Homework 1
Please note that you have to typeset your assignment (using LaTeX, Word, etc).
Hand-written assignment will not be graded.
You should submit a pdf to Gradescope before the lecture on September 3rd. Be careful not to move questions or swap their numbers when using this template.

1. (3×4) Check whether the follow sentences are propositions or not:

(a) Maine is the only one-syllable state.

(b) Is everything on schedule?

(c) The moon is made of green cheese.

(d) Turn left soon.

Solution:

(a) Proposition

(b) Not a proposition

(c) Proposition

(d) Not a proposition

2. (3 × 4) Rewrite each of the following statements in the form "If A, then B."

(a) You can enter, therefore the door is not locked.

(b) Healthy houseplant growth follows from being given enough sunlight and water.

(c) Not being hungry is a necessary condition for eating a whole pint of ice cream.

(d) Increased availability of information is a necessary condition for further technological advances.

Solution:

(a) If the door is not locked, then you can enter.

(b) If given enough sunlight and water, a houseplant will grow healthily.

(c) If you eat a whole pint of ice cream, you will not be hungry.

(d) If there are further technological advances, then there will be increased availability of information.

3. (3 × 2) Several forms of negation are given for each of the following statements. Which are correct? It may be that more than one option is correct.

(a) If you build it, they will come.

i. You build it, but they don't come.

ii. If you build it, then they won't come.

iii. You don't build it, but they do come.

(b) The answer is either 2 or 3.

i. Neither 2 nor 3 is the answer.

ii. The answer is not 2 or not 3.

iii. The answer is not 2 and it is not 3.

Solution:

(a) ii

(b) i, iii

4. (10 × 2) Write the truth tables for the following propositions:

(a) (A ∧ B) → ¬A

A	B	(A ∧ B) → ¬A
F	F	T
F	T	T
T	F	T
T	T	F

(b) A → B ↔ ¬A ∨ B

A	B	A → B ↔ ¬A ∨ B
F	F	T
F	T	T
T	F	T
T	T	T

5. (5×4) Show that whether each of the following propositions is a tautology, satisfiable but not a tautology, or a contradiction. If it is a tautology or a contradiction, please give the proof. If it is satisfiable, please give a true assignment and a false assignment.

(a) A ∧ B ↔ ¬A ∨ ¬B

1. ¬A ∨ ¬B ≡ ¬(A ∧ B) DeMorgan's
2. ¬(A ∧ B) ≠ (A ∧ B)

Contradiction

(b) ¬(A ∧ ¬B) ∨ B

1. ¬(A ∧ ¬B) ≡ (¬A ∨ B) DeMorgan's
2. (¬A ∨ B) ∨ B ≡ ¬A ∨ (B ∨ B) Associative
3. ¬A ∨ B

Satisfiable

A	B	¬A	¬A ∨ B
F	F	T	T
F	T	T	T
T	F	F	F
T	T	F	T

(c) A → (B → A)

  A ∧ B → A (Deduction Method)
  ¬A ∨ ¬B ∨ A (Implication)
  T ∨ ¬B

Tautology

(d) (A ∨ B ∨ ¬C) ∧ (A ∨ ¬B ∨ D) ∧ (A ∨ ¬C ∨ ¬D) ∧ (¬A ∨ ¬B ∨ ¬D) ∧ (A ∨ B ∨ ¬D)

A	B	C	D	Full Expression
F	F	F	F	T
F	F	F	T	F
F	F	T	F	F
F	F	T	T	F
F	T	F	F	F
F	T	F	T	T
F	T	T	F	F
F	T	T	T	F
T	F	F	F	T
T	F	F	T	T
T	F	T	F	T
T	F	T	T	T
T	T	F	F	T
T	T	F	T	F
T	T	T	F	T
T	T	T	T	F

Satisfiable