Practice Slide 1

①

Build the Linear Regression model for the data given here.

1 - Calculate regression coefficients, MAE, MSE, RMSE, R^2
2 - Predict the y for $x = 38$.
3 - Use ordinary least square (OLS) estimation to calculate the regression coefficients.
4 - Use the Loss function to calculate the loss in prediction for each data points (x). $L(y,t) = \frac{1}{2}(y-t)^2$
5 - Calculate the revised regression coefficients (after 1st iteration), using Gradient descent method.

Age (x)	BP (y)
46	354
20	190
52	405
30	263
57	451

$t = \hat{y}_i$

737

x	y	$x_i - \bar{x}$	$(x_i-\bar{x})^2$	$(y_i - \bar{y})$	$(x_i-\bar{x})(y_i-\bar{y})$	$\hat{y}_i$	$\hat{y}_i - y_i$	$\hat{y}_i - \bar{y}$	$x_i y_i$	$L(y,t) = \frac{1}{2}(y-t)^2$
46	354	5		21.4	107	366.415	53.433	34.015	16284	79.569
20	190	-21		-142.6	2994.6	189.737	217.433	-142.863	3800	0.035
52	405	11		72.4	796.4	407.433	2.433	74.833	21060	2.950
30	263	-11		-69.6	765.6	257.767	-5.233	-74.833	7890	13.692
57	451	16		118.4	1894.4	441.448	-9.552	108.848	25707	45.620
205	1663	0	964	0	6558.0	1663	258.514		91025 ~~100448~~	

$\boxed{\sum_{i=1}^{n} |\hat{y}_i - y_i| = 288.084}$

Here $n = 5$;

$\sum_{i=1}^{n} (\hat{y}_i - y_i)^2 = 50256.739$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{46+20+52+30+57}{5} = \frac{205}{5} = 41$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{354+190+405+263+451}{5} = \frac{1663}{5} = 332.6$$

$$b_1 = \frac{\sum_{i=1}^{n} (x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n} (x_i-\bar{x})^2} = \frac{6558}{964} = 6.803$$

2

$b_0 = \bar{y} - b_1\bar{x}$

$= 332.6 - 6.803 \times 41$

$\boxed{b_0 = 53.677}$

$y = b_0 + b_1 x$

$\boxed{y = 53.677 + 6.803\, x}$ ⇐ use for $\hat{y}_i$ using x_i for x in the eq^{n}

Ans 2) ⇒ $y = 53.677 + 6.803 \times 38$

$\boxed{y = 312.191}$

Ans 1) ⇒ $MAE = \frac{1}{n}\sum_{i=1}^{n} |\hat{y}_i - y_i|$

$= \frac{1}{5}\, 288.084 = 57.617$

$MSE = \frac{1}{n}\sum_{i=1}^{n} (\hat{y}_i - y_i)^2 = \frac{1}{5} \times 50256.739 = 10051.348$

$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (\hat{y}_i - y_i)^2} = \sqrt{\frac{1}{5} \times 50256.739} = \sqrt{10051.348}$

$= 100.256$

Ans 3) OSL

$X'Y = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix} = \begin{bmatrix} 1663 \\ 91025 \end{bmatrix}$

$X'X = \begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix} = \begin{bmatrix} 5 & 205 \\ 205 & 9369 \end{bmatrix} \Rightarrow (X'X)^{-1}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$(X'X)^{-1} = \frac{1}{(5 \times 9369) - (205 \times 205)} \begin{bmatrix} 9369 & -205 \\ -205 & 5 \end{bmatrix}$

$= \frac{1}{46843 - 42025} \begin{bmatrix} 9369 & -205 \\ -205 & 5 \end{bmatrix}$

$= \frac{1}{4818} \begin{bmatrix} 9369 & -205 \\ -205 & 5 \end{bmatrix}$

$= \begin{bmatrix} 1.9445 & -0.042 \\ -0.042 & 0.001 \end{bmatrix}$

$b = (X'X)^{-1} X'Y$

$= \begin{bmatrix} 1.9445 & -0.042 \\ -0.042 & 0.001 \end{bmatrix} \begin{bmatrix} 1663 \\ 91025 \end{bmatrix}$

$b = \begin{bmatrix} -589.347 \\ 21.179 \end{bmatrix} \begin{matrix} \leftarrow b_0 \\ \leftarrow b_1 \end{matrix}$

Ans 4 tabulated in Last column in the ~~table~~

Ans 5 Age ⇒ independent ⇒ predictor   Gradient Descent.
BP ⇒ dependant. ⇒ outcome.   ↳ decrease slope.

b_1 slope $m = \overset{b_1}{m} - L \times \overset{\leftarrow \text{given}}{\frac{-2}{n}} \sum_{i=0}^{n} x_i (y_i - \hat{y}_i)$ $\frac{d}{dx} b_1$

b_0 q interscep $- c = \overset{b_0}{c} - L \times \frac{-2}{n} \sum_{i=0}^{n} (y_i - \hat{y}_i)$ $\frac{d}{dx} b_0$

Initially m & $c = b_0$ $\downarrow b_0\ b_1$.

$\boxed{L = 0.0001}$

$b_1 = m = 6.803 - 0.001 \times \frac{-2}{5}$ (-0.092)

$= 6.8029632$

$b_0 = c = 53.677 - 0.001 \times -2/5\ (0)$
$= 53.677.$

Ans
$y = c + mx$
$y = 53.677 + 6.8029632x$

x	y	~~$x(y_i - \hat{y}_i)$~~	~~$y(y_i - \hat{y}_i)$~~ $(y_i - \hat{y}_i)$	$\hat{y}_i$	$x_i (y_i - \hat{y}_i)$
46	354	~~21.4~~	−12.615	366.615	−580.290
20	190	~~−142.6~~	0.263	189.737	5.260
52	405	~~72.4~~	−2.433	407.433	−126.516
30	263	~~−69.6~~	5.233	257.767	156.990
57	451	~~118.4~~	9.552	441.448	544.464
205	1663	~~6~~	0		−0.092

~~$y = 6.8029632 + 53.677x$~~

Integrate the foll for limit [0,1]   Slide 2   1

1) $\int_0^1 8x^3\,dx$

$\Rightarrow 8\int_0^1 x^3\,dx$

$\Rightarrow 8\,\frac{x^4}{4}\Big|_0^1$ — $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$\Rightarrow 8\left(\frac{1^4}{4} - \frac{0^4}{4}\right)$ — $\int_a^b f x\,dx = g(x)\Big|_a^b = g(b) - g(a)$

$\Rightarrow 8\left(\frac{1}{4} - 0\right) = \frac{8}{4} = 2$ ✓

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2) $\int_0^1 (x^e + e^x + e^e)\,dx$

$\Rightarrow \int_0^1 x^e\,dx + \int_0^1 e^x\,dx + \int_0^1 e^e\,dx$

$\Rightarrow \frac{x^{e+1}}{e+1}\Big|_0^1 + e^x\Big|_0^1 + e^e\int_0^1 1\,dx$ — $\int e^x\,dx = e^x + c$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$+ (e^1 - e^0) + e^e\,x\Big|_0^1$ — $\int 1\,dx = x + c$

$+ (2.718 - 1) + e^e(1-0)$

$(e-1)$

$0.269 + 1.718 + e^e$ ✓

15.154   17.141

$-\frac{1}{2} + 2$ $-\frac{1}{2}$

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3) $[(x^3+3x+4)/\sqrt{x}]\,dx$

$= \int_0^1 \frac{x^3+3x+4}{\sqrt{x}}\,dx$ $\int uv\,dx = u\int v\,dx - \int(u'\int v\,dx)\,dx$

$= \int_0^1 \frac{x^3}{\sqrt{x}}\,dx + \int_0^1 \frac{3x}{\sqrt{x}}\,dx + \int_0^1 \frac{4}{\sqrt{x}}\,dx$ P.T.O

$= \cancel{\frac{x^4}{4}} \int_0^1 \frac{\overset{u}{x^3}\cdot \overset{v}{1}}{\sqrt{x}}\,dx = x^3\int \frac{1}{\sqrt{x}}\,dx - \int\left(\frac{d}{dx}x^3 \int \frac{1}{\sqrt{x}}\,dx\right)dx$

$= x^3\int x^{-1/2}\,dx - \int\left(3x^2 \int x^{-1/2}\,dx\right)dx$

$= x^3\,\frac{x^{+1/2}}{1/2}\Big|_0^1 - \int\left(3x^2 \cdot \frac{x^{1/2}}{1/2}\Big|_0^1\right)dx$

$= x^3\,2(1^{1/2} - 0) - \int\left(3x^2 \times 2(1^{1/2})\right)dx$

$= 2x^3 - \int(3x^2 \times 2)\,dx$

3) $\int_0^1 ((x^3+3x+4)/\sqrt{x})\,dx$.

w.k.t $\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$.

here $u = x^3+3x+4$

$v = 1/\sqrt{x}$

$\therefore \Rightarrow x^3+3x+4\int_0^1 \frac{1}{\sqrt{x}}\,dx - \int_0^1\left[\frac{d}{dx}(x^3+3x+4)\int_0^1 \frac{1}{\sqrt{x}}\,dx\right]dx$

$\Rightarrow (x^3+3x+4)\times 2\sqrt{x}\Big|_0^1 - \int_0^1\left[3x^2+3+0 \times 2\sqrt{x}\Big|_0^1\right]dx$.

$- \int\frac{1}{\sqrt{x}} = 2\sqrt{x}+c$

$\frac{d}{dx}x^n = nx^{n-1}$

$\sqrt{x} = x^{1/2}$

$\frac{d}{dx}k = 0$

$\Rightarrow (x^3+3x+4)\times 2(1^{1/2}-0^{1/2}) - \int_0^1\left[(3x^2+3)\times 2(1^{1/2}-0^{1/2})\right]dx$

$\Rightarrow (x^3+3x+4)\times 2 - \int_0^1 (6x^2+6)\,dx$

$\Rightarrow 2x^3+6x+8 - 2\frac{x^3}{3}\Big|_0^1 + 6x\Big|_0^1$

$\int x^n = \frac{x^{n+1}}{n+1}$

$\Rightarrow 2x^3+6x+8 - 2(1^3-0^3) + 6(1-0)$

$\Rightarrow 2x^3+6x+8 - 2 + 6$

$\Rightarrow 2x^3+6x+12$

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4) $\int_0^1 (2x^2+e^x)\,dx$

$= \int_0^1 2x^2\,dx + \int_0^1 e^x\,dx$

$- \int x^n\,dx = \frac{x^{n+1}}{n+1}$

$- \int e^x\,dx = e^x + c$

$= 2\frac{x^3}{3}\Big|_0^1 + e^x\Big|_0^1$

$= \frac{2}{3}(1^3-0^3) + (e^1-e^0)$

$= 2/3 + 2.718 - 1$

$= 0.667 + 1.718$

$= 2.385$

5) $\int_0^1 [(1-x)\sqrt{x}]\,dx$ (3)

$= \int_0^1 (1-x)\,x^{1/2}\,dx$.

$= \int_0^1 (x^{1/2} - x^{1+1/2})\,dx$

$= \int_0^1 (x^{1/2} - x^{3/2})\,dx$

$= \int_0^1 x^{1/2}\,dx - \int_0^1 x^{3/2}\,dx$ —— $\int x^n = \dfrac{x^{n+1}}{n+1} + c$.

$= \left.\dfrac{x^{3/2}}{3/2}\right|_0^1 - \left.\dfrac{x^{5/2}}{5/2}\right|_0^1$

$= \dfrac{2}{3}(1^{3/2} - 0^{3/2}) - \dfrac{2}{5}(1^{5/2} - 0^{5/2})$.

$= \dfrac{2}{3}(1) - \dfrac{2}{5}(1)$.

$= \dfrac{10-6}{15} = \dfrac{4}{15} = 0.267$ // ✓

$\int e^x = e^x + c$.

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6) $\int_0^1 4^x e^{2x}\,dx$.

$\int uv\,dx = u\int v\,dx - \int\left(\dfrac{d}{dx}u \int v\,dx\right)dx$.

$\cancel{\int_0^1 4^x}\,dx$ Here $u = 4^x$

$v = e^{2x}$.

$\dfrac{d}{dx}a^x = a^x \log a$.

$= 4^x \int_0^1 e^{2x}\,dx - \int_0^1\left(\dfrac{d}{dx}4^x \int_0^1 e^{2x}\,dx\right)dx$

$= 4^x \left.\dfrac{e^{2x}}{2}\right|_0^1 - \int_0^1\left(4^x \log 4 \left.\dfrac{e^{2x}}{2}\right|_0^1\right)dx$.

$= \dfrac{4^x}{2}(e^2 - e^0)$ (3.195) $- 4^x \log 4\, \dfrac{e^2 - e^0}{2}$. —— $\int a^x dx = \dfrac{a^x}{\log a} + c$

$= \dfrac{4^x}{2}\, 6.389 - 3.195 \log 4 \int_0^1 4^x\,dx$ ~~3.195~~

$= \dfrac{4^x}{2}\, 6.389 - 3.195\ \cancel{\log 4}\ \cancel{\dfrac{4^{x+1}}{}} \left.\dfrac{4^x}{\log 4}\right|_0^1$

$= \dfrac{4^x}{2}\, 3.195$

$- 3.195 \times (4^1 - 4^0)$ $= 4^x \times 3.195 - 9.584$

$= \dfrac{4^x}{2}\, 3.195 - 3.195 \times (4-1)$

3.195×3

(4) 7) $\int_0^1\int_0^1 42y^2 - 12x \, dx \, dy$.

$\Rightarrow \int_0^1 \left[42y^2 x \Big|_0^1 - 12\frac{x^2}{2} \Big|_0^1 \right] dy$.

$\Rightarrow \int_0^1 42y^2(1-0) - 6(1^2-0^2) \, dy$.

$\Rightarrow \int_0^1 (42y^2 - 6) \, dy$

$\Rightarrow 42\frac{y^3}{3}\Big|_0^1 - 6y\Big|_0^1$

$= \frac{\cancel{42}^{14}}{\cancel{3}_1}(1^3 - 0^3) - 6(1-0)$

$= 14(1) - 6(1)$

$= \underline{\underline{8}}$.

8) $\int_0^1\int_0^1 10x^2y^3 - 6 \, dx \, dy$.

$\Rightarrow \int_0^1 \left(10y^3 \frac{x^3}{3}\Big|_0^1 - 6x \Big|_0^1 \right) dy$

$\Rightarrow \int_0^1 \frac{10y^3}{3}(1^3 - 0^3) - 6(1-0) \, dy$

$\Rightarrow \int_0^1 \frac{10}{3}y^3 - 6 \, dy$

$\Rightarrow \frac{10}{3}\frac{y^4}{4}\Big|_0^1 - 6y\Big|_0^1$

$\Rightarrow \frac{10}{12}(1^4 - 0^4) - 6(1^4 - 0)$

$\Rightarrow \frac{10}{12}(1) - 6$.

$= \frac{\cancel{10}^5}{\cancel{12}_6} - 6 = \frac{5-36}{6} = -\frac{31}{6}$ //

Diffrentiate.

1) $\frac{d}{dx} 20x^{-4} + 9$.

$\frac{d}{dx} 20x^{-4} + \frac{d}{dx} 9$

$\frac{d}{dx} k = 0$

$\frac{d}{dx} x^n = nx^{n-1}$

$20(-4x^{-4-1}) + 0$.

$20(-4x^{-5}) + 0$.

$\underline{\underline{-80x^{-5}}}$

2) $x^3 + 3x + 4$

$= \frac{d}{dx} x^3 + 3x + 4$

$= \frac{d}{dx} x^3 + \frac{d}{dx} 3x + \frac{d}{dx} 4$

$= 3x^{3-1} + 3(1x^{1-1}) + 0$.

$(1x^0)$

$= 3x^2 + 3(1) + 0$.

$= \underline{\underline{3x^2 + 3}}$

3) $\ln(10)$

$\frac{d}{dx} \ln(10) = 0$

constant w.r.t x

~~ln (10)~~

4) $\frac{8}{x^3}$

$\frac{d}{dx} \frac{8}{x^3}$. $\Rightarrow \frac{d}{dx} 8x^{-3}$

$= 8(-3x^{-3-1})$

$= 8(-3x^{-4})$.

$= -24x^{-4}$ //

(6) 5) $\ln(2x)$.

$\frac{d}{dx}\ln(2x)$. $\frac{d}{dx}\log x = \frac{1}{x}$, $x>0$.

$= \frac{1}{\cancel{2}x}\cdot\cancel{2}$

$= \frac{1}{x}$

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6) $\frac{(x+1)}{x}$

$\frac{d}{dx}\frac{(x+1)}{x}$. $\because \frac{d}{du}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $(v\neq 0)$

Here $u = x+1$
$v = x$

$$= \frac{x\frac{d(x+1)}{dx} - (x+1)\frac{dx}{dx}}{x^2}$$

$$= \frac{x(1x^{1-1}+0) - (x+1)(1x^{1-1})}{x^2}$$

$$= \frac{x(1+0) - (x+1)(1x^{0})}{x^2} \quad = 1$$

$$= \frac{x-(x+1)}{x^2}$$

$$= \frac{\cancel{x}-\cancel{x}-1}{x^2} = \frac{-1}{x^2}$$ //

Slide 3 (1)

gold → Predicted

sys output	urgent	normal	spam	Total
urgent	8	10	1	19
normal	5	60	50	115
spam	3	30	200	233
Total	16	100	251	~~474~~ 367

precision = $\frac{\text{correctly predicted}}{\text{Total predicted}}$

class urgent precision = 8/16 = 0.5

class normal precision = 60/100 = 0.6

class spam precision = 200/251 = 0.797

Recall = $\frac{\text{correctly classified}}{\text{Actual}}$

class urgent recall = 8/19 = 0.421

class normal recall = 60/115 = 0.521

class spam recall = 200/233 = 0.858

accuracy = $\frac{8+60+200}{367} = \frac{268}{367} = 0.730$; $\frac{\text{Total correctly classified}}{\text{Actual}}$

weighted Avg Precision = Actual urgent instances × precision of urgent + × normal + × spam

$= \frac{19}{367} \times 0.5 + \frac{115}{367} \times 0.6 + \frac{233}{367} \times 0.797$

$= 0.720$

(2) weighted Avg Recall = Actual urgent instances × precision of urgent + normal spam

$$= \frac{19}{367} \times 0.421 + \frac{115}{367} \times 0.521 + \frac{233}{367} \times 0.858$$

$$= 0.720$$

70	20
10	20

TP = 70

FN = 20

FP = 10

TN = 20

$$TPR = \frac{TP}{TP+FN} = \frac{70}{70+20} = 0.778$$

$$FPR = \frac{FP}{FP+TN} = \frac{10}{10+20} = 0.333$$

Accuracy = (70 + 20) / (70 + 20 + 20 + 10)

recall = TPR = 0.778

$$\text{precision} = \frac{TP}{TP+FP} = \frac{70}{70+10} = 0.875$$

70	20
15	20

TP = 70

FN = 20

FP = 15

TN = 20

$$TPR = \frac{TP}{TP+FN} = \frac{70}{70+20} = 0.778$$

$$FPR = \frac{FP}{FP+TN} = \frac{15}{15+20} = 0.429$$

$$\text{Accuracy} = \frac{70+20}{70+20+20+15}$$

recall = TPR = 0.778

$$\text{precision} = \frac{TP}{TP+FP} = \frac{70}{70+15} = 0.824$$

Age of candidates appearing in an exam are represented as $N(27, 3)$. Find the % of candidates with age (1)

a) less than 25

b) at least 29

c) btw 24-31

Slide 5

$N(27, 3)$

Given ⇒ mean = 27

SD = 3

$x = 25$

$Z = \frac{x-\mu}{\sigma} = \frac{25 - 27}{3}$

$= -0.667$

$x = 29$

$Z = \frac{x-\mu}{\sigma} = \frac{29-27}{3}$

$= 0.667$

$x = 24$

$Z = \frac{x-\mu}{\sigma} = \frac{24-27}{3}$

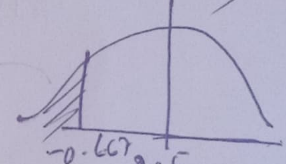
$= -1$

$x = 31$

$Z = \frac{x-\mu}{\sigma} = \frac{31-27}{3}$

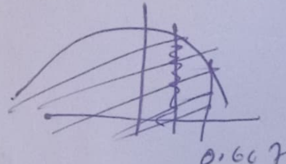
$= 1.333$

$P(X < 25) = P(Z < -0.667)$



$P(X >= 29) = P(Z > 0.667)$

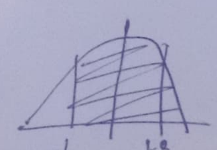
$= 1 - P(Z < -0.667)$



$P(24 < x < 31) = P(-1 < Z < 1.333)$

$= P(Z < 1.333) - P(-1 < Z)$

$=$



① PDF of $x \Rightarrow f(x) = \begin{cases} 6x(1-x), & 0<x<1 \\ 0 & \text{o.w.} \end{cases}$

PDF of $Y = x^{5/3}$.

$G(y) = P(Y \le y)$.

$g(y) = \dfrac{d\,G(y)}{dy}$.

slide 6

$f(x) = 6x(1-x) \quad 0<x<1$.

$y = x^{5/3}$. $\longrightarrow x^{m/n} = \sqrt[n]{x^m}$.

$y = \sqrt[3]{x^5}$ here $m=5$, $n=3$.

$\sqrt[3]{x^5}$

$y^3 = x^5$.

$\sqrt[5]{y^3} = x^5$

$\boxed{y^{3/5} = x}$

$x = y^{3/5}$ diff w.r.t y.

$\dfrac{dx}{dy} = \dfrac{d(y^{3/5})}{dy}$

$= \dfrac{3}{5}(y^{3/5-1})$ $\quad \dfrac{d}{dx} x^n = nx^{n-1}$.

$= \dfrac{3}{5} y^{\frac{3-5}{5}}$

$\dfrac{dx}{dy} = \dfrac{3}{5} y^{\frac{-2}{5}}$ $\quad \left|\dfrac{dx}{dy}\right| =$

$f(y) = f(x) \cdot \left|\dfrac{dx}{dy}\right|$ — Jacobian.

$= 6x(1-x) \times \dfrac{3}{5} y^{-2/5}$

$= 6y^{3/5}(1-y^{3/5}) \times \dfrac{3}{5} y^{-2/5}$ $\quad f(y) = (6y^{3/5} - 6y^{6/5}) \times \dfrac{3}{5} y^{-2/5}$

2

$f(y) = \frac{18}{5} y^{1/5} - \frac{18}{5} y^{4/5}$

$0 < x < 1$

$0 < \sqrt[5]{y^3} < 1$.

$0 < y^3 < 1$.

$0 < y < 1$.

$\Rightarrow$ $G(y) = P(Y \leq y)$.

$= P(X^{5/3} \leq y)$

$= P(X \leq y^{3/5})$

$y = x^{5/3}$.

$y^{3/5} = x$.

$= \int_0^{y^{3/5}} 6x(1-x)\,dx$.

$= \int_0^{y^{3/5}} (6x - 6x^2)\,dx$

$(a^m)^n = a^{mn}$.

$= (3x^2 - 2x^3 \Big|_0^{y^{3/5}}$.

$= 3(y^{3/5})^2 - 2(y^{3/5})^3$.

$\frac{3}{5} \times 2 = \frac{6}{5}$.

$= 3y^{6/5} - 2y^{9/5}$

$g(y) = \frac{dG(y)}{dy}$

$= \frac{d}{dy}(3y^{6/5} - 2y^{9/5})$

$= 3 \times \frac{6}{5} y^{6/5 - 1} - 2 \times \frac{9}{5} y^{9/5 - 1}$

$= \frac{18}{5} y^{\frac{6-5}{5}} - \frac{18}{5} y^{\frac{9-5}{5}} = \frac{18}{5} y^{1/5} - \frac{18}{5} y^{4/5},$

$0 < y < 1$

Let X & Y be jointly continuous random var with

① joint pdf

$$f(x,y)\begin{cases}10x^2y^3-6 & 0<x,y<1 \\ 0 & \text{OW}\end{cases}$$

$0<x<1$
$0<y<1.$

Slide 7

1) $P(X>Y)$.

2) marginal PDF $f_X(x)$ $f_Y(y)$.

X & Y independent? Find $cov(x,y)$

$$P(X>Y) = 1-\int_0^1\left[\int_0^1 10x^2y^3-6\,dx\right)dy.$$

$$=1-\int_0^1\left[\cancel{2}\,10y^3\times\frac{x^3}{3}\Big|_0^1-6x\Big|_0^1\,dy\right.$$

$$=1-\int_0^1\left[\frac{10}{3}y^3(1^3-0^3)-6(1-0)\,dy\right.$$

$$=1-\int_0^1\left(\frac{10}{3}y^3-6\right)dy$$

$$=1-\left[\frac{10}{3}\frac{y^4}{4}\Big|_0^1-6y\Big|_0^1\right]$$

$$=1-\left[\frac{5\times\cancel{2}}{6\cancel{12}}(1^4-0^4)-6(1-0)\right]$$

$$=1-[5/6-6]$$

$$=1-\left[\frac{5-36}{6}\right]$$

$$=1-\left[\frac{-31}{6}\right]$$

$$=1+\frac{31}{6}$$

$$=\frac{6+31}{6}=\frac{37}{6}$$

②

$$f_X(x) = \int_0^1 10x^2y^3 - 6\,dy = 10x^2\frac{y^4}{4}\Big|_0^1 - 6y\Big|_0^1$$

y

$$= \frac{5}{2}x^2(1^4 - 0^4) - 6(1-0)$$

$$= \frac{5}{2}x^2 - 6 \qquad 0 < x < 1.$$

$$f_Y(y) = \int_0^1 10x^2y^3 - 6\,dx.$$

x

$$= 10y^3\frac{x^3}{3}\Big|_0^1 - 6x\Big|_0^1$$

$$= \frac{10}{3}y^3(1^3 - 0^3) - 6(1^3 - 0).$$

$$= \frac{10}{3}y^3 - 6. \qquad 0 < y < 1.$$

X & Y are independent if

$10x^2y^3 - 6.$

$$f_{X,Y}(x,y) = f_X(x)\,f_Y(y).$$

$$= \left(\frac{5}{2}x^2 - 6\right)\left(\frac{10}{3}y^3 - 6\right).$$

$$= \frac{25}{3}x^2y^3 - \frac{30}{2}x^2 - \frac{60}{3}y^3 + 36.$$

$$f_{X,Y}(x,y) \neq f_X(x)\,f_Y(y)$$

$E(X) =$

~~Problem #8~~

5) cov (x,y).

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$= \int_0^1 x\left(\frac{5}{2}x^2 - 6\right) dx.$$

$$= \int_0^1 \frac{5}{2}x^3 - 6x \; dx = \frac{5}{2}\frac{x^4}{4}\Big|_0^1 - \frac{\cancel{6}^{\,3}}{\cancel{2}}x^2\Big|_0^1$$

$$= \frac{5}{8}(1^4 - 0^4) - 3(1^2 - 0^2)$$

$$= \frac{5}{8} - 3 = \frac{5-24}{8} = \frac{-19}{8} \;//$$

$$E(Y) = \int_{-\infty}^{\infty} y f_Y(y)\,dy$$

$$= \int_0^1 y\left(\frac{10}{3}y^3 - 6\right)dy = \int_0^1 \frac{10}{3}y^4 - 6y \; dy$$

$$= \frac{10}{3}\frac{y^5}{5}\Big|_0^1 - \frac{\cancel{6}^{\,3}\,y^2}{\cancel{2}}\Big|_0^1 = \frac{5\,\cancel{10}}{3\,\cancel{15}}(1^5 - 0^5) - 3(1^2 - 0^2)$$

$$= \frac{5}{3} - 3 = \frac{5-9}{3} = \frac{-4}{3}$$

$$E(XY) = \int_0^1\left(\int_0^1 xy(10x^2y^3 - 6)\,dy\right)dx$$

$$= \int_0^1\left[\int_0^1 10x^3y^4 - 6xy\;dy\right]dx = \int_0^1 \cancel{10}^{\,2}x^3\frac{y^5}{\cancel{5}}\Big|_0^1 - \cancel{6}^{\,3}x\frac{y^2}{\cancel{2}}\Big|_0^1 dx$$

$$= \int_0^1 2x^3(1^5 - 0^5) - 3x(1^2 - 0^2)\,dx = \int_0^1 2x^3 - 3x\,dx$$

$$= \cancel{2}\frac{x^4}{2\cancel{4}}\Big|_0^1 - 3\frac{x^2}{2}\Big|_0^1 = \frac{1}{2}(1^4 - 0^4) - \frac{3}{2}(1^2 - 0^2)$$

$$= \frac{1}{2} - \frac{3}{2} = -2/2 = -1 \;//$$

4

$$\text{cov}(X,Y) = E(X,Y) - E(X) \times E(Y)$$

$$= (-1) - \left(\frac{-19}{8}\right) \times \left(\frac{-4}{3}\right)$$

$$= -1 - 3.167$$

$$\text{cov}(X,Y) = \underline{\underline{-4.167}}$$