University of Utah
School of Computing

CS 6210 **Homework #7** **Fall 2014**

Due December 4, 2014 at the start of class.

The Physical Problem

Solve for the steady state temperature profile for the conductive heat flow problem in the L-shaped region shown in the figure below. The general heat equation is a time-dependent parabolic problem. However, the steady-state heat equation reduces to solving Laplace's equation ($\nabla^2 u = 0$). As shown in the figure, there are two types of boundary conditions, the Dirichlet condition, where the value is given along a portion of the boundary ($u = 1$ or $u = 0$), and the Neumann condition, in which there is no loss of heat outside of the boundary. A no loss condition is the same as saying the flux across the boundary is zero, or that the normal component of the gradient is zero ($\nabla u \cdot \hat{n} = 0$).

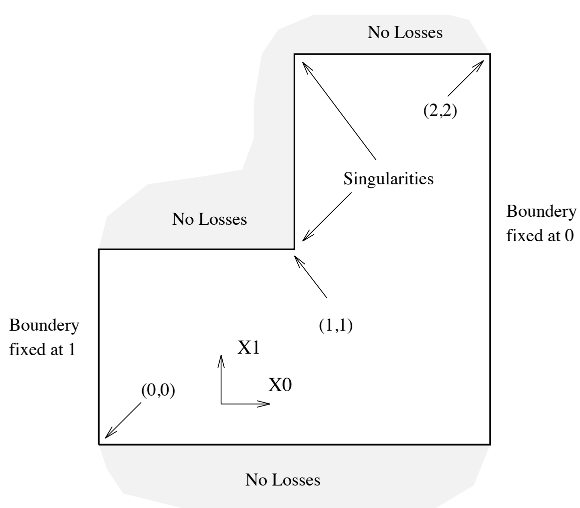

Figure 1: L-Shaped Region.

Assignment

For this assignment, you will need to read Chapter 3 of my CS 6220 notes.

1. Write a program to generate a "finite difference grid" for the problem. This program should accept a value of h as input. To generate a grid means to break up the solution domain into uniform subregions and specifying the node points. It is acceptable to require that $1/h$ be an integer in order to avoid problems that arise when the boundary is not on a grid line.
2. Write a program to convert this grid into a matrix problem (a linear system $Ax = b$) using a 5 point centered difference finite difference approximation.
3. Solve the resulting linear system using Jacobi, Gauss-Seidel, and Conjugate Gradient iterative solvers, and determine which solver worked best for this system. Solve the problem (from end-to-end) using several different values of h. Graph these results. Discuss the relationship between the number of elements and the time to solve the linear system.
4. Rework your code so that it now uses the nine-point finite difference approximation for the Laplacian operator. Solve the previous problem (from end-to-end) using several different values of h. Again, graph the results. Compare the results from the nine-point and five-point difference formulas in terms of accuracy and computational costs.