ALL CALCULATIONS – 01APR2021

Variance

Variance describes how much a random variable differs from its expected value. It entails computing squares of deviations.

$$s^2 = \frac{\sum_{1}^{n=1} (x_i - \bar{x})^2}{n}$$

x : Individual data points
n : Total number of data points
$\bar{x}$: Mean of data points

Standard Deviation

Deviation is the difference between each element from the mean.

Deviation = $(x_i - \mu)$

Population Variance is the average of squared deviations.

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} = (x_i - \mu)^2$$

Sample Variance is the average of squared differences from the mean.

$$s^2 = \frac{1}{(n-1)} \sum_{i=1}^{N} = (x_i - \bar{x})^2$$

Standard Deviation is the measure of the dispersion of a set of data from its mean.

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}$$

The formula for computing the covariance of the variables X and Y is

$$\text{COV} = \frac{\sum_{i=1}^{n}(X_i - \bar{x})(Y_i - \bar{y})}{n-1},$$

with $\bar{x}$ and $\bar{y}$ denoting the means of X and Y, respectively.

Covariance Matrix

$$C = \begin{pmatrix} \sigma(x,x) & \sigma(x,y) \\ \sigma(y,x) & \sigma(y,y) \end{pmatrix}$$

$$COV(x,y) = \sigma_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$VAR(x) = \sigma_x^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$$

$$CORR(X,Y) = \frac{COV(X,Y)}{SD(X)\times SD(Y)}$$

3.19 Theorem. *The covariance satisfies:*

$$\mathsf{Cov}(X,Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y).$$

The correlation satisfies:

$$-1 \leq \rho(X,Y) \leq 1.$$

If $Y = aX + b$ for some constants a and b then $\rho(X,Y) = 1$ if $a > 0$ and $\rho(X,Y) = -1$ if $a < 0$. If X and Y are independent, then $\mathsf{Cov}(X,Y) = \rho = 0$. The converse is not true in general.

Input Data:

Day	x	y
1	30	5
2	35	8
3	40	8
4	25	4
5	35	5

Solution:

X	Y	Z	XI - MEAN(X)	YI - MEAN(Y)	ZI - MEAN(Z)	E * F	E * G	F * G	SQRDEV(X)	SQRDEV(Y)	SQRDEV(Z)
4	2	0.6	-0.1	-0.08	-0.004	0.008	0.0004	0.00032	0.01	0.0064	0.000016
4.2	2.1	0.59	0.1	0.02	-0.014	0.002	-0.0014	-0.00028	0.01	0.0004	0.000196
3.9	2	0.58	-0.2	-0.08	-0.024	0.016	0.0048	0.00192	0.04	0.0064	0.000576
4.3	2.1	0.62	0.2	0.02	0.016	0.004	0.0032	0.00032	0.04	0.0004	0.000256
4.1	2.2	0.63	0	0.12	0.026	0	0	0.00312	0	0.0144	0.000676
4.1	2.08	0.6			SUM VALUE	0.03	0.007	0.0054	0.1	0.028	0.00172
				COV VALUES	SUM/(n-1)	0.0075	0.00175	0.00135	0.025	0.007	0.00043
						COV(X,Y)	COV(X, Z)	COV(Y, Z)	VAR(X)	VAR(Y)	VAR(Z)
			covariance Mat	COV(X, X)	0.025	0.15811388	CORR(X, Y)				
0.16				COV(X, Y)	0.008						
				COV(X, Z)	0.002						
				COV(Y, X)	0.008						
				COV(Y, Y)	0.007	0.083666	CORR(Y, Z)				
				COV(Y,Z)	0.001						
				COV(Z, X)	0.002						
				COV(Z, Y)	0.001						
				COV(Z, Z)	0.00043	0.02073644	CORR(X, Z)				

Input Data:

Market outcome	P(X,Y)	X	Y	X-E(X)	Y-E(Y)	(X-E(X))(Y-E(Y))
Booming	0.15	10	-3			
Favourable	0.35	6	-1			
Even	0.25	3	2			
Unfavourable	0.2	-8	4			
Crash	0.05	-15	10			

Mean		
Var		
ST Dev		
COV(X,Y)		
CORR(X,Y)		

Solution:

VARIANCE-COVARIANCE CALCULATION - DISCRETE PROBABILISTIC DATA

p(X, Y)	X	Y		X * P(X,Y)		Y * P(X, Y)	X - E(X)	Y - E(Y)	(X - E(X)) * (Y - E(Y))	COV(X, Y) = [(X - E(X)) * (Y - E(Y))] * P(X, Y)		SQUARE (X - E(X))	M * P(X, Y)	SQUARE (Y - E(Y))	O * P(X, Y)
0.15	10	-3		1.5		-0.45	8	-4	-32	-4.8		64	9.6	16	2.4
0.35	6	-1		2.1		-0.35	4	-2	-8	-2.8		16	5.6	4	1.4
0.25	3	2		0.75		0.5	1	1	1	0.25		1	0.25	1	0.25
0.2	-8	4		-1.6		0.8	-10	3	-30	-6		100	20	9	1.8
0.05	##	10		-0.75		0.5	-17	9	-153	-7.65		289	14.45	81	4.05
			MEAN(X) = SUM(X * P(X, Y))	2	MEAN(Y) = SUM(Y * P(X,Y))	1				-21	SUM	470	49.9	111	9.9
					X	Y									
				MEAN	2	1									
				VAR	49.9	9.9									
			SQRT(VAR)	STDEV	7.064	3.15									
			STDEV(X) * STDEV(Y) * P(X, Y)	COV(X, Y)	-21										
			COV(X, Y) /(STDEV(X) * STDEV(Y)	CORR(X, Y)	-0.94482511										

Input Data:

1. Calculate the correlation coefficient between the two variables x and y shown below:

X:	1	2	3	4	5	6
Y:	2	4	7	9	12	14

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$

Solution:

	X	Y	X * Y	X * X	Y * Y
	1	2	2	1	4
	2	4	8	4	16
	3	7	21	9	49
	4	9	36	16	81
	5	12	60	25	144
	6	14	84	36	196
SUM	21	48	211	91	490

n * sum(xy)	1266
sum(x) * sum(y)	1008
n * sum(x* x)	546
sum(x) * sum(x)	441
n * sum(y * y)	2940
sum(y) * sum(y)	2304

Numerator =	258
dr1 =	105
dr2 =	636
dr1 * dr2	66780
sqrt(dr1 * dr2)	258.42
R Value =	0.998