**Worst/Best/Average vs $O/\Omega/\Theta$**

Students often confuse the concepts of worst case, best case and average case analysis with the three kinds of bounds ($O$,$\Omega$,$\Theta$). The purpose of this exercise is to understand the interplay between these two concepts.

Suppose an algorithm takes 50, 60, 70, 80, and 100 units of time on inputs $A$, $B$, $C$, $D$ and $E$, respectively (and suppose these are the only inputs possible for the problem)

1. What is the best case time of the algorithm and on which input?
   50 units of time on input $A$.

2. What is the worst case time of the algorithm and on which input?
   100 units of time on input $E$.

3. What is the average case time of the algorithm and on which input (trick question)?
   The average case time is 72 units of time over all of the inputs.

4. For the best case time (from item 1 above), provide an integer that
   (a) serves as an upper bound ($O$ analogy),
   51

   (b) serves as a lower bound ($\Omega$ analogy) and
   49

   (c) *simultaneously* serves as an upper and lower bound ($\Theta$ analogy).
   50

5. Why is this example an "analogy" and not the real thing?
   A real algorithm will almost certainly have more than five inputs

6. Repeat for the other two cases; i.e., worst (from item 2) and average (from item 3).
   Worst: $O(101)$, $\Omega(99)$, $\Theta(100)$.
   Average: $O(73)$, $\Omega(71)$, $\Theta(72)$.

## Practical Analysis

Assume array $A$ is indexed from 1 to $n$.

INEFFICIENT_SORT($A$, $n$)

$n!$ **for** $i = 1$ **to** $n!$ **do**<br>
    Boolean *sortedSoFar* = TRUE;<br>
    $j = 1$;<br>
    $P$ = nextPermutation($A$); $\Theta(n)$<br>
    **while** $j < n$ **and** *sortedSoFar* **do** $O(n)$<br>
        **if** $P[j] > P[j+1]$<br>
        **then** *sortedSoFar* = FALSE<br>
        $j$++<br>
    **if** (*sortedSoFar*) **then output** $P$ $\Theta(n)$

Analyze the worst-case complexity of INEFFICIENT_SORT assuming that the nextPermutation function always takes $\Theta(n)$ time.

$\Theta(n! \cdot n)$

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Your answer should fit above the line!

## Log Worksheet

1. If $\log_{100} x = y$, express $\log_{10} x^3$ in terms of $y$?

   $\log_{10} x^3 = 3\log_{10} x$
   $3(\frac{\log_{100} x}{\log_{100} 10}) = 3(\frac{y}{\frac{1}{2}})$
   $6y$

2. Prove that $\log(n!) = O(n \log n)$.

   $n! \leq n^n, n \geq 1$
   $\log n! \leq \log n^n, n \geq 1$
   $\log n! \leq n \log n, n \geq 1$

   Therefore, $\log(n!) = O(n \log n)$.

3. Prove that $\log(n!) = \Omega(n \log n)$ (difficult).

   $n! \geq (\frac{n}{2})^{\frac{n}{2}}, n \geq 2$
   $\log n! \geq \frac{n}{2} \log \frac{n}{2}$
   $\log n! \geq cn \log n, c < \frac{1}{2}$