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CMPSC456
Homework 7

1.

1. $x' = -x + t\sqrt{x}$ $x(0)=1 \Rightarrow y_0 = 1$ $h = 1/4$

$y_1 = y_0 + h/2[f(t,y) + f(t_{n+1}, y_n + hf(t_n,y_n))]$

$= 1 + 1/8[f(0,1) + f(1/4, 1 + 1/4 f(0,1))]$

$= 1 + 1/8[-1 + f(1/4, 3/4)]$

$= 1 + 1/8[-1 + 0.5535] = .8083$

$y_2 = y_1 + h/2[f(t_1,y_1) + f(t_{n+1}, y_n + hf(t_n,y_n))]$

$= .8083 + 1/8[f(1/4, .8083) + (1/2, .8083 + 1/8 f(1/4, .8083))]$

$= .8083 + 1/8[-0.6062 + f(1/2, 0.7325)]$

$= .8083 + 1/8[-0.6062 - 0.3046] = \boxed{0.69445}$

2.

$K_1 = f(t_n, y_n)$

$K_2 = f(t_n + 2h, y_n + 2hK_1)$

$y_{n+1} = y_n + h/4(3K_1 + K_2)$

→ by taylor expansion,

$y(t_i + h) = y(t_i) + hy'(t_i) + 1/2h^2 y''(t_i) + O(h^3)$

$\Rightarrow (t_i+h) = y(t_i) + hf(t_i, y(t_i)) + h^2/2[f_t(t_i,y(t_i)) + f(t_i,y(t_i)) f_y(t_i,y(t_i))] + O(h^3)$

$= y_i + hK_1 + \frac{h^2}{2}[f_t(t_i,y_i) + K_1 f_y(t_i,y_i)] + O(h^3)$

∴ we find the Runge Kutta method to have 3rd order since $O(h^3)$

3. Driver file:

```
%% part 1
h = 0.1;
t = [0,3];
y0 = [-1.2, 0.8];

[t2,y2] = RK3(t, y0, h, @func0);
[t2x,y2x] = RK3(t, y0, h, @func2);

%plot
figure(1);
plot(t2, y2);
hold on
plot(t2x, y2x)
```

```
y0 = [-1.2, 0.8]; h = 0.01; t = [0,100];

F_tx = @(x,y) x.^2 - y.^2;
F_ty = @(x,y) x*y + y + x + 1;

t(1)=t(1);
y(1,:)=y0;
n=round((t(2)-t(1))/h);

x = zeros(1, length(t));
y = zeros(1, length(t));

for i=1:n
    t(i+1)=t(i)+h;
    kx1=F_tx(t(i),x(i,:)');
    kx2=F_tx(t(i)+h/2,x(i,:)'+(h*kx1)/2);
    kx3=F_tx(t(i) + h,x(i,:)'+h*kx1);
    x(i+1,:)=x(i,:)+h/6*(kx1'+4*kx2'+kx3');

    ky1=F_ty(t(i),y(i,:)');
    ky2=F_ty(t(i)+h/2,y(i,:)'+(h*ky1)/2);
    ky3=F_ty(t(i) + h,y(i,:)'+h*ky1);
    y(i+1,:)=y(i,:)+h/6*(ky1'+4*ky2'+ky3');
end
%plot
figure(2)
plot(t,x)
hold on
plot(t,y)
```

Runge-kutta:

```
function [t,y]=RK3(tint,y0,h,func)

% the 3nd-order Runge-Kutta
% tint: time interval
% y0: initial value [should be a row vector]
% h:  step size
% func: the function on the right hand side y=func(x)
%           x  and y are column vectors
%

t(1)=tint(1); y(1,:)=y0;
n=round((tint(2)-tint(1))/h);

for i=1:n
    t(i+1)=t(i)+h;
    k1=func(t(i),y(i,:)');
    k2=func(t(i)+h/2,y(i,:)'+(h*k1)/2);
    k3 = func(t(i) + h, y(i,:)'+h*k1);
    y(i+1,:)=y(i,:)+h/6*(k1'+4*k2'+k3');
end
```

Functions:

```
function f=func0(t,x)
   f(1) = exp(t-x(1));
end
```

```
function f=func2(t,x)
   f(1) = log(exp(t)+exp(1)-1);
end
```

Output: (first graph is for part 1, second is for part 2)