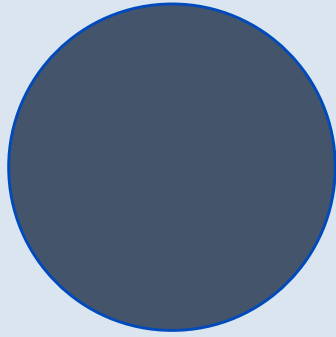


Exploratory Project



Bubble Point

Bubble point is the temperature at which the first bubble of vapor is formed when a liquid-vapor mixture is heated at constant pressure. This occurs during the phase transition from a liquid to a two-phase liquid-vapor mixture.

At the bubble point, the liquid begins to vaporize.

Bubble point is a critical parameter that provides insights into the thermodynamic and fluid dynamic behavior of substances. Its significance extends to various scientific and engineering applications, including process optimization, reservoir engineering, and ensuring the safety and reliability of systems and processes.



Dew Point for a gas

- Dew point is the temperature at which a gas (such as air) becomes saturated with moisture and undergoes a phase change, transitioning from a gaseous state to a liquid state.
- It is the temperature at which the partial pressure of water vapor in the air reaches the saturation pressure
- The dew point is an important parameter in various applications, including HVAC (heating, ventilation, and air conditioning) systems, meteorology, and industrial processes where controlling humidity is essential.
- Meteorologists often use the dew point as a measure of humidity. A higher dew point indicates higher humidity, and when the dew point is close to the actual air temperature, it suggests that the air is nearly saturated with moisture. The concept is crucial in weather forecasting and understanding atmospheric conditions.



MATLAB Code(Determining Bubble/Dew Point)



Bisection Method

The bisection method works by repeatedly dividing an interval that contains a root into two smaller subintervals, and then choosing the subinterval that has a sign change of the function. The process is continued until the subinterval is sufficiently small or the function value is sufficiently close to zero. The bisection method is simple and reliable, but it is also slow and requires that the function is continuous and has a sign change in the initial interval. The bisection method requires the following steps :

- Choose an initial interval $[a, b]$ such that $f(a)$ and $f(b)$ have opposite signs and $f(x)$ is continuous $\forall x \in (a, b)$. This means that there is at least one root in the interval by the intermediate value theorem.
- Calculate the midpoint of the interval, $c = \frac{a+b}{2}$ and the function value at the midpoint, $f(c)$.
- Check if c is a root, i.e., if $f(c) = 0$, or if the interval is small enough, i.e., if $|b - a| < \epsilon$, where ϵ is a small positive number. If either of these conditions is true, then stop the method and return c as the approximate root.
- Otherwise, check if $f(a)$ and $f(c)$ have opposite signs. If they do, then replace b with c and repeat the steps from 2 if they don't then replace a with c and repeat the steps from 2.
- Continue this process until a root is found or the interval is small enough.

MATLAB Code(Bisection)



Newton Raphson Method

- It is an iterative numerical technique for finding approximate roots (or zeros) of a real-valued function. It's commonly used for solving non-linear equations of the form $f(x)=0$. The method starts with an initial guess for the root and then iteratively refines that guess to get closer to the true root.

The iterative formula for Newton-Raphson method is given by

$$X_{n+1} = X_n - \frac{f(x_n)}{f'(x_n)}$$

- x_n is the current approximation of the root,
- $f(x_n)$ is the function value at x_n
- $f'(x_n)$ is the derivative of the function at x_n
- x_{n+1} is the next approximation of the root.

MATLAB Code(Newton Raphson)



Regression(Least Square Parameter Estimation)

A **Regression Model** is a mathematical function that relates the dependent variable to the independent variables and the parameters of the model. The model can be deterministic or probabilistic, depending on whether it incorporates random errors or not. A deterministic model assumes that the relationship between the variables is exact, while a probabilistic model allows for some randomness or variation in the data. A probabilistic model is more realistic and can account for measurement errors, natural variability, and other sources of uncertainty

A **linear regression model** is a type of regression model that assumes that the dependent variable is a linear function of the independent variables and the parameters. A linear function is one that has the form $y = a + bx$, where a and b are constants. A linear regression model can be written as $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \epsilon$, where $\beta_0, \beta_1, \dots, \beta_k$ are the parameters and ϵ is the random error term.

Regression(Least Square Parameter Estimation)

- The goal of linear regression is to find the values of β that minimize the sum of squared errors (SSE), which is given by:

$$SSE = \sum_{i=1}^n (y_i - x_i^T \beta)^2$$

where, n is the number of observations, y_i is the i^{th} element of $[Y]$ & x_i^T is the i^{th} row of $[X]$.

- To find the minimum of SSE, we take the partial derivatives of SSE with respect to each element of β and set them to zero. This gives us the normal equations:

$$X^T X \beta = X^T Y$$

where X^T is the transpose of X .

- To solve for β , we multiply both sides of the normal equations by the inverse of $X^T X$, assuming that it exists and is unique. This gives us:

$$(X^T X)^{-1} (X^T X) \beta = (X^T X)^{-1} X^T Y \quad \Rightarrow \quad \beta (X^T X)^{-1} X^T Y$$

This is the formula for β in linear regression. It shows how the parameters depend on the independent and dependent variables.

Regression(Least Square Parameter Estimation)

Let the $(n \times 1)$ vector of observations of the dependent variable be denoted as:

$$y = (y_1, y_2, \dots, y_n)^T$$

The $(k \times 1)$ vector of (unknown) parameters to be estimated, is given as:

$$\beta = (\beta_1, \beta_2, \dots, \beta_n)^T$$

The $(n \times k)$ matrix of observations of the independent variables is denoted as:

$$A = \begin{bmatrix} X_{11} & \dots & X_{1k} \\ \vdots & & \vdots \\ X_{n1} & \dots & X_{nk} \end{bmatrix}$$

Let the $(n \times 1)$ vector, \hat{y} denote the estimates of the dependent variable, based on the functionality of y and x , \hat{y} is given as:

$$\hat{y} = A\beta$$

Let the $(n \times 1)$ vector, e represent the error between the experimental and predicted values of the dependent variable, denoted as:

$$e = y - \hat{y} = y - A\beta$$

Regression(Least Square Parameter Estimation)

- The method of Least Squares (LS) based on the minimization of the sum of squares of errors. The (scalar) objective function, $\phi(\beta)$ is therefore represented as :

$$\phi(\beta) = ee^T$$

(Represents variance/square of errors)

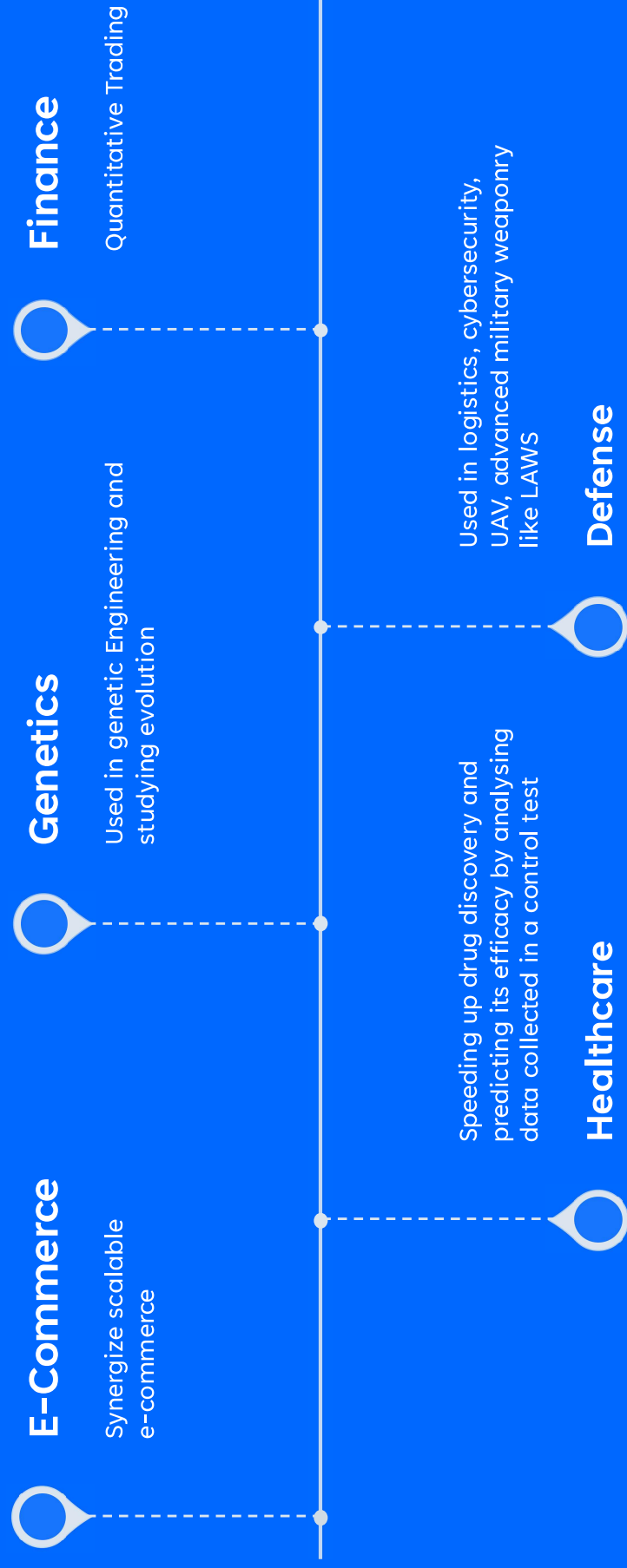
- To minimize $\phi(\beta) \Rightarrow \frac{\partial \phi}{\partial \beta} = 0 \Rightarrow \frac{\partial \phi}{\partial \beta} = [A^T(y - A\beta) + (y - A\beta)^T(-A)] = 0$

By Jacobian Vector differentiation $\Rightarrow \frac{\partial \phi}{\partial \beta} = [A^T(y - A\beta) + (y - A\beta)^T(-A)] = 0$
 $\Rightarrow [-2 A^T(y - A\beta)] = 0$

- Solving the equation we get,

$$\beta = (X^T X)^{-1} X^T Y$$

Applications of Artificial Intelligence

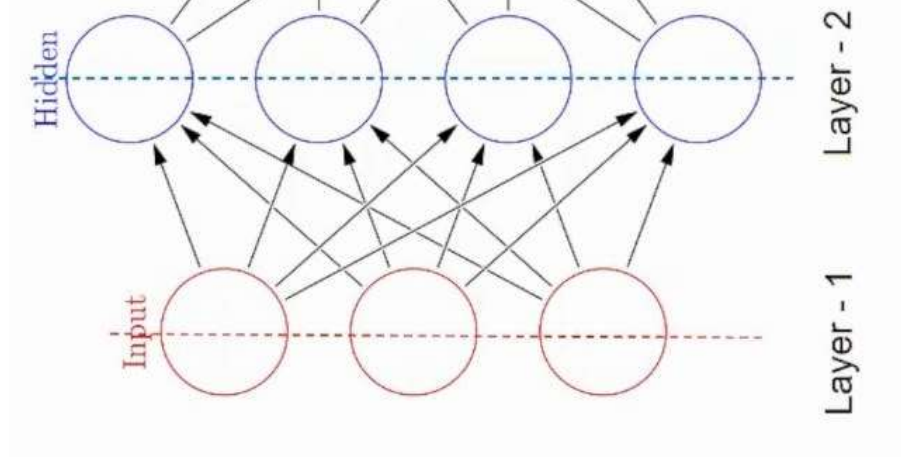


Machine Learning & AI

Artificial Neural Networks

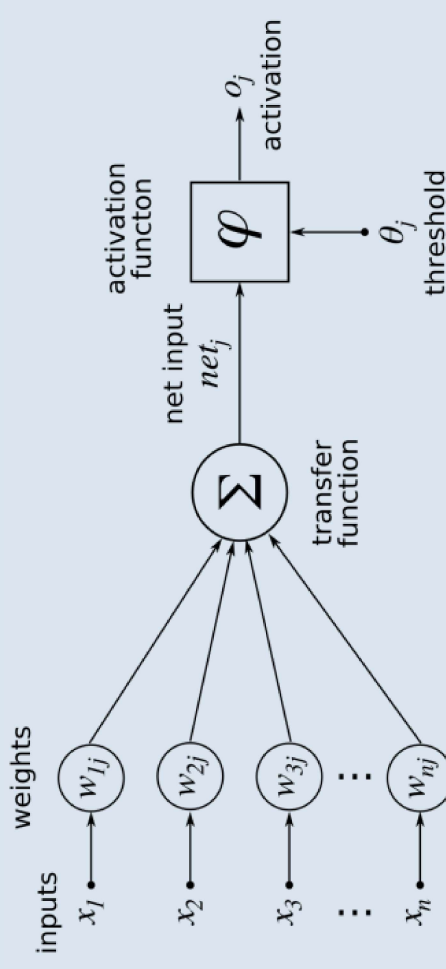
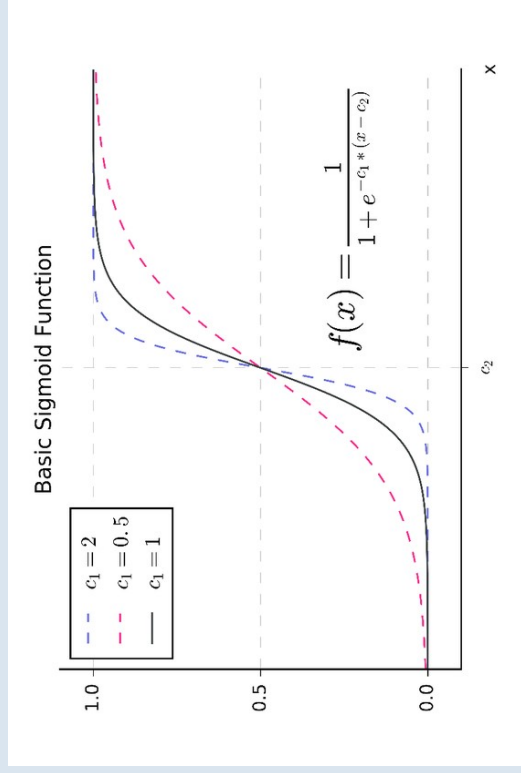
An Artificial Neural Network (ANN) is a mathematical model that tries to simulate the structure and functionalities of biological neural networks. Basic building block of every artificial neural network is artificial neuron, that is, a simple mathematical model (function).

Such a model has three simple sets of rules: multiplication, summation and activation. At the entrance of artificial neuron the inputs are weighted what means that every input value is multiplied with individual weight. In the middle section of artificial neuron is sum function that sums all weighted inputs and bias. At the exit of artificial neuron the sum of previously weighted inputs and bias is passing through activation function that is also called transfer function.



Artificial Neurons

An artificial neuron is a mathematical function conceived as a model of biological neurons, a neural network. Artificial neurons are elementary units in an artificial neural network. The artificial neuron receives one or more inputs (representing excitatory postsynaptic potentials and inhibitory postsynaptic potentials at neural dendrites) and sums them to produce an output (or activation, representing a neuron's action potential which is transmitted along its axon). Usually each input is separately weighted, and the sum is passed through a non-linear function known as an activation function or transfer function. The transfer functions usually have a sigmoid shape, but they may also take the form of other non-linear functions, piecewise linear functions, or step functions. They are also often monotonically increasing, continuous, differentiable and bounded. The thresholding function has inspired building logic gates referred to as threshold logic; applicable to building logic circuits resembling brain processing



Types of Learning

Supervised Learning

This learning process is dependent. During the training of ANN under supervised learning, the input vector is presented to the network, which will give an output vector. This output vector is compared with the desired output vector. An error signal is generated, if there is a difference between the actual output and the desired output vector. On the basis of this error signal, the weights are adjusted until the actual output is matched with the desired output.



Unsupervised Learning

During the training of ANN under unsupervised learning, the input vectors of similar type are combined to form clusters. When a new input pattern is applied, then the neural network gives an output response indicating the class to which the input pattern belongs. There is no feedback from the environment as to what should be the desired output and if it is correct or incorrect. Hence, in this type of learning, the network itself must discover the patterns and features from the input data, and the relation for the input data over the output

Reinforcement

This learning process under supervised learning might have very less. During the training under reinforcement, the network receives some information from the environment, which is somewhat similar to supervised learning. However, the information obtained here is even more instructive, which makes it act as a teacher in supervised learning. After receiving feedback, the network makes adjustments to become a better critic in reinforcement learning.

Neural Network Activation Function

In a neural network, inputs, which are typically real values, are fed into the neurons in the network. Each neuron has a weight, and the inputs are multiplied by the weight and fed into the activation function. Each neuron's output is the input of the neurons in the next layer of the network, and so the inputs cascade through multiple activation functions until eventually, the output layer generates a prediction. Neural networks rely on nonlinear activation functions—the derivative of the activation function helps the network learn through the backpropagation process.

Sigmoid Activation Function: It is of two types as follows –

1. **Binary sigmoidal function :** This activation function performs input editing between 0 and 1 and is positive in nature. It is always bounded, which means its output cannot be less than 0 and more than 1. It is also strictly increasing in nature, which means more the input higher would be the output. It can be defined as $f(x) = \frac{1}{1+e^{-x}}$
2. **Bipolar sigmoidal function :** This activation function performs input editing between -1 and 1 and can be positive or negative in nature. It is always bounded, which means its output cannot be less than -1 and more than 1. It is also strictly increasing in nature like sigmoid function. It can be defined as $f(x) = -1 + \frac{2}{1+e^{-x}}$



Pattern Recognition using ANN

Pattern recognition is the automated recognition of patterns and regularities in data. Pattern recognition is closely related to artificial intelligence and machine learning, together with applications such as data mining and knowledge discovery in databases (KDD), and is often used interchangeably with these terms. However, these are distinguishing machine learning is one approach to pattern recognition, while other approaches include hand-crafted (not learned) rules or heuristics; and pattern recognition is one approach to artificial intelligence, while other approaches include symbolic artificial intelligence. A modern definition of pattern recognition is: The field of pattern recognition is concerned with the automatic discovery of regularities in data through the use of computer algorithms and with use of these regularities to take actions such as classifying the data into different categories. Pattern recognition is generally categorized according to the type of learning procedure used to generate the output value. Supervised learning assumes that a set of training data (the training set) has been provided, consisting of a set of instances that have been properly labeled by hand with the correct output. Unsupervised learning, on the other hand, assumes training data that has not been hand-labeled, and attempts to find inherent patterns in the data that can then be used to determine the correct output value for new data instances. Algorithms for pattern recognition based on statistical modelling of data. With statistical model in hand, one applies probability theory and decision theory to design an algorithm. This is opposed to using heuristics/common sense to design an algorithm

The following learning types are associated with pattern recognition using ANN.

1. Supervised learning
2. Unsupervised learning
3. Generative model
4. Discriminative model.

Pattern recognition can be implemented by using a feed-forward neural network that has been trained accordingly. During training, the network is trained to associate outputs with input patterns. When the network is used, it identifies the input pattern and tries to output the associated output pattern. The power of neural networks comes to life when a pattern has no output associated with it, is given as an input. In this case, the network gives the output that corresponds to a input pattern that is least different from the given pattern. During training, the network is trained to associate outputs with input patterns. When the network is used, it identifies the input pattern and tries to output the associated output pattern. The power of neural network comes to life when a pattern that has no output associated with it, is give it as an input. In this case, the network gives the output that corresponds to a taught input pattern that is least different from the given pattern. During training, the network is trained to associate outputs within put patterns. When the network is used, it identifies the input pattern and tries to output the associated output pattern. The power of neural network comes to life when a pattern comes to life when a pattern that has no output associated with it, is give it as an input. In this case, the network gives the output that corresponds to a taught input pattern that is least different from the given pattern.

Conclusion

In this project we learnt about various numerical techniques and methods and applied them in real life problems and we also learned about artificial neural networks and its applications in real life.



Thank you