PRACTICE - 1

3rd Sem, MCA

- Intro to Machine Learning
- Probability distribution
- Linear Regression
- Logistic Regression

Practice

Build the Linear Regression model for the data given here.

- Calculate regression coefficients, MAE, MSE, RMSE, R^2
- Predict the y for x = 38.
- Use Ordinary Least Square (OLS) estimation to calculate the regression coefficients.
- Use the Loss function to calculate the loss in prediction for each data points (x).
- Calculate the revised regression coefficients (after 1st iteration), using Gradient descent method.

Age (x)	BP (Y)
46	354
20	190
52	405
30	263
57	451

$$y = b_0 + b_1 x$$

$$b_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |\hat{y}_i - y_i|$$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2}$$

$$\mathcal{L}(y, t) = \frac{1}{2}(y - t)^2$$

$$R^2 = \frac{SSR}{SST} = \frac{\sum (\hat{y}_i - \bar{y})^2}{\sum (y_i - \bar{y})^2}$$

$$b = (X'X)^{-1} X'Y$$

$$X'Y = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix}$$

$$X'X = \begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{bmatrix}$$

$$m = m - L \times D_m$$

$$c = c - L \times D_c$$

$$D_m = \frac{-2}{n} \sum_{i=0}^{n} x_i (y_i - \bar{y}_i)$$

$$D_c = \frac{-2}{n} \sum_{i=0}^{n} (y_i - \bar{y}_i)$$

Practice

Integrate the following for limit [0,1]:

- $8x^3\,dx$
- $(x^e + e^x + e^e)\,dx$
- $[(x^3+3x+4)/\sqrt{x}]\,dx$
- $(2x^2+e^x)\,dx$
- $[(1-x)\sqrt{x}]\,dx$
- $4^x e^{2x}\,dx$

$$\iint 42y^2 - 12x$$

$$\iint 10x^2y^3 - 6$$

$$\int 1\,dx = x + C$$

$$\int a\,dx = ax + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C;\ n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C;\ a > 0, a \neq 1$$

Differentiate the following:

- $20x^{-4} + 9$
- x^3+3x+4
- $\ln(10)$
- $8/x^3$
- $\ln(2x)$
- $(x+1)/x$

d/dx (k) = 0, where k is any constant

d/dx(x) = 1

d/dx(x^n) = nx^{n-1}

d/dx (kx) = k, where k is any constant

d/dx ($\sqrt{x}$) = $1/2\sqrt{x}$

d/dx (1/x) = $-1/x^2$

d/dx (log x) = 1/x, x > 0

d/dx (e^x) = e^x

d/dx (a^x) = a^x log a

Practice

Use the two confusion matrix given here to answer the following;.

- Calculate the TP, TN, FP, FN, TPR, FPR.
- Calculate model accuracy, recall, precision.

Another model predicts 15 to be wrongly as pass.

Which of the two models is a better one.

		gold labels		
		urgent	normal	spam
system output	urgent	8	10	1
	normal	5	60	50
	spam	3	30	200

	Predicted Pass	Predicted Fail
Actual Pass	70	20
Actual Fail	10	20

		gold labels			
		urgent	normal	spam	
system output	urgent	8	10	1	**precision**u$=\frac{8}{8+10+1}$
	normal	5	60	50	**precision**n$=\frac{60}{5+60+50}$
	spam	3	30	200	**precision**s$=\frac{200}{3+30+200}$
		recallu $=\frac{8}{8+5+3}$	**recall**n $=\frac{60}{10+60+30}$	**recall**s $=\frac{200}{1+50+200}$	

		Predicted Class		
		Positive	**Negative**	
Actual Class	**Positive**	True Positive (TP)	False Negative (FN) **Type II Error**	**Sensitivity** $\frac{TP}{(TP+FN)}$
	Negative	False Positive (FP) **Type I Error**	True Negative (TN)	**Specificity** $\frac{TN}{(TN+FP)}$
		Precision $\frac{TP}{(TP+FP)}$	**Negative Predictive Value** $\frac{TN}{(TN+FN)}$	**Accuracy** $\frac{TP+TN}{(TP+TN+FP+FN)}$

Practice

A coin is tossed 'n' number of times.

- If the coin is 'Fair', probability of the occurrence of a head 6 times is the same as the probability that a head comes 8 times, then find the value of n.
- Uncertain about coin's 'fairness', use the maximum likelihood estimation to find the probability of head on a single toss, if there were 10 heads for this computed 'n'.
- Also apply Log-likelihood to find the answer. Are the both answer same?
- For this 'n' & 'p' find the probability of getting minimum 9 heads.
- If the probability of the occurrence of a head maximum 3 times is the same as the probability that a head comes 4 times, then find the value of n.

$$P(x{:}n,p) = {}^{n}C_{x}\, p^{x}\, (1-p)^{n-x}$$

$$_{n}C_{r} = \frac{n!}{r!(n-r)!}$$

Practice

Age of candidates appearing in an exam are represented as N(27,3). Find the percentage of candidates with age (a) Less than 25, (b) at least 29, (c) between 24-31.

$$Z = \frac{X - \mu}{\sigma}$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831

Practice

Let the probability density function of X be given by:

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Find the probability density of $Y = X^{5/3}$

$$G(y) = P(Y \le y)$$

$$g(y) = \frac{dG(y)}{dy}$$

Practice

Let X and Y be jointly continuous random variables with joint PDF

$$f(x,y) = 10x^2y^3 - 6$$ for $0 < x, y < 1$

zero otherwise.

- Find P(X>Y)
- Find the marginal PDFs fX(x) and fY(y)
- Are X and Y independent? Find Cov(x,y).

$$P(X_1 \in [a_1, b_1], \ldots, X_K \in [a_K, b_K]) = \int_{a_1}^{b_1} \cdots \int_{a_K}^{b_K} f_X(x_1, \ldots, x_K) dx_K \ldots dx_1$$

X and Y are independent, if

$$f_{X,Y}(x,y) = f_X(x) f_Y(y)$$

$$E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x) dx$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y) dy$$

$$\mathrm{Cov}(X,Y) = E(XY) - E(X) \times E(Y)$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y) dx$$

END !!