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CMPSC456
Homework 3

1.

$$1. (t, y) = \{(0, 1), (1/4, 3), (1/2, 2), (3/4, 0)\}$$

$$y = c_1 + c_2 \cos 2\pi t + c_3 \sin 2\pi t$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix} \quad A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 3 \end{bmatrix} \quad a = 3/2 \quad c = 3/2$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 3/2 \\ -1/2 \\ 3/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}$$

$$SE = 0^2 + 0^2 + 0^2 + 0^2 = 0$$

2.

$$2. \begin{bmatrix} 1 & 1 \\ 1 & 9 \\ 1 & 9 \\ 1 & 1 \end{bmatrix} \quad \|y\| = \sqrt{1^2 + 1^2 + 1^2 + 1^2} = 2 = r_1$$

$$\vec{v}_1 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix} \quad y = \begin{bmatrix} 1 \\ 9 \\ 9 \\ 1 \end{bmatrix}$$

$$r_{12} = (\vec{a}_2, \vec{v}_1) = (1 \cdot 1/2 + 9 \cdot 1/2 + 9 \cdot 1/2 + 1 \cdot 1/2) = 10$$

$$\vec{y} = \begin{pmatrix} 1 \\ 9 \\ 9 \\ 1 \end{pmatrix} - 10 \cdot \vec{v}_1 = \begin{pmatrix} 1 \\ 9 \\ 9 \\ 1 \end{pmatrix} - 10 \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 4 \\ -4 \end{pmatrix}$$

$$r_{22} = \sqrt{(-4)^2 + (4)^2 + (4)^2 + (-4)^2} = 8$$

$$\vec{v}_2 = r_{22} \cdot y = \frac{1}{8} \begin{pmatrix} -4 \\ 4 \\ 4 \\ -4 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 1/2 \\ 1/2 \\ -1/2 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 9 \\ 1 & 9 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \\ 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 2 & 10 \\ 0 & 8 \end{bmatrix}$$

3.

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% Homework 3 - Question 3

x = [1.0, 1.1, 1.3, 1.5, 1.9, 2.1]; % given x data values
y = [1.84, 1.96, 2.21, 2.45, 2.94, 3.18]; % given y data values

% least squares polynomials
deg1 = polyfit(x, y, 1); % degree 1
deg2 = polyfit(x, y, 2); % degree 2
deg3 = polyfit(x, y, 3); % degree 3

% From this we can see the following polynomials are produced
P1 = @(x)0.6209 + 1.2196*x;
P2 = @(x)0.5966 + 1.2533*x - 0.0109.*x.^2;
P3 = @(x)0.6290 + 1.1850*x + 0.0353*x.^2 - 0.0100*x.^3;

% error
deg1errSum = sum((y - plotp1).^2); % = 2.7194e^-05
deg2errSum = sum((y - plotp2).^2); % = 1.8015e^-05
deg3errSum = sum((y - plotp3).^2); % = 1.7407e^-05
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% plot data and polynomials
plotp1 = polyval(deg1, x);
figure(1)
plot(x,y,'o')
hold on
plot(x,plotp1)
hold off
title('Degree 1 Polynomial and Data Points')

plotp2 = polyval(deg2, x);
figure(2)
plot(x,y,'o')
hold on
plot(x,plotp2)
hold off
title('Degree 2 Polynomial and Data Points')

plotp3 = polyval(deg3, x);
figure(3)
plot(x,y,'o')
hold on
plot(x,plotp3)
hold off
title('Degree 3 Polynomial and Data Points')

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Graph outputs:

