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CMPSC456
Homework 6

1.

1. Consider $x = \frac{1}{\lambda_1 v_i}(a_{i1}, a_{i2}, \dots, a_{in})^t$

$\therefore x^t v = \frac{1}{\lambda_1 v_i}[a_{i1}, a_{i2}, \dots a_{in}](v_1, v_2, \dots, v_n)^t = \frac{1}{\lambda_1 v_i}\sum_{j=1}^{n} a_{ij} v_j$

Since $Av = \lambda_1 v$,

$$\sum_{j=1}^{n} a_{ij} v_j = \lambda_1 v_i$$

$$\therefore \quad x^t v = \frac{1}{\lambda_1 v_i}(\lambda_1 v_i) = 1$$

Thus the ith row of $B = A - \lambda_1 v x^t$ must consist of 0 entries and $B = A - \lambda_1 v_1 v_1^T$ has $\lambda_2, \lambda_3, \dots, \lambda_n, 0$ as its eigenvalues

2.

2. $A = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix} \quad \lambda_1 = 6 \quad \vec{v}_1 = \begin{bmatrix} 1 \\ 6 \\ 16 \end{bmatrix}$

$x = \frac{1}{6}\begin{bmatrix} -2 & -4 & 2 \end{bmatrix} = \begin{bmatrix} -1/3 & -2/3 & 1/3 \end{bmatrix}$

$vx^T = \begin{bmatrix} 1 \\ 6 \\ 16 \end{bmatrix}\begin{bmatrix} -1/3 & -2/3 & 1/3 \end{bmatrix} = \begin{bmatrix} -1/3 & -2/3 & 1/3 \\ -2 & -4 & 2 \\ -16/3 & -32/3 & 16/3 \end{bmatrix}$

$B = A - \lambda_1 v x^T = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix} - 6\begin{bmatrix} -1/3 & -2/3 & 1/3 \\ -2 & -4 & 2 \\ -16/3 & -32/3 & 16/3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 10 & 25 & -10 \\ 36 & 66 & -27 \end{bmatrix}$

$\tilde{B} = \begin{bmatrix} 25 & -10 \\ 66 & -27 \end{bmatrix}$

$\lambda 2 = (\lambda - 25)(\lambda + 27) - (66)(-10)$

$\lambda 2 = 3, -5$

$\lambda = 3$

$\begin{bmatrix} 22 & -10 \\ 66 & -30 \end{bmatrix} \Rightarrow \begin{bmatrix} 5/11 \\ 1 \end{bmatrix}$

$\lambda = -5$

$\begin{bmatrix} 30 & -10 \\ 66 & -22 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$

3. Driver file:

```
%% Homework 6

A = [4 1 1 1;
    1 3 -1 1;
    1 -1 2 0;
    1 1 0 2];

tol = 10^-5;
x = [1;0;0;0];

ans1 = inv_pow(A, tol, x)
```

Inverse Power method:

```
function ans = inv_pow(A, tol, x)

k = 1;
N = 4;
lamda_old = 1;

while k < N
    y = inv(A)*x;
    lamda_new = max(abs(y));
    x = y/lamda_new;
    err = abs(lamda_old-lamda_new);
    lamda_old = lamda_new;
    if err < tol
        disp(x);
        break
    end

end
ans = 1/lamda_old;
end
```

Output:

```
>> HW6_456
    0.4721
   -0.7639
   -1.0000
    0.2360


ans1 =

    0.7639
```