Discrete Math Cram Sheet

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Propositional Logic 1

Truth Tables 1.1

p	T	${ m T}$	\mathbf{F}	\mathbf{F}	
q	Т	\mathbf{F}	\mathbf{T}	F	
F	F	F	F	F	contradiction
$p \vee q$	F	\mathbf{F}	\mathbf{F}	Τ	joint denial
$p \not\leftarrow q$	F	\mathbf{F}	\mathbf{T}	F	converse nonimplication
$\neg p$	F	\mathbf{F}	\mathbf{T}	Τ	left negation
$p \nrightarrow q$	F	${\rm T}$	\mathbf{F}	F	nonimplication
$\neg q$	F	${\rm T}$	\mathbf{F}	Τ	right negation
$p\oplus q$	F	\mathbf{T}	\mathbf{T}	F	exclusive disjunction
$p \overline{\wedge} q$	F	\mathbf{T}	\mathbf{T}	Τ	alternative denial
$p \wedge q$	T	\mathbf{F}	\mathbf{F}	F	conjunction
$p \leftrightarrow q$	T	\mathbf{F}	\mathbf{F}	Τ	biconditional
q	T	\mathbf{F}	${\rm T}$	F	right projection
$p \rightarrow q$	T	\mathbf{F}	\mathbf{T}	Τ	implication
p	Т	\mathbf{T}	\mathbf{F}	F	left projection
$p \leftarrow q$	Т	\mathbf{T}	\mathbf{F}	Τ	converse implication
$p \lor q$	\mathbf{T}	\mathbf{T}	\mathbf{T}	F	disjunction
Т	T	\mathbf{T}	\mathbf{T}	Τ	tautology

Propositional Equivalences

Identity

- $p \wedge T \equiv p$
- $p \vee F \equiv p$
- De Morgan's
 - $\bullet \neg (p \land q) \equiv \neg p \lor \neg q$
 - $\bullet \neg (p \lor q) \equiv \neg p \land \neg q$

Domination

- $p \vee T \equiv T$
- $p \wedge F \equiv F$

Absorption

- $p \wedge (p \vee q) \equiv p$
- $p \lor (p \land q) \equiv p$

Idempotent

- $p \wedge p \equiv p$
- $\bullet \ \ p \vee p \equiv p$

Negation

- $p \lor \neg p \equiv T$
- $p \land \neg p \equiv F$

Commutative

- $p \wedge q \equiv q \wedge p$
- **Double Negation**
- $p \lor q \equiv q \lor p$
- $\neg (\neg p) \equiv p$

Associative

- $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$
- $\bullet \ p \lor (q \lor r) \equiv (p \lor q) \lor r$

Distributive

- $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Involving Biconditionals

- $p \leftrightarrow q \equiv (p \to q) \land (q \to p)$
- $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
- $p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$
- $\bullet \neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Involving Conditional Statements

- $p \to q \equiv \neg p \lor q$
- $p \to q \equiv \neg q \to \neg p$
- $\bullet \ \ p \lor q \equiv \neg p \to q$
- $p \land q \equiv \neg (p \rightarrow \neg q)$
- $(p \to q) \land (p \to r) \equiv p \to (q \land r)$
- $(p \to r) \land (q \to r) \equiv (p \lor q) \to r$
- $\bullet \ (p \to q) \lor (p \to r) \equiv p \to (q \lor r)$
- $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

1.3 Rules of Inference

- 2 Proofs
- 2.1 Mathematical Induction
- 2.2 Strong Induction
- 3 Recurrence Relations
- 4 Number Theory
- 5 Graph Theory
- 6 Linear Algebra
- 7 Combinatorics
- 8 Probability