15 tuples (x y)

PCA of:

Mean (x) = 108.5 cm

Mean (y) = 38.3 cm

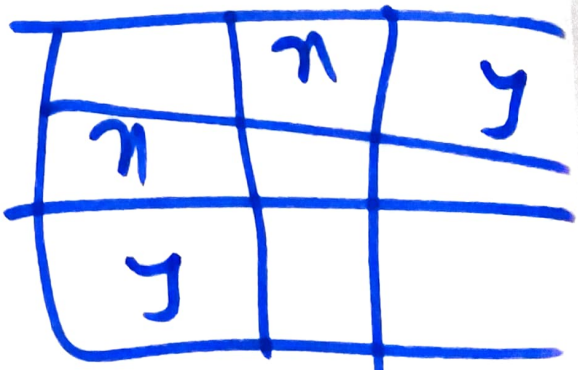
Rainfall (x) ; Runoff (y)

(1) cov (x, y) = 141.0857

(2) cov (x, x) = 250.0357

(3) cov (y, y) = 117.5357

Covariance Matrix



$$\begin{bmatrix} cov(x,x) & cov(x,y) \\ cov(y,x) & cov(y,y) \end{bmatrix}$$

$$\begin{bmatrix} cov(x,x) & cov(x,y) \\ cov(y,x) & cov(y,y) \end{bmatrix}$$

Cov Matrix

$$= \begin{bmatrix} 250.0357 & 141.0857 \\ 141.0857 & 117.5357 \end{bmatrix}$$

Now calculate Eigen values & Eigen vectors of Cov. matrix.

$$\begin{bmatrix} 250.0357 & 141.0857 \\ 141.0857 & 117.5357 \end{bmatrix}$$

Eigen values: $|A - \lambda I| = 0$

$$\begin{vmatrix} (250.0357 - \lambda) & 141.0857 \\ 141.0857 & (117.5357 - \lambda) \end{vmatrix}$$

$$= 0$$

$$(250.0357 - \lambda)(117.5357 - \lambda)$$

$$- (141.0857 * 141.0857)$$

$$= 0$$

$$(141.0857)^2 = 19905.17474$$

$$(250.0357) * (117.5357)$$

$$= 29388.12102$$

$= 29388.12102 - 250.0357\lambda$

$- 117.5357\lambda + \lambda^2$

$- 19905.17474$

$= 0$

$\Rightarrow \lambda^2 - 367.5714\lambda + 9482.946284 = 0$

$\lambda_1 = 339.65178$

$\lambda_2 = 27.91961$

Eigen values

Eigen vecho : $|A - \lambda I| = 0$

$$A = \begin{bmatrix} 250.0357 & 141.0857 \\ 141.0857 & 117.5357 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|A - \lambda I| = \begin{bmatrix} 250.0357 - \lambda & 141.0857 \\ 141.0857 & 117.5357 - \lambda \end{bmatrix}$$

$$= 0$$

$T_1 = 339.65178$

$$\Rightarrow \begin{bmatrix} 89.61608 & 141.0857 \\ 141.0857 & -222.11608 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} -89.61608\, x_1 + 141.0857\, y_1 &= 0 \\ 141.0857\, x_1 - 222.11608\, y_1 &= 0 \end{aligned}$$

$T_2 = 27.91961$

$$\begin{bmatrix} 222.11609 & 141.0857 \\ 141.0857 & 89.61609 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$222.11609x_1 + 141.0857y_1 = 0$$

$$141.0857x_1 + 89.61609y_1 = 0$$

Final Answer :

(1) Eigen values :

$$27.919612;$$

$$339.6517875$$

(2) Eigen Vectors :

(i) $\lambda = 27.919612$

$$\begin{bmatrix} -0.635189 \\ 1 \end{bmatrix}$$

(ii) $T = 339.65178 75$

$$\begin{bmatrix} 1.57433451 \\ 1 \end{bmatrix}$$