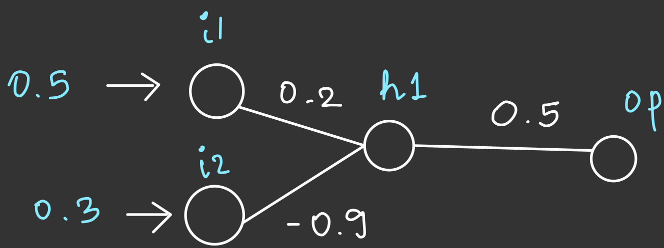
1. Neural Networks

i) An observation with two variables (0.5 , 0.3) is input into the given neural network what is the predicted output using sigmoid activation function ?



$i1 = 0.5$ $\quad$ $w1 = 0.2$

$i2 = 0.3$ $\quad$ $w2 = -0.9$

$w3 = 0.5$

$$h1 = (i1 \times w1 + i2 \times w2)$$

$$= 0.5 \times 0.2 + 0.3 \times -0.9$$

$$= -0.17$$

$$h1_f = \frac{1}{1 + \frac{1}{e^h}}$$ (applying sigmoid fn)

$$= \frac{1}{1 + \frac{1}{e^{-0.17}}}$$

$$= 0.457$$

$$op = h_{1f} \times w_3$$

$$= 0.457 \times 0.5$$

$$= 0.2285$$

$$op_f = \frac{1}{1 + \frac{1}{e^{op}}}$$

$$= \frac{1}{1 + \frac{1}{e^{0.2285}}}$$

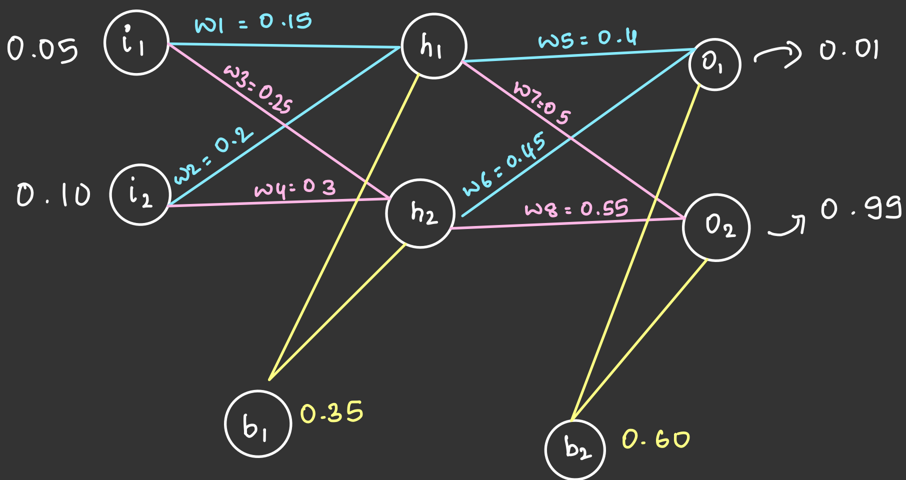
$= \underline{\underline{0.557}}$ is the output

2) Given inputs for neural network are

$i1 = 0.05 \quad i2 = 0.10$

Required output : 0.01 and 0.99

$0.01 \to t_1$, $0.99 \to t_2$



Step 1 : Forward pass

Calculate net input at each hidden layer node

$$h_1 = (w1 \times i1) + (w2 \times i2) + b1$$

$$= (0.15 \times 0.05) + (0.2 \times 0.1) + 0.35$$

$$= \underline{\underline{0.3775}}$$

Apply sigmoid activation function

$$h_{1_f} = \frac{1}{1 + \frac{1}{e^{h}}} = \frac{1}{1 + \frac{1}{e^{0.3775}}}$$

$$= \underline{\underline{0.593}}$$

IIIly for h_2 :

$$h_2 = (w_3 \times i_1) + (w_4 \times i_2) + b_1$$

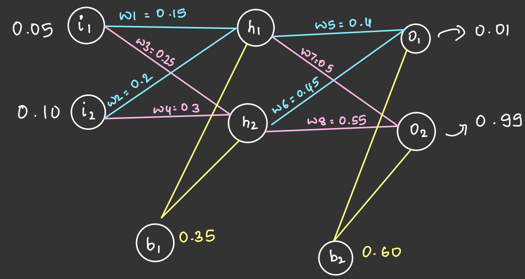
$$= (0.25 \times 0.05) + (0.3 \times 0.1) + 0.35$$

$$= 0.3925$$

$$h_{2_f} = \frac{1}{1 + \frac{1}{e^{h_2}}} = \frac{1}{1 + \frac{1}{e^{0.3925}}}$$

$$h_{2_f} = \underline{\underline{0.597}}$$

$|||^{ly}$ for O_1



$$O_1 = (w5 \times h1) + (w6 \times h_2) + b_2$$

$$= (0.4 \times 0.593) + (0.45 \times 0.597) + 0.6$$

$$= \underline{\underline{1.106}}$$

$$O_{1_f} = \frac{1}{1 + \frac{1}{e^{O_1}}} = \frac{1}{1 + \frac{1}{e^{1.106}}}$$

$$O_{1f} = \underline{\underline{0.751}}$$

$$O_2 = (w_7 \times h_1) + (w_8 \times h_2) + b_2$$

$$= (0.5 \times 0.593) + (0.55 \times 0.597) + 0.6$$

$$= 1.225$$

$$O_{2_f} = \frac{1}{1 + \frac{1}{e^{O_2}}} = \underline{\underline{0.773}}$$

Now calculate total error

$$E_{total} = \sum \frac{1}{2}(\text{target} - \text{output})^2$$

Given target for $O_1 = 0.01 = t_1$

- " - $O_2 = 0.99 = t_2$

Obtained $O_{1f} = 0.751$

$O_{2f} = 0.773$

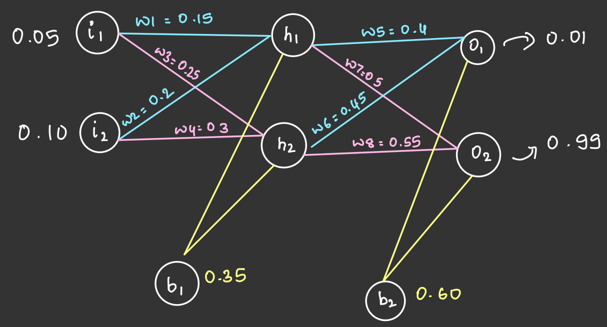
$$E_{total} = \frac{1}{2}(0.01 - 0.751)^2 + \frac{1}{2}(0.99 - 0.773)^2$$

$$= \underline{\underline{0.298}}$$

<u>Step 2</u> Backward pass

- to update weights in order to reduce error and obtain required output

i) $\dfrac{\partial E_{total}}{\partial w_5}$



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use chain rule

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial o_{1f}} \times \frac{\partial o_{1f}}{\partial o_1} \times \frac{\partial o_1}{\partial w_5}$$

wkt $E_{total} = \frac{1}{2}\left[(t_1 - o_{1f})^2 + (t_2 - o_{2f})^2\right]$

$$\therefore \frac{\partial E_{total}}{\partial o_{1f}} = \frac{1}{2}\cdot 2\left((t_1 - o_{1f})\cdot(-1)\right)$$

$$= -(t_1 - o_{1f})$$

$$= -(0.01 - 0.751)$$

$$= 0.741 \quad \text{———} \quad 1$$

$$\frac{\partial O_{1f}}{\partial O_1} \rightarrow ?$$

wkt $O_{1f} = \frac{1}{1+e^{-O_1}} = (1+e^{-O_1})^{-1}$

Chain rule:

$$\frac{\partial O_{1f}}{\partial O_1} = (-1)(1+e^{-O_1})^{-2} \cdot e^{-O_1} \cdot (-1)$$

$$= \frac{e^{-O_1}}{(1+e^{-O_1})^2}$$

Subs

$$\frac{\partial O_{1f}}{\partial O_1} = \frac{e^{-1.106}}{(1+e^{-1.106})^2} = \underline{\underline{0.186}} \quad \text{—} 2$$

$$\frac{\partial O_1}{\partial w_5} \rightarrow ?$$

$$O_1 = (w_5 \times h_{1_f}) + (w_6 \times h_{2_f}) + (b_2)$$

$$\frac{\partial O_1}{\partial w_5} = h_{1_f} = \underline{\underline{0.593}} \quad — ③$$

$$\therefore \frac{\partial E_{total}}{\partial w_5} = ① \times ② \times ③$$

$$= 0.741 \times 0.186 \times 0.593$$

$$= \underline{\underline{0.082}} \quad — ④$$

Now to decrease error, subtract ④ from w_5. Repeat same for all weights