Experiment. Olympics Testing

Assume you were hired by the World Anti-Doping Agency to test olympic athletes for performance enhancing drugs. You are given results from eleven olympic champions, all of which test negative. What do you believe the true probability of olympic champions using performance enhancing drugs would be?

Calculate the maximum likelihood estimator of p the probability of olympic champions using performance enhancing drugs.
Let y_i be 1 if test i is positive and zero otherwise, then $y = \sum_i y_i$.

How does this estimator match up with your intuition?

What if no "cheaters" were caught in the testing, does this estimator still match your intuition?

Now we will talk about the mechanics of Bayesian statistics in the context of the olympic testing problem.

Sampling Model:

Ex. Binomial Model.

Likelihood Function:

Ex. Binomial Model.

Prior Distribution:

Ex.

Posterior Distribution: Given a prior distribution and a likelihood function, or sampling model, the posterior distribution of the parameters can be calculated using Bayes' rule.

In Bayesian statistics, inferences are made from the posterior distribution. In cases where analytical solutions are possible, the entire posterior distribution provides an informative description of the uncertainty present in the estimation. In other cases credible intervals are used to summarize the uncertainty in the estimation.

Experiment. Olympic Testing (with Bayes).
Now reconsider the olympic testing program from a Bayesian perspective. Use the Beta(α, β) as the prior distribution for p and compute the posterior distribution for p.

first the integration in the denominator.

Now the posterior distribution follows as:

Use a Beta(1,1) distribution as the prior for p(p) and assume thirteen gold medal winners were tested and zero tested positive for performance enhancing drugs. Compute $p(p|y)$.

What is the expectation, or mean, or your posterior distribution? Hint $E[X] = \frac{\alpha}{\alpha+\beta}$ if $X \sim Beta(\alpha, \beta)$.

How do these results compare with your intuition which we stated earlier?

How about the MLE estimate?

What impact did the prior distribution have on the posterior expectation?

Classical, or frequentist, statistical paradigm:

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Bayesian statistical paradigm

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