University of Utah
School of Computing

CS 6220 — Final Project — Spring 2016

Due COB – 5:00 pm MDT – on April 29, 2016

On the subsequent two pages are two different final projects. The first project involves solving the one-way wave equation in 1D using a variety of methods. The second project involves solving an elliptic problem in 2D using finite elements. The two projects are approximately comparable in the amount of effort required. For the final project, you are to select **one** of these two projects, complete it, write it up in report form using at most **15** pages, and submit your final report via Canvas.

Final Project: Selection 1

Suppose that a distant galactic culture is attempting to contact us via something that resembled Morse Code. For argument sake, let us assume that we can infer the message as a collection of on/off signals (or 1/0 binary signals). Suppose that the message that they are attempting to send can be encoded as follows on the interval $[0, 10]$: the signal is "on" (a value of 1.0) on $[0.5, 0.7]$, $[0.9, 1.0]$, $[2, 3]$, $[4, 4.3]$, $[7, 8]$,$[8.7, 8.9]$, and is zero everywhere else. Assuming that we believe that signals are sent in 0.1 increments, this function would equate to the following message:

00000110010000000000111111111100000000001110000000
00000000000000000000111111111100000001100000000000

Assume that the equation which dictates the propagation of this information is the one-way wave equation: $u_t + u_x = 0$. Let us further assume that a periodic domain is a good approximation of the propagation of this information. In order for the signal above to travel from our distant friends to us, the information must travel ten convective units (i.e. until final time $T = 10$).

For all experiments, assume an evenly-spaced spatial grid centered on each 0.1 interval. If the mesh refinement is such that you have many cells per 0.1 interval (and hence data values at the center of each cell), assume that the value of the interval is the arithmetic mean of the data. When examining the resulting signal, assume the following convention: if the average value on any particular interval is greater than or equal to 0.5, then the signal is considered "on" (1). If the signal is below 0.5, the signal is considered "off".

- This is a CS 6210-type assignment. If you formed an interpolant over the entire interval (assuming a grid in which points are centered inside of each 0.1 interval), what signal would you get if you plotted it at a higher sampling than the interpolating points? Is this a good idea or not? Why?
- If you formed an interpolant over each unit interval (hence ten different interpolating functions), what signal would you get? What principle(s) do these examples show?
- Solve the one-way wave equation above on $[0, 10]$ with periodic conditions up to $T = 10$. Use the following four methods:
  1. Euler-Forward/First-Order Upwind
  2. Euler-Forward/Second-Order Upwind
  3. Lax-Friedrichs
  4. Lax-Wendroff

  Use a spatial grid spacing of $h = 0.1, 0.05, 0.025, 0.01, 0.001$. Use a time step appropriate for the CFL condition of the smallest mesh (i.e. use one small time step for comparison of all methods). For each grid spacing, specify what is the signal obtained.
- For each method, attempt to determine what is the maximum grid spacing that will still maintain the signal. Is this number different per method? How important is the time step choice? Obeying the CFL restriction satisfies stability. Is more needed for accuracy?

Final Project: Selection 2

Consider the following system:

$$
\begin{aligned}
-\nabla^2 u(x,y) &= f(x,y) \;\; (x,y) \in (0,1) \times (0,1) \\
u(0,y) &= g_1(y); \;\; y \in [0,1] \\
u(1,y) &= g_2(y); \;\; y \in [0,1] \\
u(x,0) &= g_3(x); \;\; x \in [0,1] \\
u(x,1) &= g_4(x); \;\; x \in [0,1]
\end{aligned}
$$

Develop a finite element solver to solve the problem above. Assume that piecewise linear C^0 global basis functions constructed on triangular tessellations are used. Use a union-jack mesh configuration for all experiments.

Demonstrate the convergence rate of your method using the following known solution:

$$u(x,y) = sin(2\pi x)\, sin(2\pi y)$$

from which the right-hand-side and boundary conditions can be derived. Provide both the rate of convergence when you compute the right-hand-side exactly (to machine precision) and when you approximate the right-hand-side integrals by only sampling the forcing function at the center of the triangles.

Then solve the above system with:

$$
\begin{aligned}
f(x,y) &= 1.0; \;\; (x,y) \in (0,1) \times (0,1) \\
g_1(y) &= 0.0; \;\; y \in [0,1] \\
g_2(y) &= 1.0; \;\; y \in [0,1] \\
g_3(x) &= x; \;\; x \in [0,1] \\
g_4(x) &= x; \;\; x \in [0,1]
\end{aligned}
$$

with spacings of 1/5, 1/10, 1/20, 1/40 and 1/80, with the two right-hand-side computations mentioned above.