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CMPSC456
Homework 9

1.

1. $y_{n+1} - 3y_n + 2y_{n-1} = \frac{h}{12}[13f_{n+1} - 20f_n - 5f_{n-1}]$

$y_{n+1} - 3y_n + 2y_{n-1} = 0$

$\frac{\mathcal{E}^{n+1} - 3\mathcal{E}^n + 2\mathcal{E}^{n-1} = 0}{\mathcal{E}^{n-1}}$

$\mathcal{E}^2 - 3\mathcal{E} + 2 = 0 \Rightarrow (\mathcal{E}-2)(\mathcal{E}-1)$

$\mathcal{E} = 2$ or $\mathcal{E} = 1$

$y_n = c_1 2 + c_1(1)^n$

unbounded

$\rightarrow$ Not zero stable because $2 \geq 1$ and therefore unbounded

2.

2. $y_{n+1} - 3y_n + 2y_{n-1} = \frac{h}{12}[13f_{n+1} - 20f_n - 5f_{n-1}]$

$e_n = x(t_{n+1}) - 3x(t_n) + 2x(t_{n-1}) = \frac{h}{12}[13x'(t_{n+1}) - 20x'(t_n) - 5x'(t_{n-1})]$

$x(t_n+h) - 3x(t_n) + 2x(t_n-h) - h/12[13x'(t_n+h) - 20x'(t_n) - 5x'(t_n-h)]$

$= O(h^4)$

3.

3. $y_{n+1} - y_n = h[b_0 f_{n+1} + b_1 f_n + b_2 f_{n-1}]$

$e_n = x(t_{n+1}) - x(t_n) - hb_0 x'(t_{n+1}) - hb_1 x'(t_n) - hb_2 x'(t_{n-1})$

$= x(t_n+h) - x(t_n) - hb_0 x'(t_n+h) - hb_1 x'(t_n) - hb_2 x'(t_n-h)$

$x(t_n+h) = x(t_n) + hx'(t_n) + 1/2 x''(t_n)h^2 + 1/6 x'''(t_n)h^3$

$(-1)\, x(t_n) = x(t_n)$

$(-hb_0)\, x'(t_n+h) = x'(t_n) + x''(t_n)h + \frac{h^2}{2}x'''(t_n)$

$(-hb_1)\, x'(t_n) = x'(t_n)$

$(-hb_2)\, x'(t_n-h) = x'(t_n) - x''(t_n)h + \frac{h^2}{2}x'''(t_n)$

$= x(t_n) - x(t_n) + x'(t_n)h(1 - b_0 - b_1 - b_2) + x''(t_n)h^2(1/2 - b_0 + b_2) + x'''(t_n)h^3(1/6 - 1/2 b_0 - \frac{1}{2}$

$-1 = b_0 + b_1 + b_2 \quad -1/2 = b_0 - b_2 \quad -1/6 = 1/2 b_0 + 1/2 b_2$

$\boxed{b_0 = -5/12 \quad b_1 = -2/3 \quad b_2 = 1/12}$