

1. A CRD having 5 treatments with 6 replications per treatment was run. You are given that  $\sum \sum y_{ij}^2 = 5185.5$ ,  $y_{1.} = 50$ ,  $y_{2.} = 75$ ,  $y_{3.} = 100$ ,  $y_{4.} = 90$ , and  $y_{5.} = 60$ .
  - (a) (3pt) Based on these values, generate an ANOVA table that includes the Sources of Variation, sums of squares, degrees of freedom, mean squares, the  $F$ -statistic.
  - (b) (1pt) What are the critical  $F$  values using  $\alpha = .01$ ,  $\alpha = .05$ , and  $\alpha = .10$ ?
  - (c) (1pt) For which (if any) of these  $\alpha$  values would you reject  $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ ?
  - (d) (1pt) Suppose that one observation from treatment 3 was removed because of improper handling by a technician. The resulting sum of the remaining 5 replicates is  $y_{3.} = 95$  (instead of 100). Briefly describe why the  $MS_{trt}$  will decrease. (I am looking for conceptual reasoning and not new calculation of an ANOVA table.)
2. (1.5pt) Assume significance level for testing is  $\alpha = .01$ . For a oneway ANOVA, you are given
  - (a) The  $p$ -value from the  $F$ -test is .4917. Your friend Justin Case states that your conclusion should be to accept the null hypotheses that all treatment means are equal. Assuming there were not violations for performing an ANOVA, do you agree or disagree with Justin? Justify your answer in at most 2 sentences.
  - (b) The  $p$ -value from the  $F$ -test is .0233. Your friend Justin Time states that your conclusion should be to reject the null hypotheses that all treatment means are equal. Assuming there were not violations for performing an ANOVA, do you agree or disagree with Justin? Justify your answer in at most 2 sentences.
3. Use the following data from Problem 3-12 (page 132). There are 3 dosages of a new drug and the response ( $y$ ) is a measure of the bioactivity of the drug.

Dosage	Bioactivity			
20 g	24	28	37	30
30 g	37	44	34	35
40 g	42	47	52	38

- (a) (1pt) Write out the least squares normal equations in terms of the data values. That is, replace dot notation with numbers.
- (b) (1pt) Solve the normal equations for estimating  $\hat{\mu}, \hat{\tau}_1, \hat{\tau}_2, \hat{\tau}_3$  assuming  $\tau_1 + \tau_2 + \tau_3 = 0$ .
- (c) (1pt) Solve the normal equations for  $\hat{\mu}, \hat{\tau}_1, \hat{\tau}_2, \hat{\tau}_3$  assuming  $\tau_2 = 0$ .
- (d) (2pt) Estimate  $\mu + \tau_3$ ,  $2\tau_3 - \tau_1 - \tau_2$ ,  $3\tau_1 - 2\tau_2 - \tau_3$ , and  $\mu + \tau_1 + \tau_2 + \tau_3$  using the solutions in (b) and (c).
- (e) (1pt) If any estimates differ, provide a reason why.

4. Use the following data from Problem 3-31 (page 135). The table below contains a random sample of oven temperatures from three ovens which are supposed to operate at the same temperature.

Oven	Temperature					
1	491.50	498.30	498.10	493.50	493.60	
2	488.50	484.65	479.90	477.35		
3	490.10	484.80	488.25	473.00	471.85	478.65

- (a) (2pt) Fill in an ANOVA table like the one given on the top of page 8 of the course notes. You can either do the computations by hand or using statistical software.
- (b) (1pt) State the means model including model assumptions. Describe the model parameters in the context of this problem.
- (c) (1pt) State the effects model including model assumptions. Describe the model parameters in the context of this problem.
- (d) For (b) and (c), provides estimates of all model parameters assuming the  $\sum_{i=1}^3 n_i \tau_i = 0$ .
5. (2pt) Suppose there are  $a = 4$  treatments, and you are given the values for  $n_1, n_2, n_3, n_4, s_1^2, s_2^2, s_3^2$ , and  $s^2$  (the sample variance of the  $N = n_1 + n_2 + n_3 + n_4$  responses). Note that  $s_4^2$  is not given.
- (a) Is this enough information to determine  $SS_E$ ? If yes, than provide a formula for calculating  $SS_E$ . If not, what minimal additional information (not  $y_{ij}$  values) is needed and why?
- (b) Is this enough information to determine  $SS_{trt}$ ? If yes, than provide a formula for calculating  $SS_{trt}$ . If not, what minimal additional information (not  $y_{ij}$  values) is needed and why?
- (c) Is this enough information to determine  $SS_T$ ? If yes, than provide a formula for calculating  $SS_T$ . If not, what minimal additional information (not  $y_{ij}$  values) is needed and why?

#### For Stat 541 Students

6. (1.5pt) Return to Problem 5. Can the normal equations be solved for  $\hat{\mu}, \hat{\tau}_1, \hat{\tau}_2, \hat{\tau}_3$  assuming  $\mu = -10$ ? If yes, provide the solutions. If no, then why not?
7. Use the data from Problem 3-12 (page 132). There are 3 dosages of a new drug and the response ( $y$ ) is a measure of the bioactivity of the drug. Assume, however, the first response was missing.

Dosage	Bioactivity			
20 g	.	28	37	30
30 g	37	44	34	35
40 g	42	47	52	38

- (a) (1pt) Write out the least squares normal equations in terms of the data values. That is, replace dot notation with numbers.
- (b) (1.5pt) Solve the normal equations for estimating  $\hat{\mu}, \hat{\tau}_1, \hat{\tau}_2, \hat{\tau}_3$  assuming

$$n_1 \tau_1 + n_2 \tau_2 + n_3 \tau_3 = 0.$$