

1. (a) Type II Sums of Squares were used.

SSE	$6.2505657 \times 10^7$
MSE	$4.1670438 \times 10^6$
$SS_{trt}$	$1.0319149 \times 10^8$
$MS_{trt}$	$2.5797872 \times 10^7$
F-stat	6.190929
p-value	0.003786

- (b) Let  $i$  and  $j$  index the 1-5 voltage levels.

$H_0$ :  $\mu_i = \mu_j$  for all  $i \neq j$

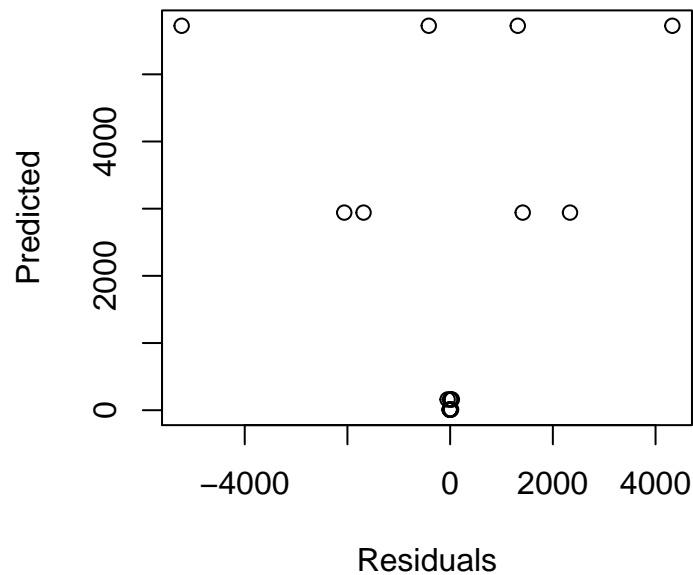
$H_A$ : at least one  $\mu_i \neq \mu_j$  for all  $i \neq j$

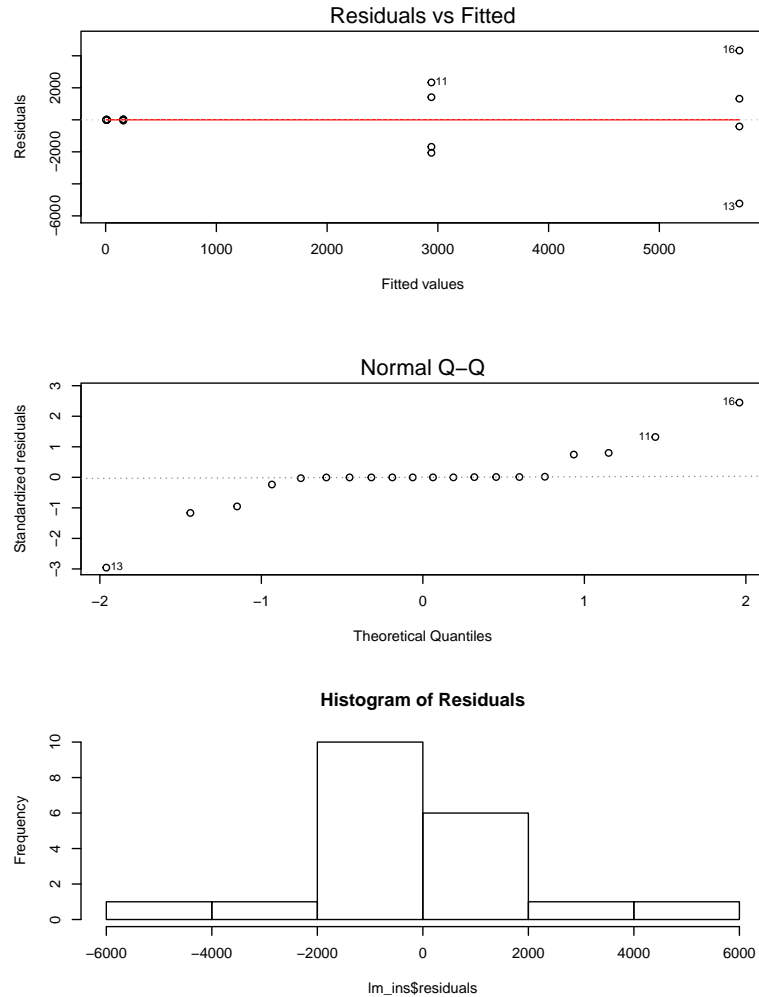
- (c) Below are the five sample variances.

$s_1^2$	1329.58
$s_2^2$	62.92
$s_3^2$	4846188.25
$s_4^2$	15987486.00
$s_5^2$	152.25

- (d) Below are diagnostic plots.

### Insulating Material Resids vs. Predicted





The constant variance assumption is the most concerning violation, but the normal Q-Q plot also shows the tails of residuals are more extreme than would be expected under normality. Without correcting for the non-constant variance, the p-value from the F-test will be too small because there is extra variation we aren't accounting for, making us more likely to find evidence of significant differences that do not exist.

- (e) The constant variance assumption violation is the most concerning, and without any more work, I would most likely not report these results. If I *were* to make a conclusion, it would be there is strong evidence the mean failure time differs between at least two voltage levels.

2. (a) Assume  $\tau_2 = 0$ .

$$X = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 12 & 4 & 4 \\ 4 & 4 & 0 \\ 4 & 0 & 4 \end{bmatrix}$$

$$(X^T X)^{-1} = \begin{bmatrix} 0.25 & -0.25 & -0.25 \\ -0.25 & 0.5 & 0.25 \\ -0.25 & 0.25 & 0.5 \end{bmatrix}$$

**fix this – somethings wrong** `tapply(dfy, as.factor(df$x), mean)`

$$X^T Y = \begin{bmatrix} 445 \\ 119 \\ 179 \end{bmatrix}$$

$$(b) \theta = \begin{bmatrix} \mu \\ \theta_{dosage=20} \\ \theta_{dosage=40} \end{bmatrix}$$

$\mu$  = the true mean observation for dosage 20

$\theta_{dosage=30}$  = the true change in mean observational units between the mean for dosage 20 and the mean for dosage 30

$\theta_{dosage=40}$  = the true change in mean observational units between the mean for dosage 20 and the mean for dosage 40

$$(c) \hat{\theta} = \begin{bmatrix} 36.75 \\ -7 \\ 8 \end{bmatrix}$$

- (d) Assume  $\Sigma\tau_i = 0$ .

Let  $\tau_3 = \tau_1 + \tau_2$

$$X = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 4 \\ 0 & 4 & 8 \end{bmatrix}$$

$$(X^T X)^{-1} = \begin{bmatrix} 0.083 & 0 & 0 \\ 0 & 0.167 & -0.083 \\ 0 & -0.83 & 0.167 \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} 445 \\ -60 \\ -32 \end{bmatrix}$$

$$(e) \quad \theta = \begin{bmatrix} \mu \\ \theta_{dosage=30} \\ \theta_{dosage=40} \end{bmatrix}$$

$\mu$  = the true mean observation over all dosages

$\theta_{dosage=30}$  = the true change in mean observational units between the overall mean and the mean for dosage 30

$\theta_{dosage=40}$  = the true change in mean observational units between the overall mean and the mean for dosage 40

$$(f) \quad \hat{\theta} = \begin{bmatrix} 37.0833333 \\ -7.3333333 \\ -0.3333333 \end{bmatrix}$$

Remaining problems are attached.