

1.

$$1. \quad w(x) = e^{-x} \quad [0, \infty)$$

$$\boxed{\phi_0 = 1}$$

$$\phi_1 = x + c$$

$$\int_0^\infty \phi_0 \cdot \phi_1 \cdot w(x) dx = 0 \Rightarrow \int_0^\infty (x+c) e^{-x} dx = 0$$

$$= \int_0^\infty x e^{-x} dx + c \int_0^\infty e^{-x} dx$$

$$\Rightarrow 1 + c = 0 \Rightarrow c = -1$$

$$\boxed{\phi_1 = x - 1}$$

$$\phi_2 = x^2 + c_1 \phi_1 + c_0 \phi_0$$

$$\int_0^\infty (x^2 + c_1 \phi_1 + c_0 \phi_0) \phi_0 e^{-x} dx$$

$$\int_0^\infty x^2 e^{-x} dx + 0 + c_0 \int_0^\infty e^{-x} dx = 0$$

$$\Rightarrow 2 + c_0 = 0, \quad c_0 = -2$$

$$\int_0^\infty (x^2 + c_1 \phi_1 + c_0 \phi_0) \phi_1 e^{-x} dx = 0$$

$$\int_0^\infty x^2 e^{-x} (x-1) dx + c_1 \int_0^\infty \phi_1^2 e^{-x} dx = 0$$

$$+ c_1 \int_0^\infty (x-1)^2 e^{-x} dx = 0 \Rightarrow c_1 = -4$$

$$\boxed{\phi_2 = x^2 - 4x + 2}$$

2. Composite trapezoid method: (used in both computer questions)

Code modified from trapezoidal method on mathworks.com

```
function ans = trapezoid(func, a, b, n)
```

```
    h = (b-a) / n;
```

```
    ans = 0;
```

```
    for i = 1:n-1
```

```
        p1 = func(a + i * h);
```

```
        p2 = func(a + (i+1)*h);
```

```
        ans = ans + (h/2)*sum(p1+p2);
```

```
    end
```

Driver file:

```
%% Question 2
```

```
a = -1;  
b = 1;  
n = 100;  
func = @(x) log(x+2);
```

```
t = trapezoid(func, a, b, n)
```

```
% polynomials:
```

```
syms x  
poly = legendreP([0 1 2 3], x)
```

Outputs: (polynomial approximation and legendre polynomials)

```
t =
```

```
1.2956
```

```
poly =
```

```
[1, x, (3*x^2)/2 - 1/2, (5*x^3)/2 - (3*x)/2]
```

3. Driver file:

```
%% Question 3
```

```
a = -1;  
b = 1;  
func = @(x) heaviside(x);
```

```
t10 = trapezoid(func, a, b, 10)  
t20 = trapezoid(func, a, b, 20)  
t30 = trapezoid(func, a, b, 30)
```

```
figure(1);  
fplot(func);  
title('Heaviside function')
```

```
figure(2);  
fplot(t10);  
title('Polynomial for n = 10')
```

```
figure(3);  
fplot(t20);  
title('Polynomial for n = 20')
```

```
figure(4);  
fplot(t30);  
title('Polynomial for n = 30')
```

Outputs:

```
t10 =
```

```
1
```

```
t20 =
```

```
1.0000
```

```
t30 =
```

```
1.0000
```

