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CMPSC456
Homework 5

1.

1a. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $x_0 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ $p = 1$

$\vec{y} = A\vec{x}_0 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$, $x_1 = \begin{bmatrix} 3/4 \\ 1/4 \\ 1 \end{bmatrix}$

$\vec{y} = A\vec{x}_1 = \begin{bmatrix} 11/4 \\ 9/4 \\ 3 \end{bmatrix}$, $x_2 = \begin{bmatrix} 11/12 \\ 3/4 \\ 1 \end{bmatrix}$

$\vec{y} = A\vec{x}_2 = \begin{bmatrix} 43/12 \\ 41/12 \\ 11/3 \end{bmatrix}$, $x_3 = \begin{bmatrix} 43/44 \\ 41/44 \\ 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $x_0 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $p = 3$

$\vec{y} = A\vec{x}_0 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ $x_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

$\vec{y} = A\vec{x}_1 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ $x_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

$\vec{y} = A\vec{x}_2 = \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}$ $x_3 = \begin{bmatrix} -1 \\ -1 \\ -1/2 \end{bmatrix}$

2.

2a. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $x_0 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ $\|x_0\|_2 = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6} \Rightarrow x_0 = \begin{bmatrix} 1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}$

$\vec{y} = A\vec{x}_0 = \begin{bmatrix} 3/\sqrt{6} \\ 1/\sqrt{6} \\ 4/\sqrt{6} \end{bmatrix}$ $\mu_0 = (x_0, y) = \frac{5}{3}$, $x_1 = \begin{bmatrix} \sqrt{3}/\sqrt{26/3} \approx 0.5883 \\ 0.1961 \\ 2\sqrt{2}/\sqrt{13} \approx 0.7845 \end{bmatrix}$

$\vec{y} = Ax_1 = \begin{bmatrix} 2.1572 \\ 1.765 \\ 2.3534 \end{bmatrix}$ $\mu_1 = (x_1, y) = 3.461$, $x_2 = \begin{bmatrix} 0.5914 \\ 0.4838 \\ 0.6451 \end{bmatrix}$

$\vec{y} = Ax_2 = \begin{bmatrix} 2.3117 \\ 2.2041 \\ 2.3654 \end{bmatrix}$ $\mu_2 = (x_2, y) = 3.959$ $x_3 = \begin{bmatrix} 0.5816 \\ 0.5546 \\ 0.5951 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $x_0 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\|x_0\|_2 = \sqrt{-1^2 + 1^2} = \sqrt{2}$ $x_0 = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$

$\vec{y} = Ax_0 = \begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$, $\mu_0 = (x_0, y) = 0$ $x_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

$y = Ax_1 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ $\mu_1 = (x_1, y) = 1$ $x_2 = \begin{bmatrix} -\sqrt{2}/2 \\ -\sqrt{2}/2 \\ 0 \end{bmatrix}$

$y = Ax_2 = \begin{bmatrix} -\sqrt{2} \\ -\sqrt{2} \\ -\sqrt{2}/2 \end{bmatrix}$ $\mu_2 = (x_2, y) = 2$ $x_3 = \begin{bmatrix} -2/3 \\ -2/3 \\ -1/3 \end{bmatrix}$

3. Driver file:

```
%% Homework 5

A = [4 1 1 1;
    1 3 -1 1;
    1 -1 2 0;
    1 1 0 2];

tol = 10^-5;
x0 = [1;0;0;0];

ans1 = pow(A, tol, x0)
ans2 = symmpow(A, tol, x0)
```

Power method:

```
%% Power Method

function x = pow(A, tol, x)

k = 1;
N = 4;
lamda_old = 1;

while k < N
    y = A*x;
    lamda_new = max(abs(y));
    x = y/lamda_new;
    err = abs(lamda_old-lamda_new);
    lamda_old = lamda_new;
    if err < tol
        disp(x);
        break
    end

end
end
```

Symmetric Power method:

```matlab
%% Symmetric Power Method

function x = symmpow(A, tol, x)

k = 1;
N = 4;
lamda_old = 1;

while k < N
    y = A*x;
    lamda_new = norm(y);
    x = y/lamda_new;
    err = abs(lamda_old-lamda_new);
    lamda_old = lamda_new;
    if err < tol
        disp(x);
        break
    end

end
end
```

Output: ans1 is from power method, ans2 is from symmetric power method

```
ans1 =

    1.0000
    0.6180
    0.1180
    0.5000


ans2 =

    0.7800
    0.4809
    0.0929
    0.3895
```