MA503: Homework 6

Use wolfram alpha for modular exponentiation.

Exercise 6.1. [2pts] Solve the quadratic congruence $x^2 + 7x + 1 \equiv_{11} 0$.

Exercise 6.2. [5pts] Find ALL square roots of 11 modulo 35.

Exercise 6.3. [2pts] Find the values of the following Legendre symbols:

(a) (19/23),
(b) (18/43).

Exercise 6.4. [1pts] Assume that r is a quadratic residue of an odd prime p and $ab \equiv_p r$. Prove that either both a and b are quadratic residues of p, or both quadratic nonresidues of p.

Exercise 6.5. [10pts] For a remote coin toss, Alice selects $p = 47$, $q = 79$ and Bob chooses $x = 123$. Of the four numbers Alice then calculates, which two represent loosing calls? Which two represent winning calls? [Hint. You are allowed to use **wolfram alpha** to solve a system of **linear congruences**. E.g., try "solve x=1 mod 47, x=2 mod 79".]