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Please prepare your solutions using L<sup>A</sup>T<sub>E</sub>X or another word processing software.

1. Read the article titled “Stein’s Paradox in Statistics” a copy can be found on D2L.

(a) (5 points) Briefly describe your thoughts on the article.

(b) (5 points) How does this article illustrate the concept of shrinkage?

(c) (5 points) Summarize Stein’s Paradox.

2. (30 points) For the hierarchical normal model specified as

$$\begin{aligned}p(y|\theta_j, \sigma^2) &= \text{normal}(\theta_j, \sigma^2) \text{ within-group model} \\p(\theta_j|\mu, \tau^2) &= \text{normal}(\mu, \tau^2) \text{ between-group model}\end{aligned}$$

with the following prior distributions

$$\begin{aligned}\sigma^2 &\sim \text{InvGamma}(\nu_0/2, \nu_0\sigma_0^2/2) \\ \tau^2 &\sim \text{InvGamma}(\eta_0/2, \eta_0\tau_0^2/2) \\ \mu &\sim \text{normal}(\mu_0, \gamma_0^2)\end{aligned}$$

derive the full conditional distributions for  $\mu, \sigma^2, \tau^2$  and  $\theta_j$ . You can use tricks of normal kernels and moments, but include all necessary work. Paper and pencil is fine for this question.

3. (30 points) Generate data from a two-dimensional hierarchical normal model (e.g. students within a school). Write code for a Gibbs sampler and convince me that your code returns the correct answer.
4. (15 points) Show that  $Pr(H_0|Data) = \left(1 + \frac{1-\pi_0}{\pi_0}BF^{-1}\right)^{-1}$ .
5. (10 points) Summarize the paper you have selected for the final project. What about this paper/method to you still not understand (be specific!)?