ML- Eigen values & Eigen vectors

(1) A = a 3×3 Matrix

X = A non-zero eigen vector

λ = a scalar. = Eigen value of A.

I = Identity matrix of A

$$AX = \lambda X$$

$$\Rightarrow AX = \lambda \cdot I \cdot X$$

$$\Rightarrow AX - \lambda IX = 0$$

$$\Rightarrow (A - \lambda I) \cdot x = 0$$

$$\Rightarrow \boxed{\det |A - \lambda I| = 0}$$

Note: The matrix $(A - \lambda I)$ should be singular.

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(A) Which are the values of λ for which $(A - \lambda I)$ is singular?

i.e. $\det |A - \lambda I| = 0$?? (characteristic eqn)

eg: $A = \begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$

The characteristic eqn is:

$$\begin{pmatrix} (-2-\lambda) & 4 & 2 \\ -2 & (1-\lambda) & 2 \\ 4 & 2 & (5-\lambda) \end{pmatrix}$$

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$\Rightarrow$

$$(-2-\lambda)\left[(1-\lambda)(5-\lambda)-4\right]+$$

$$4\left[(-2)(5-\lambda)-8\right]+$$

$$2\left[-4-4(1-\lambda)\right]=0$$

On Simplification:

$$-\lambda^3+4\lambda^2+27\lambda-90=0$$

$$\Rightarrow \{\lambda^3-4\lambda^2-27\lambda+90=0\}$$

By trial & error:

$$3^3-(4\times3^2)-(27\times3)+90=0$$

$$\Rightarrow \lambda^3-4\lambda^2-27\lambda+90$$

$$=(\lambda-3)(\lambda^2-\lambda-30)$$

$$=(\lambda-3)(\lambda+5)(\lambda-6)$$

∴ Eigen Values = $\{3, -5, 6\}$

We now go on to solve:

$$\begin{pmatrix} (-2-\lambda) & 4 & 2 \\ -2 & (1-\lambda) & 2 \\ 4 & 2 & (5-\lambda) \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

for each eigen value λ.

Eigenvector corresponding to eigenvalue 3

i.e. $\lambda = 3$

P.T.O

$\lambda = 3$

$$\begin{pmatrix} -5 & -4 & 2 \\ -2 & -2 & 2 \\ 4 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Set $x = 1$ (& subsequently $y = 1$ & $z = 1$)

$$-5 - 4y + 2z = 0 \rightarrow (I)$$

$$-2 - 2y + 2z = 0 \rightarrow (II)$$

$(II) - (I)$

$$\Rightarrow 3 + 2y = 0$$

$$\Rightarrow y = -\frac{3}{2}$$

$$\Rightarrow -2 - 2\left(-\frac{3}{2}\right) + 2z = 0$$

$$\Rightarrow -2 + 3 + 2z = 0 \Rightarrow 1 + 2z = 0$$

$$\Rightarrow z = -\frac{1}{2}$$

$4x + 2y + 2z = 0$ → (III)

$y = -\frac{3}{2}$; $z = -\frac{1}{2}$

$4x + 2\left(-\frac{3}{2}\right) + 2\left(-\frac{1}{2}\right) = 0$

$4x + (-3) + (-1) = 0$

$\Rightarrow 4x - 3 - 1 = 0 \Rightarrow 4x = 4$

$\Rightarrow \boxed{x = 1}$

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Eisen values $= \{3, -5, 6\}$

Eisen Vectors $= \{1, -\frac{3}{2}, -\frac{1}{2}\}$