

Homework 1

1. Prove the correctness of the recursive algorithm

Base case ($z = 0$): $multiply(y, 0) = y \times 0 = 0$

Inductive step:

$$cy \times floor(\frac{z}{c}) + y \times (z \% c)$$

$$cy \times \frac{(z - z \% c)}{c} + y \times (z \% c)$$

$$y \times (z - z \% c) + y \times (z \% c)$$

$$y \times z - (z \% c) + (z \% c)$$

$$y \times z$$

2. Prove by induction that for $n \geq 1$, $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$

Base case ($n = 1$): $\frac{1}{1(1+1)} = \frac{1}{2}$

Assume that $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{1}{2} + \frac{1}{6} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Therefore, we can assume $\sum_{i=1}^{n+1} \frac{1}{i(i+1)} = \frac{1}{2} + \frac{1}{6} + \dots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} = \frac{n+1}{n+2}$

Substitute in from our previous assumption, and the rest is algebra.

$$\begin{aligned} \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} &= \frac{n+1}{n+2} \\ \frac{n(n+2)}{(n+1)(n+2)} + \frac{1}{(n+1)(n+2)} &= \frac{n+1}{n+2} \\ \frac{n^2+2n+1}{(n+1)(n+2)} &= \frac{n+1}{n+2} \\ \frac{(n+1)^2}{(n+1)(n+2)} &= \frac{n+1}{n+2} \\ \frac{n+1}{n+2} &= \frac{n+1}{n+2} \end{aligned}$$

Therefore, we have proven that for any case $n + 1$ the summation holds true, proving our original statement.

3. A sorting algorithm takes 1 second to sort 1,000 items on your local machine.
How long will it take to sort 10,000 items. . .

- (a) if you believe that the algorithm takes time proportional to n^2 , and

$$C(1,000)^2 = 1 \text{ second}$$

$$C = \frac{1}{1,000^2}$$

$$C(10,000)^2 = \frac{1}{1,000,000} \times 100,000,000 = \mathbf{100 \text{ seconds}}$$

- (b) if you believe that the algorithm takes time roughly proportional to $n \log n$.

Assume the base of our logarithm is 10.

$$C \times 1000 \log 1000 = 1$$

$$C \times 3000 = 1$$

$$C = \frac{1}{3000}$$

$$C \times 10000 \log 10000 = ?$$

$$\frac{1}{3000} \times 10000 \times 4 = \frac{40}{3} \text{ of a second}$$

4. There are 25 horses. At most, 5 horses can race together at a time. You must determine the fastest, second fastest, and third fastest horses. Find the minimum number of races in which this can be done.

Begin by dividing the horses into five groups, with each horse assigned to a group lettered A-E. Race these groups and throw out the bottom two horses from each race, as they will never be fast enough to podium.

Our next race will be between the winners of our first five races. This gets us our fastest horse, which we will say came from group A, and because it was the fastest in that group, it will be called a_1 . This race also allows us to eliminate the groups from which the last two horses from the 6th race came, call them D and E. This is because we have three horses, a_1, b_1 , and c_1 that are faster than the fastest horses from those groups, d_1 and e_1 .

In our final race, we will choose a_2, a_3, b_1, b_2 , and c_1 as our racers, giving us the results of our fastest 3 horses, and we accomplished this in **7 races**.