

# Time Series HW 7

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*Revisit your  $CO_2$  concentration time series from HW 4. If you want to switch groups from that assignment you can or you can work in the same ones. If you switch groups, pick a time series to analyze from those you worked with. Groups of up to 3. And just one time series per group.*

As in HW4, we will use the High Altitude Global Climate Observation Center, Mexico (MEX) dataset because we thought the High Altitude aspect may show interesting features of  $CO_2$  concentrations not available in other datasets. The site is located at the coordinates 18.984, -97.311 near the summit of a 15,000 ft mountain.

The information page on these data indicates measured responses are on the X2007  $CO_2$  mole fraction scale. The excerpt from the information page gives insightful information about  $CO_2$  and the data:

Carbon dioxide ( $CO_2$ ) in ambient and standard air samples is detected using a non-dispersive infrared (NDIR) analyzer. The measurement of  $CO_2$  in air is made relative to standards whose  $CO_2$  mole fraction is determined with high precision and accuracy. Because detector response is non-linear in the range of atmospheric levels, ambient samples are bracketed during analysis by a set of reference standards used to calibrate detector response. Measurements are reported in units of micromol/mol ( $10^{-6}$  mol  $CO_2$  per mol of dry air or parts per million (ppm)). Measurements are directly traceable to the WMO  $CO_2$  mole fraction scale.

Uncertainty in the measurements of  $CO_2$  from discrete samples has not yet been fully evaluated. Key components of it are our ability to propagate the WMO X $CO_2$  scale to working standards, the repeatability of the analyzers used for sample measurement, and agreement between pairs of samples collected simultaneously. Zhao and Tans (2006) determined that the internal consistency of working standards is  $\pm 0.02$  ppm (68% confidence interval). The typical repeatability of the analyzers, based on repeated measurements of natural air from a cylinder, is  $\pm 0.03$  ppm. Average pair agreement across the entire sampling network is  $\pm 0.1$  ppm.

The Pacific Ocean Cruise (POC, travelling between the US west coast and New Zealand or Australia) data have been merged and grouped into 5 degree latitude bins. For the South China Sea cruises (SCS) the data are grouped in 3 degree latitude bins.

Sampling intervals are approximately weekly for the fixed sites and average one sample every 3 weeks per latitude zone for POC and about one sample every week per latitude for SCS.

Historically, samples have been collected using two general methods: flushing and then pressurizing glass flasks with a pump, or opening a stopcock on an evacuated glass flask; since 28 April 2003, only the former method is used. During each sampling event, a pair of flasks is filled.

1. *We are going to be exploring a more complete model that includes a long-term trend and a seasonal component. We need to pick the type of trend and the form of the seasonal component. For the long term trend, consider the following options: no trend, linear trend, or quadratic trend. For the seasonal component, consider no seasonal component, seasonal means, single harmonic pair ( $m=1$ ), and 5th order harmonic ( $m=5$ ). Consider all combinations of these components, fit using “lm”. Create a table that contains the model description, model used DF (so the count of free parameters), AICs, and  $\Delta$ AICs, sorting the table by AICs. Use this information to discuss the top model selected (what was in it), the strength of support for that model versus the others, and the strength of evidence for a long-term trend and seasonal component (of the type selected) versus not including them in the model.*

- *You should be creating a table that contains twelve models.*

Model	Free DF	AIC	deltaAIC
Linear Year, 5th Order Harmonic	12	202.38	0.00
Linear Year, Seasonal Means	13	203.08	0.71
Quadratic Year, 5th Order Harmonic	13	204.37	1.99
Quadratic Year, 5th Order Harmonic	14	205.07	2.70
Linear Year, 1st Order Harmonic	4	287.20	84.82
Quadratic Year, 1st Order Harmonic	5	289.20	86.82
Linear Year, No Seasonality	2	387.39	185.02
Quadratic Year, No Seasonality	3	389.39	187.02
No Year, 1st Order Harmonic	3	500.30	297.93
No Year, No Seasonality	1	510.90	308.52
No Year, 5th Order Harmonic	11	511.64	309.27
No Year, 5th Order Harmonic	12	513.61	311.23

No Year, No Seasonality  $y_t = \beta_o + e_t$

Linear Year, No Seasonality  $y_t = \gamma_o + \gamma_1 \text{year} + e_t$

Quadratic Year, No Seasonality  $y_t = \gamma_o + \gamma_1 \text{year} + \gamma_2 \text{year}^2 + e_t$

No Year, Seasonal Means  $y_t = \beta_1 \text{jan.ind} + \beta_2 \text{feb.ind} + \beta_3 \text{mar.ind} + \beta_4 \text{apr.ind} + \beta_5 \text{may.ind} + \beta_6 \text{june.ind} + \beta_7 \text{july.ind} + \beta_8 \text{aug.ind} + \beta_9 \text{sept.ind} + \beta_{10} \text{oct.ind} + \beta_{11} \text{nov.ind} + \beta_{12} \text{dec.ind} + e_t$

Linear Year, Seasonal Means  $y_t = \gamma_1 \text{year} + \beta_1 \text{jan.ind} + \beta_2 \text{feb.ind} + \beta_3 \text{mar.ind} + \beta_4 \text{apr.ind} + \beta_5 \text{may.ind} + \beta_6 \text{june.ind} + \beta_7 \text{july.ind} + \beta_8 \text{aug.ind} + \beta_9 \text{sept.ind} + \beta_{10} \text{oct.ind} + \beta_{11} \text{nov.ind} + \beta_{12} \text{dec.ind} + e_t$

Quadratic Year, Seasonal Means  $y_t = \gamma_1 \text{year} + \gamma_2 \text{year}^2 + \beta_1 \text{jan.ind} + \beta_2 \text{feb.ind} + \beta_3 \text{mar.ind} + \beta_4 \text{apr.ind} + \beta_5 \text{may.ind} + \beta_6 \text{june.ind} + \beta_7 \text{july.ind} + \beta_8 \text{aug.ind} + \beta_9 \text{sept.ind} + \beta_{10} \text{oct.ind} + \beta_{11} \text{nov.ind} + \beta_{12} \text{dec.ind} + e_t$

No Year, 1st Order Harmonic  $y_t = \beta_0 + \beta_1 \cos(2\pi f(t)) + \beta_2 \sin(2\pi f(t)) + e_t$

Linear Year, 1st Order Harmonic  $y_t = \gamma_0 + \gamma_1 \text{year} + \beta_1 \cos(2\pi f(t)) + \beta_2 \sin(2\pi f(t)) + e_t$

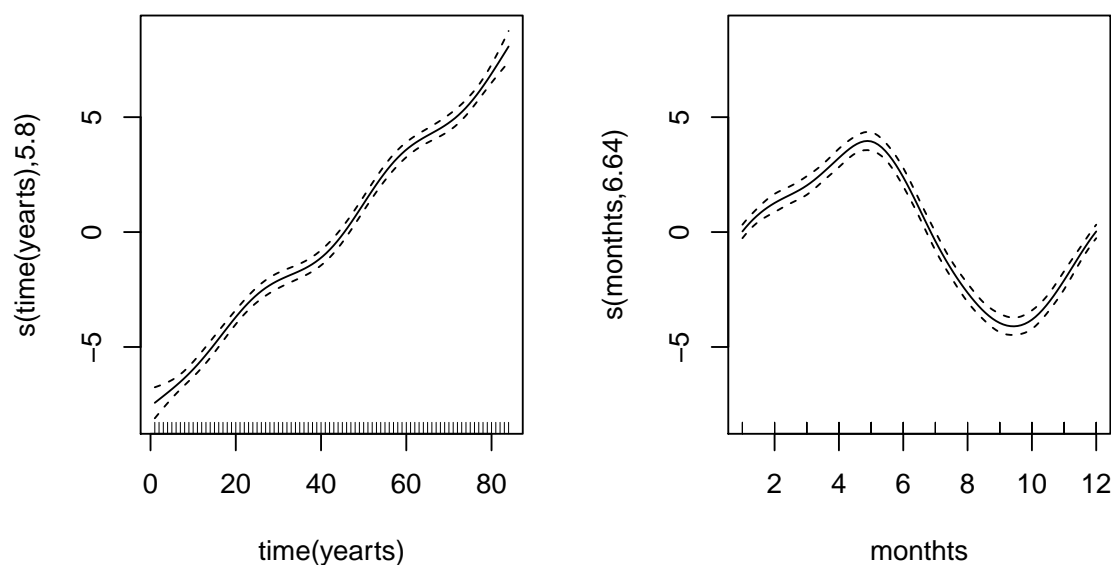
Quadratic year, 1st Order Harmonic  $y_t = \gamma_0 + \gamma_1 \text{year} + \gamma_2 \text{year}^2 + \beta_1 \cos(2\pi f(t)) + \beta_2 \sin(2\pi f(t)) + e_t$

No Year, 5th Order Harmonic  $y_t = \beta_0 + \sum_{i=1:5} \beta_i \cos(2i\pi f(t)) + \psi_i \sin(2i\pi f(t)) + e_t$

Linear Year, 5th Order Harmonic  $y_t = \gamma_0 + \gamma_1 \text{year} + \sum_{i=1:5} \beta_i \cos(2i\pi f(t)) + \psi_i \sin(2i\pi f(t)) + e_t$

Quadratic Year, 5th Order Harmonic  $y_t = \gamma_0 + \gamma_1 \text{year} + \gamma_2 \text{year}^2 + \sum_{i=1:5} \beta_i \cos(2i\pi f(t)) + \psi_i \sin(2i\pi f(t)) + e_t$

- Now fit a “gam” from the “mgcv” package that includes a long-term trend based on a thin-plate spline with shrinkage that uses ‘`k=#years,bs=”ts”`’ from the fractional year variable and a cyclic spline seasonal component. To build the cyclic spline component, use the numerically coded month variable that goes from 1 to 12 and `k=12,bs=”cc”`. Fit the model, plot the long-term trend and the seasonal component (use “`plot(gam_model)`”), and discuss the estimated components, using both the plots and the EDF of each term.



- Calculate the AIC of the GAM using the “AIC” function and discuss how that result compares to your AICs in #1. How is it similar or different in terms of information (degrees of freedom) used?

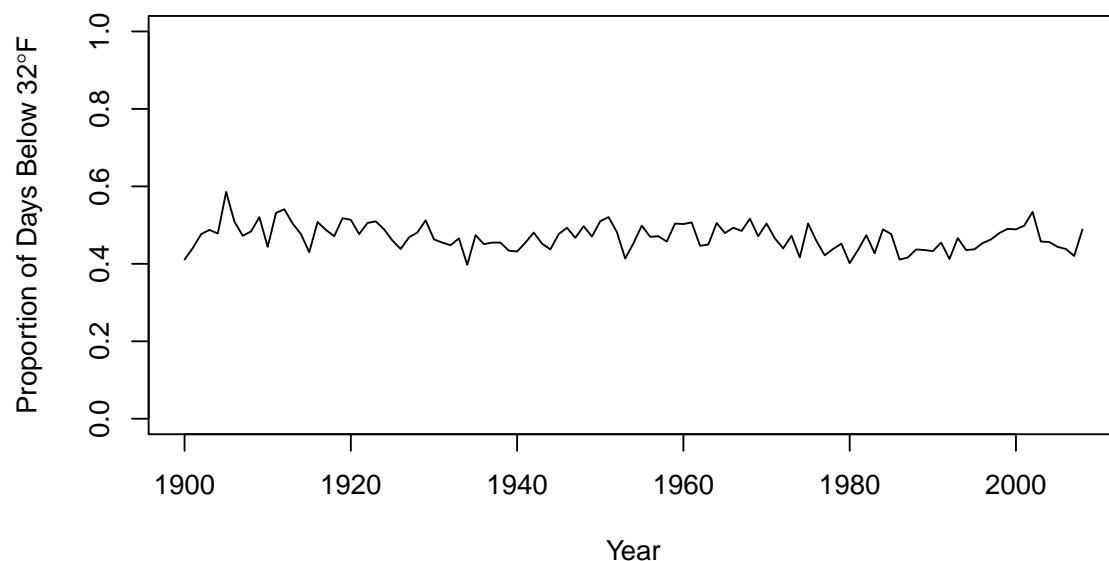
4. Compare the fitted values of your GAM to those from your top model, plotting the two models's results and the responses vs time on the same plot.

### A simulation study with autocorrelation present

5. Revisit your simulation with an  $AR(1)$  from HW 6 #8. Consider fitting a model with autocorrelation in it using “gls” from the “nlme” package that accounts for an  $MA(1)$  error and another that accounts for an  $AR(1)$  error. Run your simulation code, extracting the  $p$ -values from the two model summaries and estimate the type I error rate in each situation and compare it to what you get from the regular linear model.

He changed this to say #8 so we're supposed to do type I error rate simulations instead of permutation tests.

### Estimated Linear Trends



- In a “gls” model summary, the estimates, SEs, test statistics, and  $p$ -values are contained in the ‘tTable’ part of the summary. It has a similar layout to ‘coef’ that we pulled the [2,4] element from to get the  $p$ -value from “lm”.

Some derivation practice (these can be handwritten).

If you have not completed STAT 421 or equivalent, please try the problem and then take advantage of advanced help by stopping by to chat about your answer.

6. Answer Cryer and Chan question 2.4 (page 20)

7. Suppose that we are interested in the properties of a local average (linear filter) of two observations from an original time series,  $x_t$ . The new series is  $y_t = (0.5) * (x_{t-1} + x_t)$ . The mean of  $x_t$  is 3, the variance of  $x_t$  is 4, and the correlation between any neighboring  $x_t$ 's is 0.5 (so  $\text{cor}(x_t, x_{t-1}) = 0.5$ ).  $x_t$ 's more than two time points apart are uncorrelated (correlation is 0). Use the rules for means and variances of linear combinations to find  $E(y_t)$ ,  $\text{Var}(y_t)$ , and  $\text{Cov}(y_t, y_{t-1})$ . Do not worry about what happens at the edges of the time series (for  $t=1$  or  $t=n$ ), only worry about  $t$  in general.

- Note that you have some preliminary work to complete to go from the provided information to what you need to work on the three derivations requested.

## References

## R Code

1. *## no seasonal component*

```
lm_nt.ns <- lm(value ~ 1, data = mex)
mod_nt.ns <- paste("$y_{t}$ = $\\beta_{o}$ + $e_{t}$")
df_nt.ns <- summary(lm_nt.ns)$df[1]
aic_nt.ns <- AIC(lm_nt.ns)
nt.ns <- c("No Year, No Seasonality", df_nt.ns, aic_nt.ns)

lm_lt.ns <- lm(value ~ year, data = mex)
mod_lt.ns <- paste("$y_{t}$ = $\\gamma_{o}$ + $\\gamma_{1}$year + $e_{t}$")
df_lt.ns <- summary(lm_lt.ns)$df[1]
aic_lt.ns <- AIC(lm_lt.ns)
lt.ns <- c("Linear Year, No Seasonality", df_lt.ns, aic_lt.ns)

lm_qt.ns <- lm(value ~ poly(year, degree = 2), data = mex)
mod_qt.ns <- paste("$y_{t}$ = $\\gamma_{o}$ + $\\gamma_{1}$year + $\\gamma_{1}$year^2 + $e_{t}$")
df_qt.ns <- summary(lm_qt.ns)$df[1]
aic_qt.ns <- AIC(lm_qt.ns)
qt.ns <- c("Quadratic Year, No Seasonality", df_qt.ns, aic_qt.ns)
```

*## seasonal means*

```
lm_nt.sm <- lm(value ~ as.factor(month), data = mex)
mod_nt.sm <- paste("$y_{t}$ = $\\beta_{1}$jan.ind + $\\beta_{2}$feb.ind + $\\beta_{3}$mar.ind + $\\beta_{4}$apr.ind + $\\beta_{5}$may.ind + $\\beta_{6}$june.ind + $\\beta_{7}$july.ind + $\\beta_{8}$aug.ind + $\\beta_{9}$sep.ind + $\\beta_{10}$oct.ind + $\\beta_{11}$nov.ind + $\\beta_{12}$dec.ind + $e_{t}$")
df_nt.sm <- summary(lm_nt.sm)$df[1]
aic_nt.sm <- AIC(lm_nt.sm)
nt.sm <- c("No Year, 5th Order Harmonic", df_nt.sm, aic_nt.sm)

lm_lt.sm <- lm(value ~ year + as.factor(month), data = mex)
mod_lt.sm <- paste("$y_{t}$ = $\\gamma_{1}$year + $\\beta_{1}$jan.ind + $\\beta_{2}$feb.ind + $\\beta_{3}$mar.ind + $\\beta_{4}$apr.ind + $\\beta_{5}$may.ind + $\\beta_{6}$june.ind + $\\beta_{7}$july.ind + $\\beta_{8}$aug.ind + $\\beta_{9}$sep.ind + $\\beta_{10}$oct.ind + $\\beta_{11}$nov.ind + $\\beta_{12}$dec.ind + $e_{t}$")
```

```

df_lt.sm <- summary(lm_lt.sm)$df[1]
aic_lt.sm <- AIC(lm_lt.sm)
lt.sm <- c("Linear Year, Seasonal Means", df_lt.sm, aic_lt.sm)

lm_qt.sm <- lm(value ~ poly(year, degree = 2) + as.factor(month), data = mex)
mod_qt.sm <- paste("$y_{t}$ =  $\gamma_1 \text{year} + \gamma_2 \text{year}^2 + \beta_1 \text{jan.ind} + \beta_2 \text{feb.ind} + \beta_{10} \text{oct.ind} + \beta_{11} \text{nov.ind} + \beta_{12} \text{dec.ind} + e_t$ ")
df_qt.sm <- summary(lm_qt.sm)$df[1]
aic_qt.sm <- AIC(lm_qt.sm)
qt.sm <- c("Quadratic Year, 5th Order Harmonic", df_qt.sm, aic_qt.sm)

#harmonic() generates sines and cosines
#note: time unit affects frequency measure
#discuss: what does m do? Ho: the number of cycles
# Ha: number of terms
# pvalue = 0.12 (number of terms is related to number of cycles)
#discuss: is there a phi component? the help file doesn't have one and I don't understand what is meant by "p
# phase is physics-talk for shifting the cos left or right
# eg cos(t - phi) shifts the graph of cos(t) to the right by phi
# harmonic doesn't include phi because it uses b_1*cos(t) + b_2*sin(t)
# which is equivalent to b*cos(t+phi) because of the sum identity
# source Esty (2012) section 7.3 :P
#discuss: what is t here?
# I think t is number of years, starting at year=1
# Before we added frequency = 12, t was the number of months
mex$monthts <- ts(mex$month, frequency = 12)

lm_nt.sh1 <- lm(value ~ harmonic(monthts, m = 1), data = mex)
mod_nt.sh1 <- paste("$y_{t}$ =  $\beta_0 + \beta_1 \cos(2\pi f(t)) + \beta_2 \sin(2\pi f(t)) + e_t$ ")
df_nt.sh1 <- summary(lm_nt.sh1)$df[1]
aic_nt.sh1 <- AIC(lm_nt.sh1)
nt.sh1 <- c("No Year, 1st Order Harmonic", df_nt.sh1, aic_nt.sh1)

lm_lt.sh1 <- lm(value ~ year + harmonic(monthts, m = 1), data = mex)
mod_lt.sh1 <- paste("$y_{t}$ =  $\gamma_0 + \gamma_1 \text{year} + \beta_1 \cos(2\pi f(t)) + \beta_2 \sin(2\pi f(t)) + e_t$ ")
df_lt.sh1 <- summary(lm_lt.sh1)$df[1]
aic_lt.sh1 <- AIC(lm_lt.sh1)
lt.sh1 <- c("Linear Year, 1st Order Harmonic", df_lt.sh1, aic_lt.sh1)

lm_qt.sh1 <- lm(value ~ poly(year, 2) + harmonic(monthts, m = 1), data = mex)
mod_qt.sh1 <- paste("$y_{t}$ =  $\gamma_0 + \gamma_1 \text{year} + \gamma_2 \text{year}^2 + \beta_1 \cos(2\pi f(t)) + e_t$ ")
df_qt.sh1 <- summary(lm_qt.sh1)$df[1]
aic_qt.sh1 <- AIC(lm_qt.sh1)
qt.sh1 <- c("Quadratic Year, 1st Order Harmonic", df_qt.sh1, aic_qt.sh1)

## m = 5

lm_nt.sh5 <- lm(value ~ harmonic(monthts, m = 5), data = mex)
mod_nt.sh5 <- paste("$y_{t}$ =  $\beta_0 + \sum_{i=1}^5 \beta_i \cos(2i\pi f(t)) + \sum_{i=1}^5 \psi_i \sin(2i\pi f(t)) + e_t$ ")
df_nt.sh5 <- summary(lm_nt.sh5)$df[1]
aic_nt.sh5 <- AIC(lm_nt.sh5)
nt.sh5 <- c("No Year, 5th Order Harmonic", df_nt.sh5, aic_nt.sh5)

lm_lt.sh5 <- lm(value ~ year + harmonic(monthts, m = 5), data = mex)

```

```

mod_lt.sh5 <- paste("$y_{t} = \\gamma_{0} + \\gamma_{1}year + \\Sigma_{i=1:5} \\beta_{i}\\cos(2i\\pi f(t)) + \\gamma_{1}year^2")
df_lt.sh5 <- summary(lm_lt.sh5)$df[1]
aic_lt.sh5 <- AIC(lm_lt.sh5)
lt.sh5 <- c("Linear Year, 5th Order Harmonic", df_lt.sh5, aic_lt.sh5)

lm_qt.sh5 <- lm(value ~ poly(year, 2) + harmonic(monthts, m = 5), data = mex)
mod_qt.sh5 <- paste("$y_{t} = \\gamma_{0} + \\gamma_{1}year + \\gamma_{1}year^2 + \\Sigma_{i=1:5} \\beta_{i}\\cos(2i\\pi f(t)) + \\gamma_{1}year^2")
df_qt.sh5 <- summary(lm_qt.sh5)$df[1]
aic_qt.sh5 <- AIC(lm_qt.sh5)
qt.sh5 <- c("Quadratic Year, 5th Order Harmonic", df_qt.sh5, aic_qt.sh5)

df_all <- data.frame(rbind(nt.ns, lt.ns, qt.ns, nt.sm, lt.sm, qt.sm, nt.sh1, lt.sh1, qt.sh1, nt.sh5, lt.sh5, qt.sh5))
colnames(df_all) <- c("Model", "Free DF", "AIC")
df_all$`Free DF` <- as.numeric(as.character(df_all$`Free DF`))
df_all$AIC <- as.numeric(as.character(df_all$AIC))

df_all <- arrange(df_all, AIC)

df_all$deltaAIC <- df_all$AIC - df_all$AIC[1]

print(xtable(df_all, digits = c(0, 0, 0, 2, 2)), include.rownames = FALSE)

all_mod <- data.frame(rbind(mod_qt.sh5, mod_lt.sh5, mod_nt.sh5, mod_qt.sh1, mod_lt.sh1, mod_nt.sh1, mod_qt.sm, mod_lt.sm, mod_nt.sm))
row.names(all_mod) <- c("Quadratic Year, 5th Order Harmonic", "Linear Year, 5th Order Harmonic", "No Year, 5th Order Harmonic")
colnames(all_mod) <- " "

#print(xtable(all_mod))

```

2. `mex$yearts <- ts(mex$year)`  
`gam1 <- gam(value ~ s(time(yearts), k = 7, bs = "ts") + s(monthts, k = 12, bs = "cc"), data = mex)`  
`par(mfrow = c(1,2))`  
`plot(gam1)`

3. `aic_gam1 <- AIC(gam1)`

5. `Bozeman<-read.csv("Bozeman.csv",header=T)`  
`monthsF<-sort(unique(Bozeman$MonthRE))`  
`countfun<-function(x) c(sum(x<32),sum(!is.na(x)))`  
`monthcountMINF<-aggregate(Bozeman$TMIN..F.,by=list(Bozeman$MonthRE),FUN=countfun)`  
`yearcountMINF<-aggregate(Bozeman$TMIN..F.,by=list(Bozeman$Year),FUN=countfun)`  
`Data1<-data.frame(Year=yearcountMINF[,1],DaysBelow32=yearcountMINF$x[,1],`  
`MeasuredDays=yearcountMINF$x[,2],`  
`PropDays=yearcountMINF$x[,1]/yearcountMINF$x[,2])`

```

plot(PropDays ~ Year, data = Data1, type = "l", ylim = c(0,1),
     main = "Estimated Linear Trends",
     ylab = expression("Proportion of Days Below 32"*degree*F))
#abline(lm.Year, lwd = 2)
#abline(model1s, col = "purple", lty = 2, lwd = 2)
#abline(a = zyp.Year1coefficients[1], b =
#       zyp.Year1coefficients[2], col = "red", lty = 3, lwd = 2)
#legend("topright", lwd = 2, lty = 1:3, col = c("black", "purple", "red"),
#       legend = c("OLS", "Sen", "Zhang"))

#corAR1 == AR1 process
#corARMA == autoregressive moving avg
gls_ar1_obs <- gls(PropDays ~ Year, data = Data1, correlation = corAR1())

gls_ar1_out <- coef(gls_ar1_obs)[2]

gls_ma_obs <- gls(PropDays ~ Year, data = Data1, correlation = corARMA(q = 1))

gls_ma_out <- coef(gls_ma_obs)[2]

gls_ar1 <- replicate(1000, coef(gls(PropDays ~ shuffle(Year), data = Data1, correlation = corAR1()))[2])

gls_ma <- replicate(1000, coef(gls(PropDays ~ shuffle(Year), data = Data1, correlation = corARMA(q = 1)))[2])

pvalue_ar1 <- (length(which(gls_ar1 <= gls_ar1_out)) + length(which(gls_ar1 >= -gls_ar1_out)))/length(gls_ar1)

pvalue_ma <- (length(which(gls_ma <= gls_ma_out)) + length(which(gls_ma >= -gls_ma_out)))/length(gls_ma)

```