1)

5060551

$\rightarrow \varphi_1 : (p \rightarrow q)$

$\rightarrow \varphi_2 : (q \rightarrow (s \wedge t))$

$\rightarrow \varphi_3 : (r \rightarrow (s \wedge t))$

$\rightarrow \varphi_4 : (p \vee r)$

The formula $s \wedge t$ is a logical consequency of the theory.

$$\Phi = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$$

Using Resolution to delive empty causes from CNF (conjunctive normal form)

$$(\varphi_1, \varphi_2, \varphi_3, \varphi_4) \wedge \neg\varphi$$

$\rightarrow \varphi_1 : (p \rightarrow q) \rightarrow \neg p \vee q' \rightarrow \{\neg p, q)$

$\rightarrow \varphi_2 : (q \rightarrow (s \wedge t)) \rightarrow \neg q \vee (s \wedge t) \rightarrow (\neg q \vee s) \wedge (\neg q \vee t) \rightarrow \{\neg q, s\}$ $\{\neg q, t\}$

$\rightarrow \varphi_3 : (r \rightarrow (s \wedge t) \rightarrow \neg r \vee (s \wedge t)$

$\rightarrow (\neg r \vee s) \wedge (\neg r \vee t)$

$\rightarrow \{\neg r, s\}$

$\{\neg r, t\}$

$\rightarrow \varphi_4 : p \vee r' \rightarrow (p, r)$

$\neg\varphi : \neg(s \wedge t) \rightarrow \neg s \vee \neg t$

$\rightarrow (\neg s, \neg t)$

1) $(\neg p, q)$

2) $(\neg q, s)$

3) $(\neg q, t)$

4) $(\neg r, s)$

5) $(\neg r, t)$

6) (p, r)

7) $(\neg r, \neg t)$

8) $(1,6) \rightarrow (q, r)$

9) $(8,7) \rightarrow (q, \neg t)$

10) $(9,3) \rightarrow (\ )$

5060551

8) $(\neg p, q)$
(p, r)
$\overline{(q, r)}$

9) (q, r)
$(\neg t, \neg r)$
$\overline{(q, \neg t)}$

10 $(q, \neg t)$
$(\neg q, t)$
$\overline{(\ )}$

$s \wedge t$ is a logical consequence of $\phi = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$

5) where an "x" in the cell x,y denotes that x "likes" y.

	Abby	Bess	Cody	Dana
Abby	x	–	x	x
Bess	–	x	–	x
Cody	–	–	x	–
Dana	–	x	x	–

1) $\forall x . \text{likes}(x, x)$

likes (a,a) = True

likes (b,b) = True

likes (c,c) = True

likes (d,d) = False

$\forall x . \text{likes}(x, x)$ is false.

2) $\forall x . \exists y . \text{likes}(x, y)$ $\Rightarrow$ ~~$\exists y$~~ ~~existential~~

$\exists y$ likes (Abby, y) ~~likes (Abby)~~,

$\exists y$ likes (Bess, y)

$\exists y$ likes (Cody, y)

$\exists y$ likes (Dana, y)

likes (Abby, Cody)

likes (Bess, Bess)

likes (cody, cody)

likes (Dana, cody)

$\forall x . \exists y$ likes (x, y) is true.

3) $\exists y . \forall x . \text{likes}(x, y)$

likes (a,a) = True

likes (b,b) = True

likes (c,c) = True

likes (d,d) = False

$\therefore$ (Dana doesn't like herself)

$\exists y . \forall x$ likes (x, y) is False.

4) $\forall x . \forall y$ likes $(x, y) \Rightarrow$ likes (y, x)

This would be satisfied if likes was symmetric in the model for likes (a,c) is satisfied likes (c,a) is not.

~~$\forall x . \forall y$ likes (x,y)~~ still

5060551

$\forall x . \forall y . likes(x,y) \rightarrow likes(y,x)$ is false

5) $\forall x . \forall y . (\exists z . likes(x,z) \wedge likes(z,y)) \rightarrow likes(x,y)$

likes (a, d)

likes (d, b)

likes (a, b)

$\forall x . \forall y . (\exists z . likes(x,z) \wedge likes(z,y)) \rightarrow likes(x,y)$ is false.