Ally Racho
CMPSC456
Homework 8

1.

1. $y' = 100y(1-y)$ $\quad y(0) = 0.1 \Rightarrow x_0 = 0$ $\quad h = 0.2$

$y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$

$y_1 = y_0 + h(100\,y_1(1-y_1)) \Rightarrow (100y_1 - 100y_1^2)$

$y_1 = y_0 + h100y_1 - h100y_1^2$

$h100y_1^2 - h100y_1 + y_1 = y_0$

$0.2(100y_1)^2 - 0.2(100)y_1 + y_1 = 0.1$

$20y_1^2 - 20y_1 + y_1 = 0.1$

$y_1 = \frac{19 \pm 3\sqrt{41}}{40} \Rightarrow .95523, -.00523$ ← closer to y^0

$y_2 = y_1 + h(100\,y_2(1-y_2))$

$h100y_2^2 - h100y_2 + y_2 = y_1$

$20y_2^2 - 20y_2 + y_2 = \frac{19 - 3\sqrt{41}}{40}$

$\boxed{y_2 = .94972, .00027}$

2.

2. $x' = x^2 - y^2$ $\qquad$ equilib = $(-1,1)$ $\qquad A = \begin{bmatrix} -2 & -2 \\ 2 & 0 \end{bmatrix}$

$y' = xy + y + x + 1$

$\lambda I - A = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -2 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} \lambda+2 & -2 \\ 2 & \lambda \end{bmatrix}$

$\det(\lambda I - A) = 0 \Rightarrow (\lambda+2)(\lambda) - (-4) =$

$= \lambda^2 + 2\lambda + 4$

$= (\lambda+1)^2 + 3 = 0$

$(\lambda+1)^2 = -3$

$\lambda + 1 = \pm\sqrt{-3} = \pm\sqrt{3}\,i$

$\lambda_1 = -1 + \sqrt{3}\,i \qquad \lambda_2 = -1 - \sqrt{3}\,i$

$|1 + h(-1+\sqrt{3}i)| \le 1$

$= |1 - h + \sqrt{3}ih| \le 1$

$= \sqrt{(1-h)^2 + 3h^2} \le 1$

$= \sqrt{1 - 2h + h^2 + 3h^2} \le 1$

$-2h + 4h^2 \le 0$

$\boxed{h \le 1/2}$

$|1 + h(-1-\sqrt{3}i)| \le 1$

$= |1 - h - \sqrt{3}ih| \le 1$

$= |(1-h) - \sqrt{3}hi| \le 1$

$= \sqrt{(1-h)^2 + 3h^2} \le 1$

$h \le 1/2$

Stability: $\left|1 + h\lambda + \frac{h\lambda^2}{2}\right| \le 1$

$\Rightarrow 1.0E-16 \le 1$ ∴ stable

3.

3. First using $y_{n+1} = y_n + hf + O(h^2)$, we find

$$f(t_{n+1}, y_{n+1}) = f(t_n + h, y_n + hf + O(h^2))$$

$$\therefore = hw_1 f + hw_2 f(t_n + h, y_{n+1})$$

$$= hw_1 f + hw_2 (f + hf_t + f_y hf + \overbrace{O(h) + O(h^2)}^{O(h^3)}$$

$$= h(w_1 + w_2) f + w_2 h^2 (f_n + ff_y) + O(h^3) \leftarrow$$

where $w_1 = w_2 = 1/2$

Therefore we find the implicit trapezoidal method to be 3rd order since:

$$y_{n+1} = y_n + hf + \frac{h^2}{2}(f_n + ff_y)) + \underline{O(h^3)}$$