CPP1113

PRINCIPLES OF PHYSICS

Foundation in Information

Technology

ONLINE NOTES

Chapter 4

Oscillatory Motion

4.1 Periodic Motion

1. The most common result of such disturbances is that objects oscillate about (around) the equilibrium position from which it was disturbed.
    a. An object's motion is referred to as **periodic motion** when it repeats itself over and over again. The beating of your heart, the ticking of a clock and the movement of a child on a swing are familiar examples.
    b. When the motion comes exactly back to its original state, we say that the object has gone through a **complete cycle** or **complete oscillation**.
    c. The amount of time it takes for one complete cycle to occur is called the **period T** of the motion
    d. While the period is one way to characterize the speed for the periodic motion, another way is through the **frequency f**.
    e. Frequency is a measure of how frequently the motion repeats; it is most commonly quoted as the number of oscillations (or cycles for short) per second. The frequency relates directly to the period by
    $$f = \frac{1}{T}$$
    f. Since period measures an amount of time its SI unit is seconds; therefore, the SI unit of frequency must be the inverse second s^{-1}. A special unit for frequency is hertz(Hz).

4.2 Simple Harmonic Motion

1. One particular type of motion that is very important in physics is called **simple harmonic motion (SHM).**
2. This type of periodic motion occurs as a result of Hookes's law force. An example of simple harmonic motion is provided by the oscillations of a mass attached to a spring.

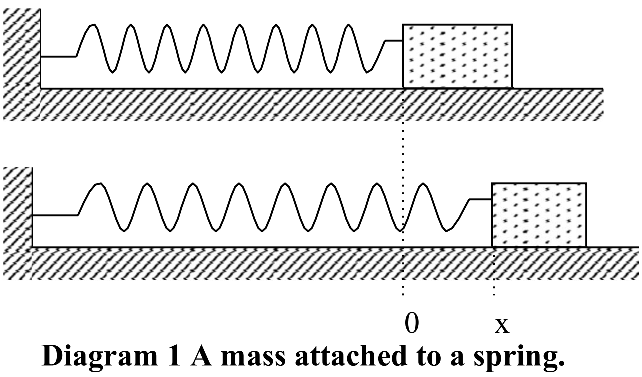

Diagram 1 A mass attached to a spring.

3. When the spring is at its equilibrium length, (x=0), the mass remains at rest.
4. If the mass is displaced from the equilibrium by a distance x, the spring exerts a restoring force given by Hooke"s law, F= -kx. In other words
   "A spring exerts a restoring force that is proportional to the displacement from Equilibrium."

5. Now, suppose the mass is released from rest at the location x = A as shown in diagram 2(a).
6. The spring exerts a restoring force on the mass to the left, causing the mass to accelerate toward the equilibrium position.
7. When the mass reaches x= 0 as shown in diagram 2(b), the net force acting on it is zero. Its speed is not zero at this point, however, so it continues to move to the left.
8. As the mass compresses the spring, it experiences a force to the right, causing it to decelerate and finally come to rest at x = -A as shown in diagram 2(c)
9. The spring continues to exert a force to the right, thus the mass immediately begins to move to the right until it comes to the equilibrium position again at x = 0 as shown in diagram 2(d).
10. The mass continues to move to the right until it comes to rest again at x =A as shown in diagram 2(e), completing one oscillation.

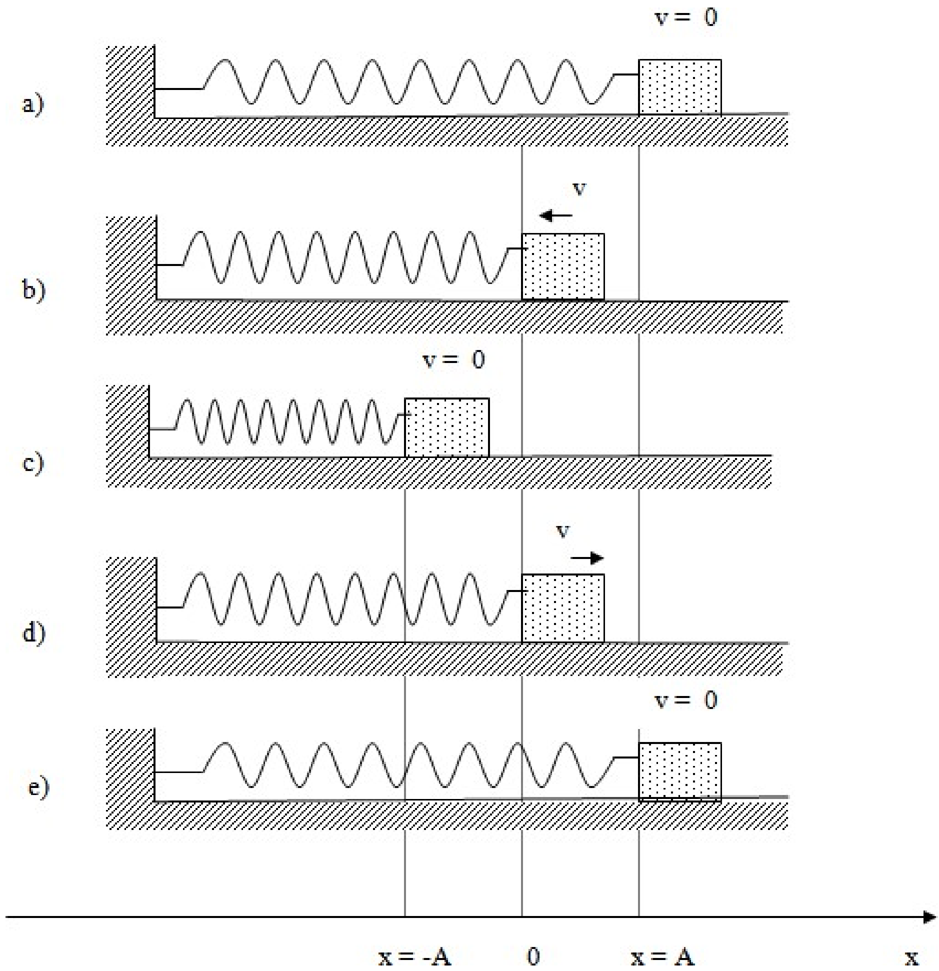

Diagram 2 A mass attached to a spring undergoes simple harmonic motion about x=0.

11. If a pen is attached to the mass, it can trace its motion on a strip of paper moving with constant speed. On this strip chart we obtain a record of mass"s motion as a function of time. The motion of the mass looks like sine or cosine function.

Equation for position, velocity and acceleration of SHM.

a.) Position

1. Mathematical analysis, using the method of calculus, shows that the position of the mass as a function of time can be represented by a sine or a cosine function.

$$x = A\cos\left(\frac{2\pi}{T}t\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$x = A\cos\omega t \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

2. For notational simplicity, we will write the position of a mass on a spring in the terms of ω.

   ω is called the **angular frequency**

   $\omega = 2\pi f \quad = \dfrac{2\pi}{T}$

   SI unit : rad/s = s^{-1}

3. In equation (2) , A is called the **amplitude** of the motion. The amplitude represents the maximum distance the object gets from equilibrium as it moves back-n-forth.
4. The above expression **assumes the object starts its motion at x = A**. It is to be understood that the argument of the cosine function is an angle in **radians**; therefore, your calculator should be in radian mode when using the above expression.
5. The diagram of position versus time in simple harmonic motion is shown in diagram 6.3(a)

b.) Velocity

1. We can obtain the speed of a particle undergoing SHM by differentiating formula (2) with respect to time :

   $v = dx/dt = -\omega A \sin \omega t$..(3)

2. The diagram of velocity versus time in simple harmonic motion is shown in diagram 3(b).
3. The speed is maximum when it goes through x=0. And the speed is zero when the mass is at
   x = A and x = -A.
4. From equation (3), the largest value of sin function is „1", so the magnitude of maximum speed of the mass is $v_{max} = \omega A$

c.) Acceleration

1. The acceleration of the particle is
   $a = dv/dt = -\omega^2 A \cos \omega t$... (4)

2. The diagram of acceleration versus time in simple harmonic motion is shown in diagram 3(c) .
3. Note that the acceleration and position vary with time in the same way, but with opposite signs.
4. That is, when the position has its maximum positive value, the acceleration has its maximum negative value, and so on.
5. By comparing equation (2) and (4), we see that the acceleration can be written as $a = -\omega^2 x$
6. Since the largest value of x is the amplitude A, the maximum acceleration is of magnitude
   $a_{max} = \omega^2 A$

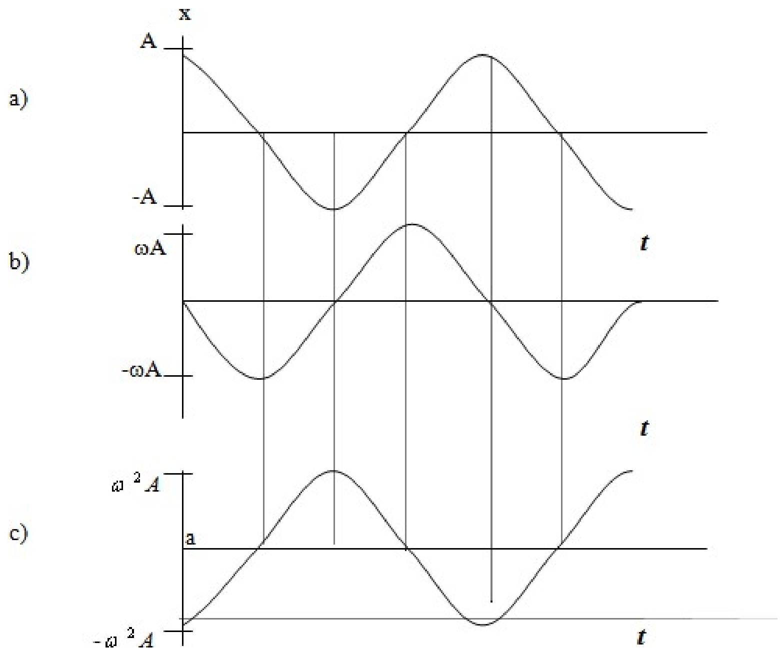

Diagram 3 Position, Velocity and Acceleration versus time

4.3 The Period of a Mass on a Spring.

1. Recall that Hooke's law is the force law that leads to simple harmonic motion $F = ma = -kx$.
2. Substituting the time dependence of x and a, we find that
   $m [-\omega^2 A \cos \omega t] = - k [A \cos \omega t]$ ………(8)

3. Canceling $- A \cos \omega t$ from each side of the equation yields
   $\omega^2 = k/m$ ………………………………
   $\omega = \sqrt{\dfrac{k}{m}}$

4. This gives us a direct relationship between the physical system (k and m) and the motion (ω). Since $\omega = 2\pi f = 2\pi/T$, we can also write the above expression in terms of period
   $T = 2\pi \sqrt{\dfrac{m}{k}}$ …………………….. (5)

5. Notice that T does not depend on the amplitude of the oscillation.

4.4 Energy Conservation in Oscillatory Motion

1. Consider mass-spring system, which the surface is frictionless and the spring is masses. So, the total mechanical energy is constant.

   The kinetic energy is given, $K = ½ mv^2$

2. Substitute the formula $v = -\omega A \sin \omega t$ and $\omega^2 = k/m$ will give
   $K = 1/2kA^2 \sin^2 \omega t$

3. While the potential energy is given, $P = 1/2kx^2$
   Substitute the formula $x = A \cos \omega t$ will give P
   $= 1/2kA^2 \cos^2 \omega t$

4. The Total Energy,
   $E = K + P$
   $= 1/2kA^2 \sin^2 \omega t + 1/2kA^2 \cos^2 \omega t$
   $= 1/2kA^2$

4.5 The Simple Pendulum

1. A simple pendulum is a particle of mass m attached to a frictionless pivot by a cable whose length is L and whose mass is negligible.
2. The small angle back-and-forth swinging of a simple pendulum is SHM, while large angle motion is not.
3. The restoring force of a simple pendulum ,F is proportional to its displacement, $F \propto x$

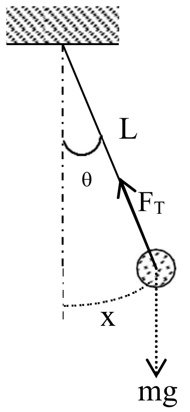

Diagram 4

4. The displacement of the pendulum along the arc is given by $x=L\theta$
5. The restoring force is the component of the weight, mg ,tangent to the arc
   $F= -mg \sin \theta$

6. When θ is small, then sin θ is very nearly equal to θ when θ is specified in radians
7. For angles less than 15 °, the difference between θ(in radians) and sin θ is less than 1 percent.
8. Thus , to a very good approximation of small angles;
   $F= -mg \sin \theta \approx -mg\,\theta$
9. Using $x = L\theta$, we have $F \approx \frac{mg}{L}x$
10. Thus, for small displacements, the motion is essentially simple harmonic, since this equation fits Hooke"s Law, $F = -kx$, where the effective force constant is $k = mg/L$
11. The period of simple pendulum can be found by using Eq.(5). Where k we substitute mg/L.

$$T = 2\pi \sqrt{\frac{m}{mg / L}}$$

$$T = 2\,\pi \sqrt{\frac{L}{g}} \qquad \text{.........................(6)}$$

12. The frequency is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

13. In other words, the period and frequency of a simple pendulum depend only on the length of the string and the value of g.

4.6 Damped Oscillations

1. The amplitude of any real oscillating spring or swinging pendulum slowly decreases in time until the oscillations stop altogether.
2. This is called damped harmonic oscillation.
3. The damping is generally due to the resistance of air and to internal friction within the oscillating system.
4. The energy that is thus dissipated to thermal energy is reflected in decreased amplitude of oscillation. Sometimes the damping is so large that the motion no longer resembles simple harmonic motion.
5. There are three common cases of heavily damped systems.
   a. **Under-damped**
   The system will oscillate but with continuously decreasing amplitude. The amplitude decreases exponentially with time and finally stops.

   b. **Critical damping**
   As the damping is increased, a point is reached where the system no longer oscillates, but simply relaxes back to the equilibrium position.
   c. **Over damped**
   If the damping is increased further, the system is said to be over-damped. In this case, the system still returns to equilibrium without oscillating, but the time required is greater.

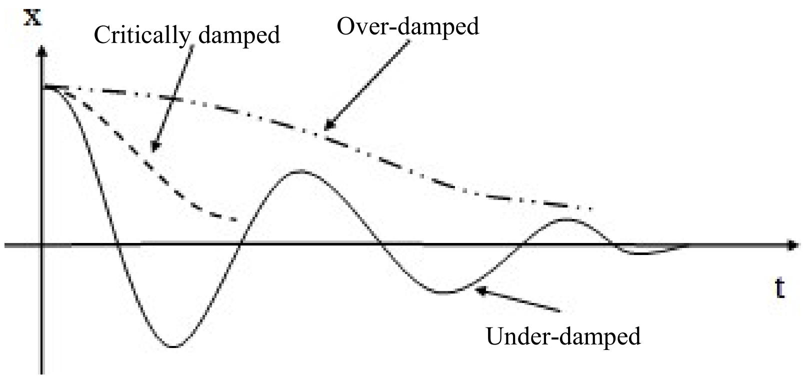

Diagram 5 Damped oscillations

4.7 Driven Oscillations

1. It is possible to compensate for the energy loss in a damped system by applying an external force that does positive work on the system.
2. At any instant, energy can be put into the system by an applied force that acts in the direction of motion of the oscillator.
3. Suppose, for example, that you hold the end of a string from which a small weight is suspended.
4. If the weight is set in motion and you hold your hand still, it will soon stop oscillating.
5. If you move your hand back and forth in a horizontal direction, you can keep the weight oscillating indefinitely.
6. The motion of your hand is said to be " driving" the weight, leading to **driven oscillations.**

End of Chapter 4