Homework 10

Please note that handwritten assignments will not be graded. To fill out your homework, use either the Latex template or the Word template (filled out in Word or another text editor). Please do not alter the order or the spacing of questions (keep them on their own pages). When you submit to Gradescope, please indicate which pages of your submitted pdf contain the answers to each question. If you have any questions about the templates or submission process, you can reach out to the TAs on Piazza. This assignment is due at 23:59 on December 11th.

1. (6×2) For each of the following binary relations R on $\mathbb{N}$, decide which of the given ordered pairs belong to R.

   a. $xRy \leftrightarrow x + y < 7$; $(1,3), (2,5), (3,3), (4,4)$

   $(1,3), (3,3)$

   b. $xRy \leftrightarrow y$ is a perfect square; $(1,1), (4,2), (3,9), (25,5)$

   $(1,1), (3,9)$

2. (4×4) Identify each relation on $\mathbb{N}$ as one-to-one, one-to-many, many-to-one, or many-to-many.

   a. $R = \{(1,2), (1,4), (1,6), (2,3), (4,3)\}$
   many-to-many

   b. $R = \{(9,7), (6,5), (3,6), (8,5)\}$
   many-to-one

   c. $R = \{(12,5), (8,4), (6,3), (7,12)\}$
   one-to-one

   d. $R = \{(2,7), (8,4), (2,5), (7,6), (10,1)\}$
   one-to-many

3. (5×3) Test the following binary relations on the given sets S for reflexivity, symmetry, antisymmetry, transitivity.

   (a) $S = \mathbb{Q}$

   $xRy \leftrightarrow |x| \leq |y|$

   Reflexive

   Not symmetrical

   Antisymmetrical

   Transitive

   (b) $S = \mathbb{N}$

   $xRy \leftrightarrow x \cdot y$ is even

   Not Reflexive

   Symmetrical

   Not antisymmetrical

   Transitive

   (c) $S = \mathbb{N} \times \mathbb{N}$

   $(x_1, y_1)\, R\, (x_2, y_2) \leftrightarrow x_1 \leq x_2$ and $y_1 \leq y_2$

   Reflexive

   Not symmetrical

   Antisymmetrical

   Transitive

4. (5×3) Which functions are one-to-one? Which functions are onto? Describe the inverse function for any bijective function.

   a. $f : \mathbb{Z} \to \mathbb{N}$ where f is defined by $f(x) = x^2 + 1$

   Not one-to-one because not every x value maps to a unique y value.

   Not onto, since not every y value has a matching x, for example 4.

   Not bijective.

   b. $f : \mathbb{N} \to \mathbb{N}$ where f is defined by $f(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ x+1 & \text{if } x \text{ is odd} \end{cases}$

   Not one-to-one because there are multiple x values that map to singular y values, such as 4 and 1 mapping to 2.

   Not onto, since every y has a possible x value.

   Not bijective.

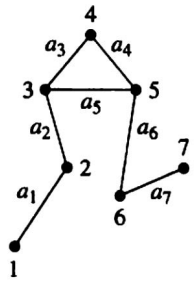
   c. $f : \mathbb{N} \to \mathbb{N}$ where f is defined by $f(x) = \begin{cases} x+1 & \text{if } x \text{ is even} \\ x-1 & \text{if } x \text{ is odd} \end{cases}$

   One-to-one since each x maps to a unique y value.

   The function is not onto because there is no y value that does not have an x value.

   Not bijective.

5. (3×7) Answer the following questions about the accompanying graph.



a. Is the graph simple? If no, why not?

   Yes.

b. Is the graph complete? If no, why not?

   No, no connection between certain nodes, for example 1 and 7.

c. Is the graph connected? If no, why not?

   Yes.

d. Can you find two paths from 3 to 6? If yes, give both paths.

   $3 \to 4 \to 5 \to 6$

   $3 \to 5 \to 6$

e. Can you find a cycle? If yes, give one cycle.

   $3 \to 4 \to 5 \to 3$

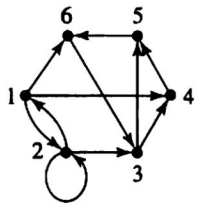
f. Can you find an edge whose removal will make the graph acyclic? If yes, give one edge.

   a_5

g. Can you find an edge whose removal will make the graph not connected? If yes, give one edge.

   a_2

6. (7×3) Use the directed graph in the figure to answer the following questions.



a. Which nodes are reachable from node 3?

   $4, 5, 6$

b. What is the length of the shortest path from node 3 to node 6?

   2

c. What is a path form node 1 to node 6 of length 8?

   $1 \to 2 \to 2 \to 2 \to 2 \to 1 \to 4 \to 5 \to 6$