CVM UNIVERSITY
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Academic Year 2023-24, Semester – I
Subject Code: 202000104
Subject Name: CALCULUS
TUTORIAL -9

Exercise-1: Evaluate the following Double Integrals

[1] $\int_0^3 \int_0^1 (x^2 + 3y^2)dydx$ Ans: 12

[2] $\int_0^1 \int_1^2 xydxdy$ Ans: $\frac{3}{4}$

[3] $\int_2^a \int_2^b \frac{dydx}{xy}$ Ans: $\log\left(\frac{a}{2}\right) \log\left(\frac{b}{2}\right)$

[4] $\int_0^{\ln 3} \int_0^{\ln 2} e^{x+y}\, dydx$ Ans: 2

[5] $\int_0^1 \int_0^x e^{\frac{y}{x}}\, dydx$ Ans: $\frac{1}{2}\, (e-1)$

[6] $\int_0^1 \int_x^{x^2} xydxdy$ Ans: $-\frac{1}{24}$

[7] $\int_1^4 \int_0^{\sqrt{x}} \frac{3}{2}\, e^{\frac{y}{\sqrt{x}}} dydx$ Ans: $7(e-1)$

Exercise-2: Evaluate the following Integrals

[1] $\iint_R (x+y)dydx$, Where R is the region bounded by $x = 0,\ x = 2,\ y = x, y = x + 2$. Ans: 12

[2] $\iint_R ydydx$, over the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$. Ans: $\frac{36}{5}$

[3] $\iint_R x^2dydx$, over the region in the first quadrant enclosed by the rectangular hyperbola $xy = 16$, the lines $y = x, y = 0$, and $x = 8$. Ans: 512

[4] $\iint_R xydydx$, over $R: y = 0, x = 0, y = x, y = 1$. Ans: $\frac{1}{8}$

[5] $\iint_R (x+y)dydx$, over $R: y = x^2$ & $y = x$ Ans: $\frac{2}{5}$

Exercise-3: Find the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. Ans: $\frac{-5a^2}{4}$

Exercise-4: Find the area bounded by parabola $y^2 = 4x$ and line $y = 2x - 4$. Ans: $\frac{25}{3}$

Exercise-5: Evaluate the following Integrals by Changing the order of Integration.

[1] $\int_0^a \int_x^a (x^2 + y^2)dydx$ Ans: $\frac{a^4}{3}$

[2] $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$ Ans: 1

[3] $\int_0^{\frac{\pi}{4}} \int_0^1 r\, dr\, d\theta$ Ans: $\frac{\pi}{8}$

[4] $\int_0^\pi \int_0^{\sin\theta} r\, dr\, d\theta$ Ans: $\frac{\pi}{4}$

Exercise-6: Evaluate the following Integration by Changing the variable to Polar.

[1] $\int_0^{\frac{a}{\sqrt{2}}}\int_x^{\sqrt{a^2-x^2}} \frac{x}{\sqrt{y^2+x^2}}\,dydx$ Ans: $\frac{a^2}{2}[1-\frac{1}{\sqrt{2}}]$

[2] $\int_0^a\int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\,dydx$ Ans: $\frac{\pi}{4}[1-e^{-a^2}]$

[3] $\int_0^1\int_0^{\sqrt{1-y^2}} (y^2+x^2)\,dydx$ Ans: $\frac{\pi}{8}$

[4] $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dydx$ Ans: $\frac{\sqrt{\pi}}{2}$

Exercise-7: Evaluate the following triple Integration.

[1] $\int_0^2\int_1^3\int_1^2 xy^2z\,dydx\,dz$ Ans: 26

[2] $\int_0^1\int_0^2\int_0^e\, dydx\,dz$ Ans: 2e

[3] $\int_0^2\int_1^z\int_0^{zy} xyz\,dydx\,dz$ Ans: $\frac{7}{2}$

[4] $\int_0^1\int_0^{1-x}\int_0^{x+y} e^z\,dydx\,dz$ Ans: $\frac{1}{2}$

[5] $\int_0^1\int_0^{x^2}\int_0^{x+y}(2x-y-z)\,dz\,dy\,dx$

[6] $\int_{-1}^1\int_0^z\int_{x-z}^{x+z}(x+y+z)dx\,dy\,dz$

[7] $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^a r^2 sin\theta\, dr\, d\theta\, d\emptyset$

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