

1.

$$\begin{aligned}
 1. \quad & 4x_1^2 - 20x_1 + \frac{x_2^2}{4} + 8 = 0 & f_1 &= 4x_1^2 - 20x_1 + \frac{x_2^2}{4} + 8 \\
 & \frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 + 8 = 0 & f_2 &= \frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 + 8 \\
 & \vec{x}^{(0)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, A_0 = I, f(\vec{x}^{(0)}) = \begin{pmatrix} 8 \\ 8 \end{pmatrix} \\
 & \vec{x}' = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - I \begin{pmatrix} 8 \\ 8 \end{pmatrix} = \begin{pmatrix} -8 \\ -8 \end{pmatrix} \\
 & f(\vec{x}') = \begin{pmatrix} -440 \\ -224 \end{pmatrix} \\
 & \vec{S}_1 = \begin{pmatrix} -8 \\ -8 \end{pmatrix}, \vec{y}_1 = \begin{pmatrix} 432 \\ -232 \end{pmatrix}, \|\vec{S}_1\|^2 = 128 \\
 & \vec{w} = \frac{A_0^{-1} \vec{y}_1 - \vec{S}_1}{\|\vec{S}_1\|^2} = \frac{\begin{pmatrix} 432 \\ -232 \end{pmatrix} - \begin{pmatrix} -8 \\ -8 \end{pmatrix}}{128} = \frac{\begin{pmatrix} 440 \\ -224 \end{pmatrix}}{128} \\
 & \vec{v}^T \vec{w} = \frac{1}{128} (-3520 + 1792) = -1728/128 \\
 & 1 + \vec{v}^T \vec{w} = -1600/128 \\
 & A_1^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{128}{-1600} \begin{pmatrix} -3520 & -3520 \\ 1792 & 1792 \end{pmatrix} \begin{pmatrix} 1 \\ 128 \end{pmatrix} = \begin{bmatrix} -4/5 & -11/5 \\ 28/25 & 53/25 \end{bmatrix} \\
 & \vec{x}^2 = \begin{pmatrix} -8 \\ -8 \end{pmatrix} - \begin{pmatrix} -4/5 & -11/5 \\ 28/25 & 53/25 \end{pmatrix} \begin{pmatrix} 440 \\ -224 \end{pmatrix} = \begin{bmatrix} 136/5 \\ -648/25 \end{bmatrix}
 \end{aligned}$$

2.

$$\begin{aligned}
 2. \quad & P = I - \vec{u}\vec{u}^T \\
 & P^2 = (I - \vec{u}\vec{u}^T)(I - \vec{u}\vec{u}^T) \\
 & = I - 2\vec{u}\vec{u}^T + (\vec{u}\vec{u}^T)(\vec{u}\vec{u}^T) \\
 & \Rightarrow \vec{u}^T \vec{u} = \|\vec{u}\|^2 = 1 \\
 & = I - 2\vec{u}\vec{u}^T + \vec{u}\vec{u}^T \\
 & P^2 = \boxed{I - \vec{u}\vec{u}^T} \\
 & P^{-1} = (I - \vec{u}\vec{u}^T)^{-1} \\
 & = I^{-1} - (\vec{u}\vec{u}^T)^{-1} \leftarrow \text{can't invert} \\
 & = \boxed{\text{DNE}}
 \end{aligned}$$

Computer Problem using qnewton.m code given:

1.

Driver file with initial guess set to [0;0;0]

%% HW2 - Question 3

```
x0 = [0 0 0]';
tol = 10^-6;
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```
ans1 = qnewton(x0, @f1_hw2, tol);
```

Function file:

```
function [f,G,H] = f1_hw2(x)

f = [15*x(1) + x(2)^2 - 4*x(3) - 13; x(1)^2 + 10*x(2) - x(3) - 11; x(2)^3 - 25*x(3) + 22];

G = [0;0];
G(1) = 15*x(1) + x(2)^2 - 4*x(3) - 13; %f1
G(2) = x(1)^2 + 10*x(2) - x(3) - 11; %f2
G(3) = x(2)^3 - 25*x(3) + 22; %f3

%jacobian matrix
H(1,1) = 15;
H(1,2) = 2*x(2);
H(1,3) = -4;
H(2,1) = 2*x(1);
H(2, 2) = 10;
H(2, 3) = -1;
H(3, 1) = 0;
H(3, 2) = 3*x(2);
H(3, 3) = -25;

end
```

Output of 6 iterations shown:

1.1013	1.1880	0.8800
1.0286	1.0786	0.9440
1.0361	1.0861	0.9317
1.0364	1.0858	0.9312
1.0364	1.0857	0.9312
1.0364	1.0857	0.9312