LINEAR THEORY WITH SNELL'S LAW

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Linear Wave Theory with Snell's Law

Description

This application provides a simple estimate for the transformation of monochromatic waves. It considers two common processes of wave transformation: refraction (using Snell's law) and shoaling using wave properties predicted by linear wave theory (Airy, 1845). Given wave properties and a crest angle at a known depth, this application predicts the values in deep water and at a subject location specified by a new water depth. An important assumption is that all depth contours are assumed to be straight and parallel. In addition to the wave transformation results, this application reports common bulk wave properties from linear wave theory. More detailed discussion of these methods can be found in the SPM (1984), Dean and Dalrymple (1984), Sarpkaya and Isaacson (1981), as well the section of this reference manual entitled **Linear Wave Theory**.

Introduction

In deep water, waves often referred to as ocean swell have a profile that is very nearly sinusoidal, with long, low crests. As the waves propagate into shallow water, they undergo a transformation, starting where the waves are affected by the seabed at a depth approximately one-half of the deepwater wavelength. The wave velocity, height, and length alter. This process is called wave shoaling.

When waves travel at an angle to underwater contours, the portion of the wave in deeper water is moving faster than the part in shallower water. This variation causes the wave crest to bend toward alignment with the contours. This process is called wave refraction.

General Assumptions and Limitations

Important assumptions and limitations made in this shoaling and refraction discussion are:

- Wave energy between wave orthogonals remains constant. A wave orthogonal is a locus of points that define the minimum time of travel for wave propagation between two points (Le Mehaute, 1976). The wave orthogonals (Figure 3-1-1) are drawn perpendicular to the wave crest.
- Direction of wave advance is perpendicular to the wave crest.
- Speed of a wave of a given period at a particular location depends only on the depth at that location.
- Changes in bottom topography are gradual.
- Waves are based on small-amplitude wave theory.
- Effects of current, winds, and reflections from beaches and underwater topographic variations are ignored.
- Offshore contours are straight and parallel to the shoreline.

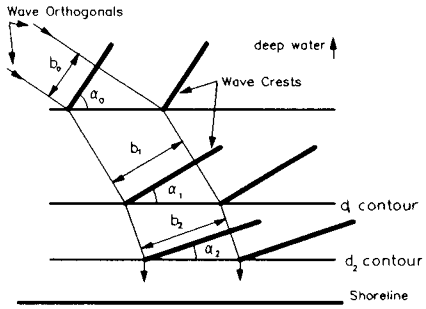

Figure 3-1-1. Snell's Law and Wave Refraction

SHOALING AND REFRACTION COEFFICIENTS

It has been observed that the decrease in wave celerity with decreasing water depth is analogous to the decrease in the speed of light with an increase in the refractive index of the transmitting medium. Using this analogy, O'Brien (1942) suggested the use of Snell's law of geometrical optics for addressing the problem of water-wave refraction by changes in depth.

Using deep water as a reference, the general form of Snell's law is:

$$\frac{c}{c_0} = \frac{\sin\alpha}{\sin\alpha_0} \tag{1}$$

where

c, c_0 = wave velocity at the depth contour

α, α_0 = angle between a wave crest and the depth contour

As stated earlier, the rate of energy transfer between wave orthogonals is assumed to remain constant and is given by average energy flux:

$$\overline{P}_0 = \overline{P} \tag{2}$$

where

Deep Water	*Shallow Water*	*Item*
$\overline{P}_o = \overline{E}_o C_{g0}$	$\overline{P} = \overline{E} C_g$	Energy flux
b_o	b	Distance between orthogonals
$\overline{E}_0 = \frac{\rho g H_0^2}{8}$	$\overline{E} = \frac{\rho g H^2}{8}$	Average energy density
C_{go}	C_g	Group velocity

Rearranging terms and solving for H/H_0 yields:

$$\frac{H}{H_o} = \sqrt{\frac{C_{go}}{C_g}} \sqrt{\left(\frac{b_o}{b}\right)} \tag{3}$$

The term $\sqrt{C_{g0}/C_g}$ is known as the *shoaling coefficient* K_S, and the term $\sqrt{b_o/b}$ is known as the *refraction coefficient* K_R.

The assumption of straight and parallel depth contours leads to a simple geometrical relationship between b and α and a resulting expression for K_R:

$$K_R = \sqrt{\frac{b_o}{b}} = \sqrt{\frac{\cos\alpha_o}{\cos\alpha}} \tag{4}$$

A final expression for this simplified wave transformation approach is then:

$$\frac{H}{H_0} = K_R K_S \tag{5}$$

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IRREGULAR WAVE TRANSFORMATION (GODA'S METHOD)

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Irregular Wave Transformation (Goda's method)

Description

This application yields cumulative probability distributions of wave heights as a field of irregular waves propagate from deep water through the surf zone. The application is based on two random-wave theories by Yoshimi Goda (1975 and 1984). The 1975 paper concerns transformation of random waves shoaling over a plane bottom with straight parallel contours. This analysis treated breaking and broken waves and resulted in cumulative probability distributions for wave heights given a water depth. It did not include refraction, however. The 1984 article details a refraction procedure for random waves propagating over a plane bottom with straight parallel contours assuming a particular incident spectrum. This ACES application combines the two approaches by treating directional random waves propagating over a plane bottom with straight parallel contours. This application also uses the theory of Shuto (1974) for the shoaling calculation. The theories assume a Rayleigh distribution of wave heights in the nearshore zone and a Bretschneider-Mitsuyasu incident directional spectrum. The processes modeled include:

- Wave refraction
- Wave shoaling
- Wave breaking
- Wave setup
- Surf beat

General Assumptions and Limitations

General assumptions and limitations in this irregular wave transformation discussion are:

- The incident wave spectrum is of the Bretschnieder-Mitsuyasu type.
- The incident wave height distribution is of the Rayleigh type.
- Waves propagate over a smooth, absorbent beach (no reflection) with straight, parallel contours.
- Irregular wave shoaling may be approximated by shoaling of monochromatic waves.
- The probability distribution of broken wave heights is proportional to that of unbroken wave heights.
- An empirical relation determines the surf beat level.

These assumptions dictate the following theoretical limitations:

- The Bretschnieder-Mitsuyasu wave spectrum is a narrow-banded spectrum. Thus, this application should not be used where broad-banded spectra or multiple-peaked spectra are present.

- This application should not be used in areas with complex bathymetry; contours should be roughly parallel to shore.

In addition, the following limitations were implemented:

- Peak period must not exceed 16 sec.
- The smallest depth of interest that can be represented is 10 ft or 3.04 m.
- Principal direction of incidence must not exceed 75 deg from shore normal.

THEORY

The development of the theory begins by considering the incident Bretschnieder-Mitsuyasu spectrum:

$$S(f) = 0.257 \quad (H_{1/3})^2 \quad T_{1/3} \quad (T_{1/3} f)^{-5} \quad e^{\left[-1.03(T_{1/3} \ f)^{-4}\right]} \tag{1}$$

where

$S(f)$ = spectral density (m^2 sec)

$H_{1/3}$ = significant wave height (m)

$T_{1/3}$ = significant wave period (sec)

f = wave frequency (rad/sec)

This incident wave spectrum is assumed to propagate over straight parallel bathymetric contours at a principal direction to shore normal. The spectrum is also assumed to have a directional spread of 135 deg, 67.5 deg to either side of the principal direction. The spreading function $G(f, \theta)$ used in this application is that of Mitsuyasu (1975):

$$G(f, \theta) = G_0 \quad \cos^{2s}\left(\frac{\theta}{2}\right) \tag{2}$$

where

θ = angular deviation from principal direction

$$G_0 = \frac{2^{2s-1}}{\pi} \ \frac{\Gamma^2(s+1)}{\Gamma(2s+1)} \tag{3}$$

s = parameter representing directional energy concentration around a peak

s_{max} = peak value of s

= 10 for wind waves

= 25 for steep swell

= 75 for flat swell

Γ = Gamma function

The effective refraction coefficient for the spectrum is defined as the *average* refraction coefficient for the entire spectrum. The *directional* Bretschnieder-Mitsuyasu spectrum is defined as (Goda, 1984):

$$S(f,\theta) = S(f) \quad G(f,\theta) \tag{4}$$

From this an expression for the effective refraction coefficient $(K_r)_{eff}$ is given as:

$$(K_r)_{eff} = \left[\frac{1}{m} \int_0^{\infty} \int_{\theta_{min}}^{\theta_{max}} S(f,\theta) \quad K_s^2(f) \quad K_r^2(f,\theta) \quad d\theta \quad df \right]^{1/2} \tag{5}$$

where

$$m = \int_0^{\infty} \int_{\theta_{min}}^{\theta_{max}} S(f,\theta) \quad K_s^2(f) \quad d\theta \quad df \tag{6}$$

$K_s(f)$ = individual shoaling coefficient

$K_r(f,\theta)$ = individual refraction coefficient

As the incident wave spectrum propagates into shallow water, individual waves within the spectrum refract, shoal, and break at rates dependent upon their individual heights, periods, and directions. Since these processes would occur at different rates (due to the irregularity of the wave field), any wave height prediction model would also have to be irregular. One way to describe the wave field during transformation and still retain irregularity is by using a probability distribution of wave heights. In this application, Goda's irregular wave height distribution model (1975) is used.

In the absence of wave breaking and assuming a Rayleigh distribution of wave heights, a probability density function of wave height normalized by H_0' is given by:

$$P_0(x) = 2a^2 x \quad e^{\left(-\frac{a^2}{x^2}\right)} \tag{7}$$

where

$P_0(x)$ = probability density function

$x = \frac{H}{H_0'}$ = normalized wave height

$$\alpha = \frac{1.416}{K_s}$$

K_s = shoaling coefficient

The Rayleigh distribution does not have an upper limit on the normalized wave height; it approaches zero asymptotically. However, normalized wave height in nature is limited by wave breaking. This is the primary difference between Goda's model and the ideal Rayleigh model.

If wave breaking is assumed to occur between x_2 and x_1, the unbroken wave heights are restricted in Equation *7* by the following equations:

$$P_r(x) = \left| \begin{array}{ll} P_0(x) & x \le x_2 \\ P_0(x) - \left(\frac{x - x_2}{x_1 - x_2}\right) P_0(x_1) & x_2 < x \le x_1 \\ 0 & x_1 \le x \end{array} \right| \qquad (8)$$

It is assumed that broken waves carry a small amount of energy. This assumption is accounted for by redistributing the energy along the top of the distribution. This redistribution leads to Goda's form of the probability density function for irregular waves:

$$P(x) = \alpha \quad P_r(x) \qquad (9)$$

where

$$\frac{1}{\alpha} = \int_0^{x_l} P_r(x) \quad dx = 1 - [1 + a^2 x_1 (x_1 - x_2)] \quad e^{\left(-a^2 x_1^2\right)} \qquad (10)$$

NOTE: $P_r(x)$ is restricted by Equation *8* and a is defined in Equation *7*.

At this point, essentially two different theories are represented in the method: Goda's refraction algorithm (Equations *1-6*) and Goda's irregular wave height distribution model (Equations *7-10*). The wave parameter in common with the two theories is the wave height, more specifically, the breaking wave height. Goda (1975) gave the following expression for normalized breaking wave height based on wave shoaling only:

$$X_b = \frac{H_b}{H_0'} = 0.17 \frac{L_o}{H_0'} \left\{ 1 - e^{\left[\frac{-1.5\pi h}{H_0'} \frac{H_0'}{L_0} \left(1 + 15 \tan^{\frac{4}{3}} n\right) \right]} \right\} \qquad (11)$$

where

H_b = breaking wave height

H'_0 = equivalent deepwater significant wave height

L_0 = deepwater wavelength

h = water depth of interest

n = bottom slope

By multiplying Equation *11* by $(K_r)_{eff}$, Goda's irregular wave distribution model is extended to include refraction effects. Shoaling, surf beat, and wave setup are also included in the application and are used to determine water depth and wave height for input into Equation *11*. The equations for surf beat, wave setup, and wave shoaling follow:

Surf beat

$$\xi_{rms} = \frac{0.01 H'_0}{\sqrt{\frac{H'_0}{L_0}\left(1 + \frac{h}{H'_0}\right)}} \tag{12}$$

Wave setup

$$\frac{d\bar{\eta}}{dx} = \frac{-1}{(\bar{\eta} + h)} \frac{d}{dx}\left[\frac{1}{8} H^2_{rms}\left(\frac{1}{2} + \frac{2kh}{\sinh 2kh}\right)\right] \tag{13}$$

where

$\frac{d\bar{\eta}}{dx}$ = set-up gradient

x = offshore-onshore coordinate

$\bar{\eta}$ = wave setup

k = wave number

Wave shoaling (Shuto, 1974)

$$\left.\begin{array}{ll} 0 < \dfrac{gH(T_{1/3})^2}{h^2} \le 30 : & \text{use linear wave theory} \\[2ex] 30 < \dfrac{gH(T_{1/3})^2}{h^2} \le 50 : & Hh^{\frac{2}{7}} = constant \\[2ex] 50 < \dfrac{gH(T_{1/3})^2}{h^2} < \infty : & Hh^{\frac{5}{2}}\left(\sqrt{\dfrac{gH(T_{1/3})^2}{h^2}} - 2\sqrt{3}\right) = constant \end{array}\right| \qquad (14)$$

where

g = gravitational acceleration

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Combined Diffraction and Reflection by a Vertical Wedge

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COMBINED DIFFRACTION AND REFLECTION BY A VERTICAL WEDGE

DESCRIPTION

This application estimates wave height modification due to combined diffraction and reflection near jettied harbor entrances, quay walls, and other such structures. Jetties and breakwaters are approximated as a single straight, semi-infinite breakwater by setting the wedge angle to zero. Corners of docks and quay walls may be represented by setting the wedge angle equal to 90 deg. Additionally, such natural diffracting and reflecting obstacles as rocky headlands can be approximated by setting a particular value for the wedge angle.

INTRODUCTION

In coastal and ocean engineering practice, it is often important to be able to determine wave height near such coastal structures as jetties, breakwaters, platforms, and docks. Such information aids engineers in evaluating coastal structure designs, especially in the areas of energy transmission, sediment transport, and structural strength. Wave heights in the vicinity of a structure have traditionally been presented in the form of dimensionless wave diffraction and reflection coefficients defined as the ratio of diffracted/reflected wave height to incident wave height. Early studies presented a dimensionless graphical solution based on theory by Penny and Price (1952) for diffraction by a semi-infinite breakwater (Wiegel, 1962). Subsequently these diagrams have been incorporated into every edition of the SPM (1984). They remain useful tools for preliminary engineering design.

Limitations of the traditional diagrams include monochromatic and unidirectional wave assumption, constant water depth assumption, no reflection, and simple structure shape with vertical walls. Recently Chen (1987) presented an analytical solution for wave height modification in the vicinity of a structure. The solution is more general than the traditional approach in that it includes reflection as well as diffraction and it allows a wedge-shaped structure with vertical walls. Other limitations of the traditional approach still apply. Chen's (1987) solution was implemented in a computer code, PCDFRAC, by Kaihatu and Chen (1988). Output consists of a wave height modification coefficient (ratio of combined diffracted and reflected height to incident height) and a wave phase for any selected point in the vicinity of the structure. The code PCDFRAC has been modified to reside in ACES.

GENERAL ASSUMPTIONS AND LIMITATIONS

Assumptions inherent in the approach include linear, monochromatic, unidirectional waves, and constant water depth. The structure is assumed to be straight and semi-infinite in length with vertical walls. The walls are treated as fully reflecting.

Modified wave heights in areas strongly affected by reflection, such as the area immediately in front of the wedge, are variable in space because of interference between the incident and reflected waves. Such variability would not be expected in natural wave conditions.

Solutions are not available beyond about 10 wavelengths from the wedge tip. The approach is based on an analytical solution which includes summation of an infinite number of terms. The summation is computed with as many terms as needed to satisfy a convergence criterion. There is a limit on the maximum number of terms allowed. More terms are required for convergence as distance from the wedge tip increases. The limit is usually reached at distances of 10 to 15 wavelengths.

THEORETICAL BACKGROUND

The general boundary value problem of linear wave reflection and diffraction by a vertical wedge of arbitrary angle has been well formulated and presented by Stoker (1957) among many other investigators. The technique to obtain an analytical solution for the problem is also given by Stoker (1957). However, analytical solutions were available only for the special case of wave diffraction by a thin semi-infinite breakwater, that is, a wedge with angle equal to zero. The more general solution by Chen (1987) is the basis for the model PCDFRAC.

A cylindrical coordinate system (r, θ, z) is adopted, where $z = 0$ represents the undisturbed free surface and the positive z-axis is positioned vertically upward. The tip of the wedge is chosen to be the origin of the coordinate system, and the two rigid walls of the wedge are at $\theta = 0$ and $\theta = \theta_0$, respectively (Figure 3-3-1). Cartesian coordinates (x,y,z), also shown in Figure 3-3-1, are used for specifying input to the routine. The wedge angle is defined as $2\pi - \theta_0$, while the water domain is defined as $0 \le \theta \le \theta_0$ and $0 \ge z \ge -h$.

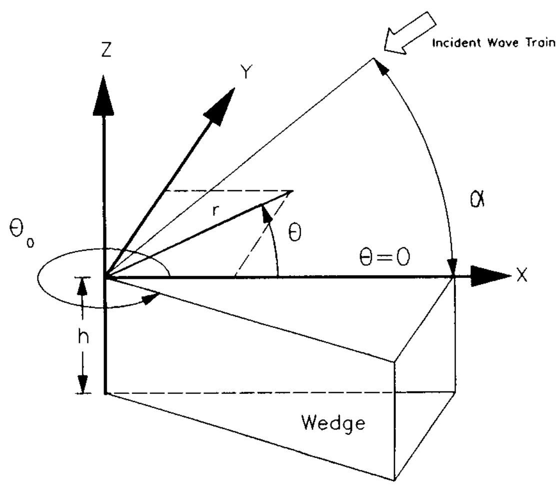

Figure 3-3-1. A Vertical Wedge of Arbitrary Angle

For the problem at hand, the velocity field for wave motion in an ideal fluid may be represented by the velocity potential $\Phi(r, \theta, z, t)$. This may be expressed as follows:

$$\Phi(r, \theta, z, t) = A_0 \frac{\cosh k(z+h)}{\cosh(kh)} \phi(r, \theta) e^{i\omega t} \tag{1}$$

where:

$\phi(r,\theta)$ = velocity potential function in the horizontal plane

t = time variable

$A_0 = \dfrac{-iga_0}{\omega}$

$i = \sqrt{-1}$

g = gravitational acceleration

a_0 = incident wave amplitude

ω = radian frequency

k = wave number

The wave number k must be real and satisfy the following linear dispersion equation:

$$\omega^2 = gk\tanh(kh) \tag{2}$$

Several properties of wave mechanics are dependent on the velocity potential component $\phi(r,\theta)$. For example, the free surface elevation η may be expressed in terms of $\phi(r,\theta)$ as follows:

$$\eta = a_0\,\phi(r,\theta)e^{i\omega t} \tag{3}$$

The flow velocity u may be expressed in polar coordinates as follows:

$$u_r = \frac{\partial\phi}{\partial r} = A_0\frac{\cosh k(z+h)}{\cosh(kh)}\frac{\partial\phi}{\partial r}e^{i\omega t} \tag{4}$$

$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = A_0\frac{\cosh k(z+h)}{\cosh(kh)}\frac{1}{r}\frac{\partial\phi}{\partial\theta}e^{i\omega t} \tag{5}$$

where the subscripts, r and θ, refer to the r- and θ- directions. The absolute values of u_r and u_θ are the maximum flow velocities for each component direction, while the phases of u_r and u_θ contain the wave phase information. It is clear the solution to Equation *1* and Equations *3-5* lies in finding $\phi(r,\theta)$.

For an incident plane wave train coming from the α direction (Figure 3-3-1), the free surface elevation of the incident wave may be described by:

$$\eta_i = a_0 e^{ikr_i\cos(\alpha+\omega t)} \tag{6}$$

where r_i defines a point on the incident wave.

The analytical solution for a wave field by a vertical wedge of arbitrary wedge angle, based on the linearized wave theory, may be written as follows (Chen, 1987):

$$\phi(r,\theta) = \frac{2}{\nu}\left[J_0(kr) + 2\sum_{n=1}^{\infty} e^{\frac{i n \pi}{2\nu}} \; J_{\frac{n}{\nu}}(kr) \; \cos\frac{n\alpha}{\nu} \; \cos\frac{n\theta}{\nu} \right] \tag{7}$$

where:

$$\nu = \frac{\theta_0}{\pi} \tag{8}$$

J_0 = zero order Bessel function of the first kind

$J_{n/\nu}$ = n/ν order Bessel function of the first kind

The semi-infinite breakwater is a special case of the diffraction/reflection problem, where the wedge angle is equal to zero ($\theta_0 = 2\pi$) and $\nu = 2$. The solution of Equation *1* for this case is:

$$\phi(r,\theta) = J_0(kr) + 2\sum_{n=1}^{\infty} e^{\frac{i n \pi}{4}} \; J_{\frac{n}{2}}(kr) \; \cos\frac{n\alpha}{2} \; \cos\frac{n\theta}{2} \tag{9}$$

NOTE: Equations *7* and *9* show a summation of an infinite number of terms. This has been accommodated in the program by carrying the summation out to a term followed by eight successive terms in which the absolute value of the Bessel function is 10^{-8} or less. If the value of the solution is of the order one, this corresponds to a truncation error of 10^{-8} or less.

The velocity potential function $\phi(r,\theta)$ in Equations *7* and *9* is a complex function and may be expressed as:

$$\phi = |\phi| \, e^{i\beta} \tag{10}$$

where:

$$|\phi|^2 = [Im(\phi)]^2 + [Re(\phi)]^2 \qquad \text{(amplitude of } \phi) \tag{11}$$

$Im \;\; \phi$ = imaginary part of $\phi(r,\theta)$

$Re \;\; \phi$ = real part of $\phi(r,\theta)$

$$\beta = \tan^{-1}\left[\frac{Im(\phi)}{Re(\phi)}\right] \qquad \text{(phase of } \phi) \tag{12}$$

Substituting Equation *10* into Equation *3*, the following is obtained:

$$\eta = a_0 \mid \phi \mid e^{i(\beta + \omega t)} \tag{13}$$

This expression represents the actual water surface elevation at a point in the water domain bounded by a vertical wedge of arbitrary wedge angle. Since the incident wave train is expressed in Equation *6*, the normalized water surface elevation in the near field may be expressed as:

$$\frac{\eta}{\eta_i} = \mid \phi \mid e^{i(\beta - k r_i \cos\alpha)} \tag{14}$$

where:

η_i = incident free surface elevation

It is clear that the term $\mid \phi \mid e^{i(\beta - k r_i \cos\alpha)}$ is a factor that modifies the incident wave elevation to account for reflection and diffraction effects. The amplitude of the normalized surface elevations, which is comparable to diffraction and reflection coefficients defined in the SPM (1984), may be expressed as the following wave diffraction/reflection coefficient:

$$\left| \frac{\eta}{\eta_i} \right| = \mid \phi \mid \tag{15}$$

The phase of Equation *14* is the phase difference between incident and modified waves.

$$\text{Phase difference} \qquad \frac{\eta}{\eta_i} = \beta - k r_i \cos\alpha \tag{16}$$

Output of PCDFRAC is composed of $\mid \phi \mid$ and β. The diffraction/reflection coefficient $\mid \phi \mid$ (modification factor) is then multiplied by the incident wave height to obtain the modified wave height. The phase difference β related to the modified wave is a quantity not usually required in engineering practice; however, it may be useful on some occasions.

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