University of Utah
School of Computing

CS 6210 **Homework #8** **Fall 2014**

Due December 18, 2014 by 1:45 p.m.

The Physical Problem

Solve for the steady state temperature profile for the conductive heat flow problem in the L-shaped region shown in the figure below. The general heat equation is a time-dependent parabolic problem. However, the steady-state heat equation reduces to solving Laplace's equation ($\nabla^2 u = 0$). As shown in the figure, there are two types of boundary conditions, the Dirichlet condition, where the value is given along a portion of the boundary ($u = 1$ or $u = 0$), and the Neumann condition, in which there is no loss of heat outside of the boundary. A no loss condition is the same as saying the flux across the boundary is zero, or that the normal component of the gradient is zero ($\nabla u \cdot \hat{n} = 0$).

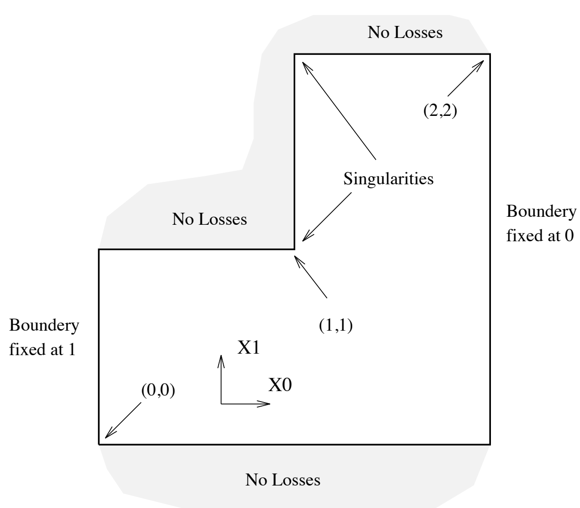

Figure 1: L-Shaped Region.

Assignment

For this assignment, you will need to read Chapter 4 of my CS 6220 notes.

1. Create triangular finite element grids of different resolution by dividing the finite difference grids squares you created in Homework 7 into two triangles.
2. Write a finite element program to solve the steady state heat problem governed by Laplace's equation. The program should use 1st order linear triangular elements. If you plan to do the extra credit problem below, you should think about how to allow your program to choose either triangular basis functions or quadrilateral basis functions.
3. Solve the steady state heat problem using the finite element method and compare the FE solution to the FD solutions you calculated in Homework 7. Compare solution times for the two methods (use the same solver). Plot the iso-value (iso-temperature) lines for both the FE and FD solutions.
4. Given the file delaunay.pts on the CS 6210 web page, use the Delaunay triangulation program in Matlab to create an unstructured triangle mesh. The format of delaunay.pts is node number, x-coordinate, y-coordinate. Note that you will need to remove a few triangles because the Delaunay triangulation method meshes the convex hull of the geometry, so it will form triangles that are outside the L-shape in the upper left quadrant (you can learn more about Delaunay triangulation in chapter 7.2 of the CS 6220 notes). Solve the steady heat problem again using the unstructured mesh. Add points to the delaunay.pts file at places where you think the most error is occurring. Re-generate the nonuniform mesh and re-calculate the solution. Continue this process until you think you have reduced the error significantly. Explain how you know your solution has become more accurate.

Extra Credit Problem

Extra Credit: Rework your finite element code to use bilinear quadrilateral elements instead of linear triangles. Solve the problem using quadrilateral finite elements and compare the accuracy with the linear triangular element solutions.