Discrete Math Cram Sheet

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Contents

1 Propositional Logic

1.1 Truth Tables

p	T	T	F	F	
q	T	F	T	F	
F	F	F	F	F	contradiction
$p \veebar q$	F	F	F	T	joint denial
$p \nleftarrow q$	F	F	T	F	converse nonimplication
$\neg p$	F	F	T	T	left negation
$p \nrightarrow q$	F	T	F	F	nonimplication
$\neg q$	F	T	F	T	right negation
$p \oplus q$	F	T	T	F	exclusive disjunction
$p \barwedge q$	F	T	T	T	alternative denial
$p \wedge q$	T	F	F	F	conjunction
$p \leftrightarrow q$	T	F	F	T	biconditional
q	T	F	T	F	right projection
$p \to q$	T	F	T	T	implication
p	T	T	F	F	left projection
$p \leftarrow q$	T	T	F	T	converse implication
$p \vee q$	T	T	T	F	disjunction
T	T	T	T	T	tautology

1.2 Logical Equivalences

Identity	$p \wedge \mathrm{T} \equiv p$ $p \vee \mathrm{F} \equiv p$
Domination	$p \vee \mathrm{T} \equiv \mathrm{T}$ $p \wedge \mathrm{F} \equiv \mathrm{F}$
Idempotent	$p \wedge p \equiv p$ $p \vee p \equiv p$
Commutative	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ $p \vee (q \vee r) \equiv (p \vee q) \vee r$
Distributive	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
De Morgan's	$\neg (p \wedge q) \equiv \neg p \vee \neg q$ $\neg (p \vee q) \equiv \neg p \wedge \neg q$
Absorption	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$
Negation	$p \vee \neg p \equiv \mathrm{T}$ $p \wedge \neg p \equiv \mathrm{F}$
Double Negation	$\neg (\neg p) \equiv p$

Involving Biconditionals

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Involving Conditional Statements

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg (p \to \neg q)$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

1.3 Rules of Inference

Modus Ponens	$\begin{array}{l} p \to q \\ p \\ \hline q \end{array}$
Modus Tollens	$\begin{array}{l} \neg q \\ p \to q \\ \hline \neg p \end{array}$
Associative	$\begin{array}{l} (p \vee q) \vee r \\ \hline p \vee (q \vee r) \end{array}$
Commutative	$\begin{array}{l} p \wedge q \\ \hline q \wedge p \end{array}$
Biconditional	$\begin{array}{l} p \to q \\ q \to p \\ \hline p \leftrightarrow q \end{array}$
Exportation	$\begin{array}{l} (p \wedge q) \to r \\ \hline p \to (q \to r) \end{array}$
Contraposition	$\begin{array}{l} p \to q \\ \hline \neg q \to \neg p \end{array}$
Hypothetical Syllogism	$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$
Material Implication	$\begin{array}{l} p \to q \\ \hline \neg p \vee q \end{array}$
Distributive	$\begin{array}{l} (p \vee q) \wedge r \\ \hline (p \wedge r) \vee (q \wedge r) \end{array}$
Absorption	$\begin{array}{l} p \to q \\ \hline p \to (p \wedge q) \end{array}$
Disjunctive Syllogism	$\begin{array}{l} p \vee q \\ \neg p \\ \hline q \end{array}$
Addition	$\begin{array}{l} p \\ \hline p \vee q \end{array}$
Simplification	$\begin{array}{l} p \wedge q \\ \hline p \end{array}$
Conjunction	$\begin{array}{l} p \\ q \\ \hline p \wedge q \end{array}$
Double Negation	$\begin{array}{l} p \\ \hline \neg\neg p \end{array}$
Disjunctive Simplification	$\begin{array}{l} p \vee p \\ \hline p \end{array}$
Resolution	$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline q \vee r \end{array}$

1.4 Satisfiability

A proposition is *satisfiable* if some setting of the variables makes the proposition true. For example, $p \wedge \neg q$ is satisfiable because the expression is true if p is true or q is false. On the other hand, $p \wedge \neg p$ is not satisfiable because the expression as a whole is false for both settings of p.

2-SAT Problem

(to follow...)

2 Proofs

2.1 Mathematical Induction

2.2 Strong Induction

3 Recurrence Relations

4 Number Theory

4.1 Divisibility

4.2 Primes and GCD

Greatest Common Divisor

This can be defined by the following recurrence relation:

$$\gcd(a, b) = \begin{cases} a & \text{if } b = 0 \\ \gcd(b, a \bmod b) & \text{else} \end{cases}$$

4.3 Modular Arithmetic

5 Graph Theory

6 Linear Algebra

7 Combinatorics

8 Probability