attraction of the sun and moon acting upon the rotating earth. The irregular distribution of land and water upon the planet, as well as the effects of friction, inertia, and the interactions of superimposed standing waves and progressive waves tend to complicate actual tidal motions induced by the gravitational forces. Around the year 1867, Lord Kelvin devised the method of reducing tides into constituents using harmonic analysis (Schureman, 1971). Harmonic analysis (as applied to tides) is the process by which observed tidal data at a location are reduced into a number of harmonic constituents. The quantities produced by such an analysis are known as harmonic constants and consist of amplitudes and phase relationships for each constituent. Harmonic prediction (the subject of this application) is accomplished by re-assembling the individual constituents in accordance with the appropriate astronomical relationships during the prediction period.

Harmonic tide prediction is a convenient tool for scheduling construction or maintenance work that may be affected by spring or neap tide conditions. It is also a useful tool in the application and verification of certain types of physical and hydrodynamic models where tide elevations are required either as boundary conditions or as stations for calibrating model parameters and verifying model performance.

General Assumptions and Limitations

The method is based upon the assumption that the tide elevation at a location can be expressed as a series of harmonic terms which are in turn dependent upon the gage location as well as the astronomical conditions during the prediction period. An individual harmonic term is represented as a simple cosine function consisting of an amplitude and phase relationship:

$$h_n = A_n \cos(a_n t + \alpha_n) \tag{1}$$

Equation *1* expresses the contribution of an individual constituent (n) to the complete tide elevation. The expression contains an amplitude (A_n) and a phase relationship (the argument of the cosine function). The phase relationship is a function of the speed of the constituent (a_n), the time (t) measured from some initial epoch, and (α_n), which represents the initial phase of the constituent at the initial epoch ($t = 0$).

An important assumption in applying this methodology is that the observed record (from which the harmonic constants were derived) was of sufficient length, resolution, and quality to resolve the harmonic data adequately. Data from site-specific studies are usually short term in duration. Permanent recording stations provide longer and often more reliable records. Typical records chosen for analysis consist of hourly data analyzed in series lengths of 29 and 369 days. The latter series length is considered highly desirable for resolving most of the short period constituents and eliminating seasonal effects. Harmonic constants are often available from site-specific studies and from the National Ocean Survey.

The accuracy of the tide prediction is dependent upon the number of constituents included in the equation. In the United States, a maximum of 37 constituents (listed in Table A-5, Appendix A) are typically considered. However, in some areas, more constituents are included because of regional complexities. In many cases, a small number of constituents will represent the significant portion of the tide. For example, tides along the eastern coast of the United States are dominated by the M_2 constituent. Care should be taken to examine the amplitudes of the individual constituents in order to select those having the most significant influence on the amplitude and phase of the tide.

THE TIDE PREDICTION EQUATION

The general equation for the height of the tide at any time t can be determined from the formula:

$$h = H_0 + \sum_{n=1}^{N} f_n H_n \cos[a_n t + (V_0 + u)_n - \kappa_n] \tag{2}$$

where

h = height of tide at time t

H_0 = mean height of water level above datum used for prediction

N = number of constituents to be considered

f_n = node factor of constituent n

H_n = amplitude of constituent n

a_n = speed of constituent n

t = time reckoned from some initial epoch

$(V_0 + u)_n$ = value of local equilibrium argument of constituent n at the initial epoch ($t = 0$)

κ_n = value of local phase lag or epoch of constituent n

In Equation 2, all quantities except h and t may be considered constant for a particular time period and location. Values of H_n and κ_n are the harmonic constants derived from analysis of past records. They must be specified (as input) for each of the constituents to be considered in the summation. The methodology implemented in ACES has algorithms for determining f_n, and $(V_0 + u)_n$ for each constituent as functions of the date and time of the initial epoch $(t = 0)$, the total record length, and the longitude of the gage record. Essentially, they are determined by various combinations of astronomical functions (which are functions of time). Additionally, the equilibrium terms are also a function of longitude. In this application, their values are determined at Greenwich and translated to the gage longitude specified. The details of the computations are well described in Schureman (1971).

It should be noted that the results of some harmonic analysis are reported yielding a value of (κ'_n) which is a modified epoch (relative to Greenwich). In these cases the value of longitude at Greenwich $(= 0)$ should be specified when using (κ'_n).

References and Bibliography

Harris, D. L. 1981. "Tides and Tidal Datums in the United States," Special Report SR-7, US Army Engineer Waterways Experiment Station, Vicksburg, MS.

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Schureman, P. 1971 (reprinted). "Manual of Harmonic Analysis and Prediction of Tides," Coast and Geodetic Survey Special Publication No. 98, Revised (1940) Edition, US Government Printing Office, Washington, DC.