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Abstract

Purpose – The purpose of this paper is to deal with the different phases of volatility behavior and the dependence of the variability of the time series on its own past, models allowing for heteroscedasticity like autoregressive conditional heteroscedasticity (ARCH), generalized autoregressive conditional heteroscedasticity (GARCH), or regime-switching models have been suggested by reserachers. Both types of models are widely used in practice.

Design/methodology/approach – Both regime-switching models and GARCH are used in this paper to model and explain the behavior of crude oil prices in order to forecast their volatility. In regime-switching models, the oil return volatility has a dynamic process whose mean is subject to shifts, which is governed by a two-state first-order Markov process.

Findings – The GARCH models are found to be very useful in modeling a unique stochastic process with conditional variance; regime-switching models have the advantage of dividing the observed stochastic behavior of a time series into several separate phases with different underlying stochastic processes.

Originality/value – The regime-switching models show similar goodness-of-fit result to GARCH modeling, while has the advantage of capturing major events affecting the oil market. Daily data of crude oil prices are used from NYMEX Crude Oil market for the period 13 February 2006 up to 21 July 2009.

Keywords Cybernetics, Prices, Stochastic processes, Forecasting, Modelling, Oil industry

Paper type Research paper

1. Introduction

Risk analysis of crude oil market has always been the core research problem that deserves lots of attention from both the practice and academia. Risks occur mainly due to the change of oil prices. During the 1970s and 1980s, there were a great deal increases in oil price. Such price fluctuations came to new peaks in 2007 when the price of crude oil doubled during the financial crisis. These fluctuations of double digit numbers in short periods of time continued between 2007 and 2008, when we see highly volatile oil prices. These fluctuations would not be worrisome if oil would not be such an important commodity in the world's economy. When the oil prices become too high and the volatility increases, it has a direct impact on the economy in general and thus affects the government decisions regarding the market regulation and thus the firm and individual consumer incomes (Bacon and Kojima, 2008).

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Price volatility analysis has been a hot research area for many years. Commodity markets are characterized by extremely high levels of price volatility. Understanding the volatility dynamic process of oil price is a very important and crucial way for producers and countries to hedge various risks and to avoid the excess exposures to risks (Bacon and Kojima, 2008).

To deal with different phases of volatility behavior and the dependence of the variability of the time series on its own past, models allowing for heteroscedasticity like autoregressive conditional heteroscedasticity (ARCH), generalized autoregressive conditional heteroscedasticity (GARCH), or regime-switching models have been suggested by researchers. The former two are very useful in modeling a unique stochastic process with conditional variance; the latter has the advantage of dividing the observed stochastic behavior of a time series into several separate phases with different underlying stochastic processes. Both types of models are widely used in practice.

Hung *et al.* (2008) employ three GARCH models, i.e. GARCH-N, GARCH-t, and GARCH-HT, to investigate the influence of fat-tailed innovation process on the performance of energy commodities value-at-risk (VaR) estimates. Narayan *et al.* (2008) use the exponential GARCH models to evaluate the impact of oil price on the nominal exchange rate. To validate cross-market hedging and sharing of common information by investors, Malik and Ewing (2009) employ bivariate GARCH models to estimate the relations between five different US sector indexes and oil prices. On the other side, regime-switching has been used a lot in modeling stochastic processes with different regimes. Alizadeh *et al.* (2008) introduce a Markov regime-switching vector error correction model with GARCH error structure and show how portfolio risks are reduced using state dependent hedge ratios. Aloui and Jammazi (1989) employ a two regime Markov-switching exponential generalized autoregressive conditional heteroscedasticity (EGARCH) model to analyze oil price change and find the probability of transition across regimes. Klaassen (2002) develops a regime-switching GARCH model to account for the high persistence of shocks generated by changes in the variance process. Oil shocks were found to contribute to a better description of the impact of oil on output growth (Cologni and Manera, 2009). There is no clear evidence regarding which approach outperforms the other one.

Fan *et al.* (2008) argue that generalized error distribution (GED)-GARCH-based VaR approach is more realistic and more effective than the well-recognized historical simulation with autoregressive moving average (ARMA) forecasts in an empirical study. The FIAPARCH model is said to outperform the other models in the VaR's prediction (Aloui and Mabrouk, 2009). GARCH models also seem to perform better than the implied volatility by inverting the Black equation. When assuming GED distributed errors GARCH was believed to perform best (Agnolucci, 2009). Clear evidence of regime-switching has been discovered in the oil market. Engel (1994) believes that regime-switching models provide a useful framework for the evolution of volatility and forecasts of exchange rates volatility. The regime-switching stochastic volatility model performs well in capturing major events affecting the oil market (Vo, 2009).

This paper will focus on volatility modeling in crude oil market using both regime-switching stochastic volatility and GARCH models. Section 2 will review the types of volatility models. We will then look at crude market data in Section 3. Computation and results analysis are presented in Section 4. Section 5 concludes the paper.

2. Volatility models

2.1 Historical volatility

We suppose that ε_t is the innovation in mean for energy log price changes or price returns. To estimate the volatility at time t over the last N days, we have:

$$\mathrm{V_{H,t}} = \left[\left(\frac{1}{N}\right) \sum_{i=0}^{N-1} \varepsilon_{t-i}^2 \right]^{1/2},$$

where N is the forecast period. This is actually an N-day simple moving average (MA) volatility, where the historical volatility is assumed to be constant over the estimation and the forecast periods. To involve the long-run or unconditional volatility using all previous returns available at time t, we have many variations of the simple MA volatility model (Fama, 1970).

2.1.1 ARMA(R,M). Given a time series of data X_t, the ARMA model is a very useful for predicting future values in time series where there are both an autoregressive (AR) part and a MA part. The model is usually then referred to as the ARMA(R,M) model where R is the order of the first part and M is the order of the second part. The following ARMA(R,M) model contains the AR(R) and MA(M) models:

$$X_t = c + \varepsilon_t + \sum_{i=1}^{R} \varphi_i X_{t-i} + \sum_{j=1}^{M} \theta_j \varepsilon_{t-j}.$$

where φ_i and θ_j are parameters for AR and MA parts, respectively.

2.1.2 ARMAX(R,M, b). To include the AR(R) and MA(M) models and a linear combination of the last b terms of a known and external time series d_t, one can have a model of ARMAX(R,M, b) with R autoregressive, M moving average, and b exogenous inputs terms:

$$X_t = c + \varepsilon_t + \sum_{i=1}^{R} \varphi_i X_{t-i} + \sum_{j=1}^{M} \theta_j \varepsilon_{t-j} + \sum_{k=1}^{b} \eta_k d_{t-k},$$

where $\eta_1, \ldots, \eta_b$ are the parameters of the exogenous input d_t.

2.1.3 ARCH(q). ARCH type modeling is the predominant statistical technique employed in the analysis of time-varying volatility. In ARCH models, volatility is a deterministic function of historical returns. The original ARCH(q) formulation proposed by Engle (1982) models conditional variance ht as a linear function of the first q past-squared innovations:

$$\sigma_t^2 = c + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2.$$

This model allows today's conditional variance to be substantially affected by the (large) square error term associated with a major market move (in either direction) in any of the previous q periods. It thus captures the conditional heteroscedasticity of financial returns and offers an explanation of the persistence in volatility. A practical difficulty with the ARCH(q) model is that in many of the applications a long length q is called for.

2.1.4 GARCH(p,q). Bollerslev's (1986) GARCH(p,q) specification generalizes the model by allowing the current conditional variance to depend on the first p past conditional variances as well as the q past squared innovations. That is:

$$\sigma_t^2 = L + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2,$$

where L denotes the long-run volatility.

By accounting for the information in the lag(s) of the conditional variance in addition to the lagged t − i terms, the GARCH model reduces the number of parameters required. In most cases, one lag for each variable is sufficient. The GARCH(1,1) model is given by: $\sigma_t^2 = L + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2$. GARCH can successfully capture thick tailed returns and volatility clustering. It can also be modified to allow for several other stylized facts of asset returns.

2.1.5 EGARCH. The EGARCH model introduced by Nelson (1991) builds in a directional effect of price moves on conditional variance. Large price declines, for instance, may have a larger impact on volatility than large price increases. The general EGARCH(p,q) model for the conditional variance of the innovations, with leverage terms and an explicit probability distribution assumption, is:

$$\log \sigma_t^2 = L + \sum_{i=1}^{p} \beta_i \log \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j \left[\frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} - E\left\{ \frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right\} \right] + \sum_{j=1}^{q} L_j \left(\frac{\varepsilon_{t-j}}{\sigma_{t-j}} \right)$$

where:

$$E\{|z_{t-j}|\} E\left\{ \frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right\} = \sqrt{\frac{2}{\pi}}$$

for the normal distribution, and:

$$E\{|z_{t-j}|\} E\left\{ \frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} \right\} = \sqrt{\frac{v-2}{\pi}} \frac{\Gamma((v-1)/2)}{\Gamma(v/2)}$$

for the Student's t distribution with degree of freedom $v > 2$.

2.1.6 GJR(p,q). GJR(p,q) model is an extension of an equivalent GARCH(p,q) model with zero leverage terms. Thus, estimation of initial parameter for GJR models should be identical to those of GARCH models. The difference is the additional assumption with all leverage terms being zero:

$$\sigma_t^2 = L + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{q} L_j S_{t-j} \varepsilon_{t-j}^2$$

where $S_{t-j} = 1$ if $\varepsilon_{t-j} < 0$, $S_{t-j} = 0$ otherwise, with constraints:

$$\sum_{i=1}^{p} \beta_i + \sum_{j=1}^{q} \alpha_j + \frac{1}{2} \sum_{j=1}^{q} L_j < 1, \quad L \geq 0, \;\; \beta_i \geq 0, \;\; \alpha_j \geq 0, \;\; \alpha_j + L_j \geq 0.$$

2.2 Regime-switching models

Markov regime-switching model has been applied in various fields such as oil and the macroeconomy analysis (Raymond and Rich, 1997), analysis of business cycles (Hamilton, 1989) and modeling stock market and asset returns (Engel, 1994).

We now consider a dynamic volatility model with regime-switching. Suppose a time series y_t follow an AR (p) model with AR coefficients, together with the mean and variance, depending on the regime indicator s_t:

$$y_t = \mu_{s_t} + \sum_{j=1}^{p} \varphi_{j,s_t} y_{t-j} + \varepsilon_t, \quad \text{where } \varepsilon_t \sim i.i.d.\ \mathrm{N}ormal\left(0, \sigma_{s_t}^2\right).$$

The corresponding density function for y_t is:

$$f(y_t|s_t, Y_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_{s_t}^2}} \cdot \exp\left[-\frac{\omega_t^2}{2\sigma_{s_t}^2}\right] = f(y_t|s_t, y_{t-1}, \ldots, y_{t-p}),$$

where:

$$\omega_t = y_t - \omega_{s_t} - \sum_{j=1}^{p} \varphi_{j,s_t} y_{t-j}.$$

The model can be estimated by use of maximum log likelihood (LLC) estimation. A more practical situation is to allow the density function of y_t to depend on not only the current value of the regime indicator s_t but also the past values of the regime indicator s_t which means the density function should takes the form of:

$$f(y_t|s_t, S_{t-1}, Y_{t-1}),$$

where $S_{t-1} = \{s_{t-1}, s_{t-2}, \ldots\}$ is the set of all the past information on s_t.

3. Data

3.1 Data and sample description

The data spans a continuous sequence of 866 days from February 2006 to July 2009, showing the closing prices of the NYMEX Crude Oil index during this time period on a day-to-day basis. Weekends and holidays are not included in our data thus considering those days as non moving price days. Using the logarithm prices changes means that our continuously compounded return is symmetric, preventing us from getting nonstationary level of oil prices which would affect our return volatility. Table I presents the descriptive statistics of the daily crude oil price changes. In Figure 1, we show a plot of the crude oil daily price movement.

To get a preliminary view of volatility change, we show in Table II the descriptive statistics on the daily crude oil index log return ranging from February 2006 to July 2009. The corresponding plot is shown in Figure 2.

Table I.
Statistics on the daily crude oil index changes from February 2006 to July 2009

Statistics	Value
Sample size	866
Mean	77.2329
Maximum	145.9600
Minimum	44.4100
SD	20.9270
Skewness	1.3949
Kurtosis	4.3800

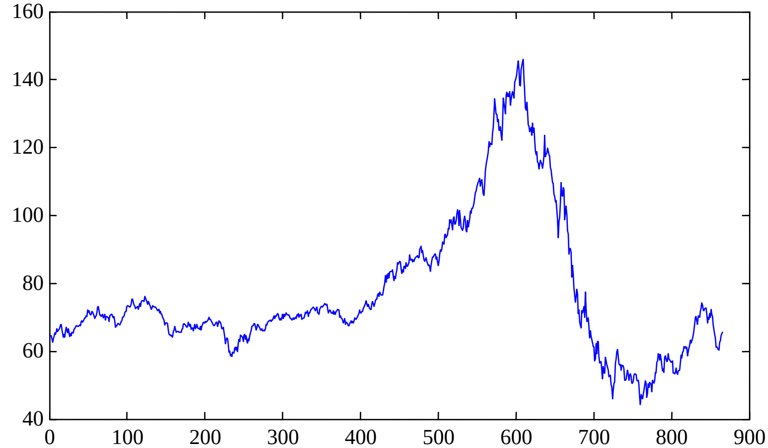

Figure 1.
NYMEX Crude Oil daily price movements

Statistics	Value
Sample size	865
Mean	1.8293×10^{-005}
Maximum	0.1003
Minimum	0.0874
SD	0.0218
Skewness	-0.0962
Kurtosis	6.1161

Table II.
The statistics on the daily crude oil index log return from February 2006 to July 2009

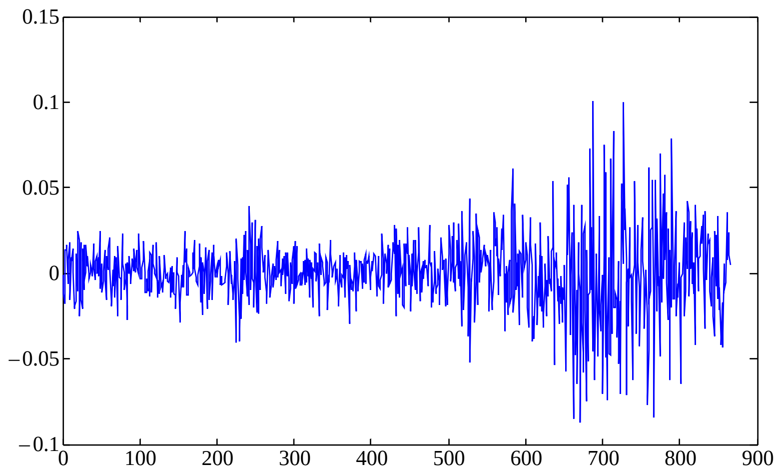

Figure 2.
NYMEX Crude Oil daily logreturn

3.2 Distribution analysis

Figure 3 shows a distribution analysis of our data ranging from February 2006 up to July 2009. The data are the log return of the daily crude oil price movements over

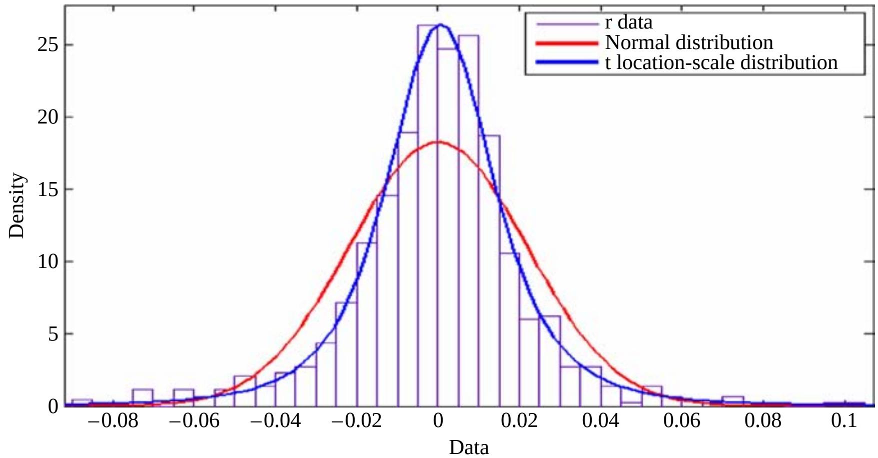

Figure 3.
Normal distribution vs t-distribution

the time period mentioned above. We can see (Figure 3) that the best distribution for our data is a t-distribution which is shown by the blue line. The red line represents the normal distribution of our data is also shown. So, a conditional t-distribution is preferred to normal distribution in our research. An augmented Dickey-Fuller univariate unit root test yields a resulted p-value of $1.0^{*}10^{-3}$, $1.1^{*}10^{-3}$ and $1.1^{*}10^{-3}$ for lags of 0, 1 and 2, respectively. All p-values are smaller than 0.05, which indicates that the time series has a trend-stationary property.

4. Results

4.1 GARCH modeling

We first estimated the parameter of the GARCH(1,1) model using 865 observations in Matlab, and then tried various GARCH models using different probability distributions with the maximum-likelihood estimation technique. In many financial time series, the standardized residuals $z_t = \varepsilon_t/\sigma_t$ usually display excess kurtosis, which suggests departure from conditional normality. In such cases, the fat-tailed distribution of the innovations driving a dynamic volatility process can be better modeled using the Student's-*t* or the GED. Taking the square root of the conditional variance and expressing, it as an annualized percentage yields a time-varying volatility estimate. A single estimated model can be used to construct forecasts of volatility over any time horizon. Table III presents the GARCH(1,1) estimation using t-distribution. The conditional mean process is modeled by use of ARMAX(0,0,0).

Substituting these estimated values in the math model, we yield the explicit form as follows:

$$y_t = 6.819 \times 10^{-4} + \varepsilon_t \quad \sigma_t^2 = 2.216 \times 10^{-6} + 0.9146\sigma_{t-1}^2 + 0.0815\varepsilon_{t-1}^2.$$

Figure 4 shows the dynamics of the innovation, standard deviation, and return using the above estimated GARCH model, i.e. the ARMAX(0,0,0) GARCH(1,1) with the LLC value of 2,284.97. We want to find a higher LLC value for other GARCH modeling, so we use the same data with different models in order to increase the robustness of

Model	AIC	BIC	lnL	Parameter	Value	SE	t-statistic
Mean: ARMAX(0,0,0); variance: GARCH(1,1)	$-4{,}559.9$	$-4{,}536.1$	2,284.97	C	6.819×10^{-4}	5.0451×10^{-4}	1.3516
				K	2.216×10^{-6}	1.306×10^{-6}	1.7011
				β_1	0.9146	0.0174	52.6514
				α_1	0.0815	0.0179	4.5539
				DoF	34.603	8.4422×10^{-7}	$4.0988 \times 10^{+7}$

Table III.
GARCH(1,1) estimation using t-distribution

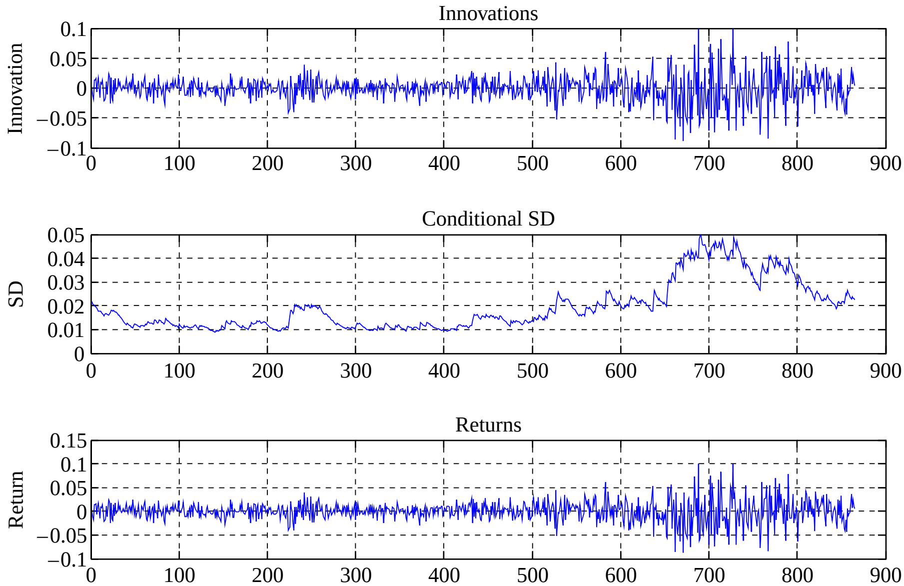

Figure 4.
Innovation, SD and return

our model. We now try different combinations of ARMAX and GARCH, EGARCH, and GJR models. Computation results are presented in Table IV.

A general rule for model selection is that we should specify the smallest, simplest models that adequately describe data because simple models are easier to estimate, easier to forecast, and easier to analyze. Model selection criteria such as Akaike (AIC) and Bayesian (BIC) penalize models for their complexity when considering best distributions that fit the data. Therefore, we can use LLC, AIC, and BIC information criteria to compare alternative models. Usually, differences in LLC across distributions cannot be compared since distribution functions can have different capabilities for fitting random data, but we can use the minimum AIC and BIC, maximum LLC values as model selection criteria (Cousineau *et al.*, 2004).

As can be seen from Table IV, the LLC value of ARMAX(1,1,0) GJR(2,1) yields the highest LLC value 2,292.32 and lowest AIC value −4,566.6 among all modeling technique. Thus, we conclude that GJR models should be our preferred model. The ARMAX(1,1,0) GJR(2,1) model was then used to do a simulation and a forecast for the standard deviation for a 30-day period using 20,000 realizations.

The forecasting horizon was defined to be 30 days (one month). The simulation uses 20,000 realizations for a 30-day period based on our fitted model ARAMX(1,1,0) GJR(2,1) and the horizon of 30 days from "Forecasting." In Figure 5, we compare the forecasts from "Forecasting" with those derived from "Simulation."

The first four panels of Figure 5 shows directly each of the forecasted outputs with the corresponding statistical result obtained from simulation. The last two panels of Figure 5 shows histograms from which we could compute the approximate probability density functions and empirical confidence bounds. When comparing forecasting with its counterpart derived from the Monte Carlo simulation, we show computation for four parameters in the first four panels of Figure 5: the conditional standard deviations

Table IV. Various GARCH modeling

Model	AIC	BIC	lnL	Parameter	Value	SE	t-statistic
Mean: ARMAX(1,1,0); variance: GARCH(1,1)	−4,561.0	−4,527.7	2,287.5	C	8.995×10^{-4}	6.6851×10^{-4}	1.3455
				ϕ_1	−0.3119	0.4386	−0.7111
				θ_1	0.2363	0.4465	0.5292
				K	2.0564×10^{-6}	1.2567×10^{-6}	1.6363
				β_1	0.9175	0.0169	54.1607
				α_1	0.0790	0.0174	4.5436
				DoF	30.107	1.6771×10^{-4}	$1.795 \times 10^{+5}$
Mean: ARMAX(1,1,0); variance: EGARCH(1,1)	−4,557.8	−4,524.5	2,286.3	C	6.6556×10^{-4}	6.2368×10^{-4}	1.0672
				ϕ_1	−0.3067	0.3895	−0.7874
				θ_1	0.2226	0.3969	0.5607
				K	−0.0396	0.0297	−1.3339
				β_1	0.9950	3.6255×10^{-3}	274.4553
				α_1	0.1459	0.0281	5.1980
				L_1	−0.0316	0.0155	−2.0337
				DoF	37.596	48.455	0.7759
Mean: ARMAX(1,1,0); variance: GJR(1,1)	−4,560.9	−4,522.8	2,288.4	C	6.912×10^{-4}	6.3923×10^{-4}	1.0813
				ϕ_1	−0.2967	0.4499	−0.6596
				θ_1	0.2218	0.4567	0.4852
				K	2.1511×10^{-6}	1.2682×10^{-6}	1.6961
				β_1	0.9189	0.0168	54.7178
				α_1	0.0592	0.0211	2.8779
				L_1	0.0344	0.0254	1.3541
				DoF	38.36	1.1967×10^{-4}	$3.2054 \times 10^{+4}$
Mean: ARMAX(1,1,0); variance: GJR(2,1)	−4,566.6	−4,523.8	2,292.3	C	5.6469×10^{-4}	6.4635×10^{-4}	0.8737
				ϕ_1	−0.3582	0.4030	−0.8891
				θ_1	0.2843	0.4138	0.6871
				K	3.5044×10^{-6}	1.9942×10^{-6}	1.7573
				β_1	0	0.0255	0.0000
				β_2	0.8682	0.0294	29.5592
				α_1	0.0910	0.0255	3.5714
				L_1	0.0677	0.0346	1.9552
				DoF	50.013	6.0689×10^{-6}	$8.2409 \times 10^{+6}$

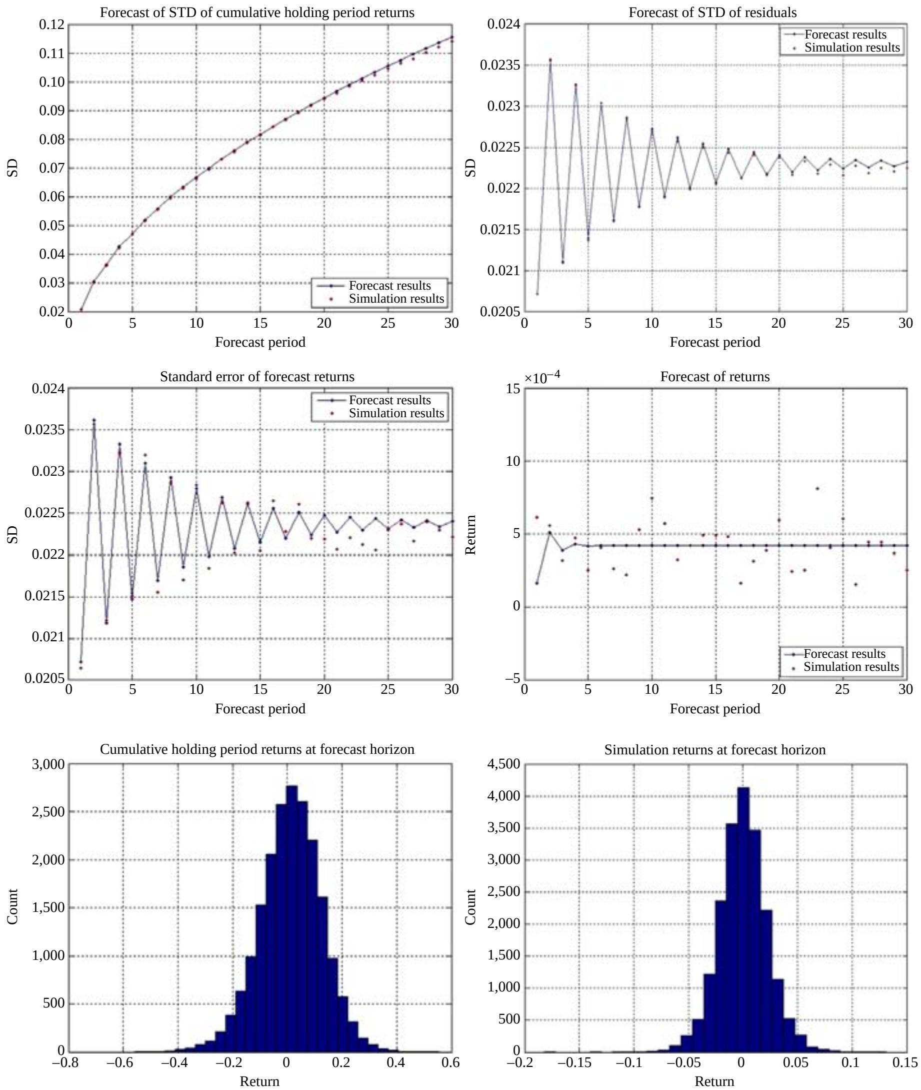

Figure 5.
Simulation and forecasting

of future innovations, the MMSE forecasts of the conditional mean of the nasdaq return series, cumulative holding period returns and the root mean square errors of the forecasted returns. The fifth panel of Figure 5 uses a histogram to illustrate the distribution of the cumulative holding period return obtained if an asset was held for the full 30-day forecast horizon. In other words, we plot the log return obtained by investing in NYMEX Crude Oil index today, and sold after 30 days. The last panel of Figure 5 uses a histogram to illustrate the distribution of the single-period return at the forecast horizon, that is, the return of the same mutual fund, the 30th day from now.

4.2 Markov regime-switching modeling

We now try Markov regime-switching modeling in this section. The purpose is twofold: first, to see if Markov switching regressions can beat GARCH models in time series modeling; second, find turmoil regimes in historical time series.

We employ a Markov regime-switching computation example in Table V to illustrate our results. The model in Table V assume normal distribution and allow all parameters to switch. We use $S = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ to control the switching dynamics, where the first elements of S control the switching dynamic of the mean equation, while the last terms control the switching dynamic of the residual vector, including distribution parameters mean and variance. A value of "1" in S indicates that switching is allowed in the model while a value of "0" in S indicates that parameter is not allowed to change states. Then, the model for the mean equation is:

$$\begin{array}{cc} \text{State 1 } (S_t = 1) & \text{State 2 } (S_t = 2) \\ y_t = -0.0015 - 0.0667 y_{t-1} + \varepsilon_t & y_t = 0.0012 - 0.0934 y_{t-1} + \varepsilon_t \\ \varepsilon_t \sim N(0, 0.0306^2) & \varepsilon_t \sim N(0, 0.0115^2), \end{array}$$

where ε_t is residual vector which follows a particular distribution. The transition matrix:

$$P = \begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix},$$

controls the probability of a regime switch from state 1(2) (column 1(2)) to state 2(1) (row 2(1)). The sum of each column in P is equal to one, since they represent full probabilities of the process for each state.

In order to yielded the best fitted Markov regime-switching models, we now try various parameter settings for traditional Hamilton's (1989) model and complicated setting using t-distribution and GED. We present computational results in Tables VI-VIII. A comparison of LLC values indicate that complicated setting using t-distribution and GED usually are preferred. The best fitted Markov regime-switching models should assume GED and allow all parameters to change states (Table VIII).

We now focus on analysis using the best fitted Markov regime-switching model, i.e. "MS model, S = [1 1 1 1 1] (GED)" in Table VIII. Figure 6 shows transitional probabilities in Markov regime switching with GED: fitted state probabilities and smoothed state probabilities. Based on such a transitional probability figure, we can classify historical data into two types according to their historical states.

Figure 7 shows the log return of two regimes in historical time series. Figure 8 shows the price of two regimes in historical time series. As can be seen from Figures 7 and 8, the total historical time series are divided into two regimes: a normal one with small change (state 2) and a turmoil one with big risk (state 1). For each state, regime-switching model identifies three periods of data. The normal regime includes two periods: 10 February to 11 December 2006, and 30 January to 14 October 2007. The turmoil regime also includes two periods: 12 December 2006 to 29 January 2007, and 15 October 2007 to 7 July 2009. The first turmoil lasts only one and a half months, but the second one covers almost the total financial crisis.

Table V.
Markov regime-switching computation example

Model (distribution assumption)	Log likelihood	Non-switching parameters	Switching parameters			Transition probabilities matrix	
				State 1	State 2		
MS model, S = [1 1 1] (normal)	2,257.36	N/A	Model's SD	0.0306	0.0115	0.99	0.01
			Indep. column 1	−0.0015	0.0012	0.01	0.99
			Indep. column 2	−0.0667	−0.0934		

Table VI. Markov regime-switching using Hamilton's (1989) model

Model (distribution assumption)	Log likelihood	Non-switching parameters	Switching parameters	State 1	State 2	Transition probabilities matrix	
Hamilton's (1989) model, S = [1 0 1] (t)	2,212.38	0.0135	Degrees of freedom (t-dist.)	100.00	1.5463	1.00	0.00
						0.00	1.00
			Indep. column 1	0.0008	−0.0002		
Hamilton's (1989) model, S = [1 1 1] (t)	2,257.34	N/A	Model's SD	0.0264	0.0113	0.99	0.01
			Degrees of freedom (t-dist.)	7.8238	112.3094	0.01	0.99
			Indep. column 2	−0.0012	0.0010		

Table VII.
Markov regime-switching using t-distribution

Model (distribution assumption)	Log likelihood	Non-switching parameters		Switching parameters	State 1	State 2	Transition probabilities matrix	
MS model, S = [1 1 0 0] (t)	2,172.41	SD	0.0128	Indep. column 1	0.0021	−0.0010	0.45	0.57
		Degrees of freedom (t-dist.)	2.9506	Indep. column 2	−0.3925	0.2553	0.55	0.43
MS model, S = [1 1 1 1] (t)	2,174.86	N/A		Model's SD	0.0130	0.0117	0.80	0.98
				Degrees of freedom (t-dist.)	3.2408	2.3637	0.20	0.02
				Indep. column 1	0.0013	−0.0034		
				Indep. column 2	−0.2015	0.9080		
MS model, S = [1 1 1 1 1] (t)	2,260.95	N/A		Model's SD	0.0262	0.0113		
				Degrees of freedom (t-dist.)	7.4904	100.000	0.99	0.01
				Indep. column 1	−0.0012	0.0011	0.01	0.99
				Indep. column 2	−0.0736	−0.0915		
				Indep. column 3	−0.0121	0.0422		

Table VIII. Markov regime-switching using GED

Model (distribution assumption)	Log likelihood	Switching parameters	State 1	State 2	Transition probabilities matrix	
MS model, S = [1 1 1 1] (GED)	2,172.16	Model's SD	0.0029	0.0094	0.06	0.26
		Value of k (GED dist)	1.4987	0.8011	0.94	0.74
		Indep. column 1	0.0020	0.0013		
		Indep. column 2	0.8905	0.2207		
MS model, S = [1 1 1 1 1] (GED)	2,263.06	Model's SD	0.0203	0.0120		
		Value of k (GED dist.)	0.7122	0.4675	0.99	0.01
		Indep. column 1	0.0014	0.0010	0.01	0.99
		Indep. column 2	0.0706	0.0848		
		Indep. column 3	0.0287	0.0384		

5. Conclusion

We have examined crude oil price volatility dynamics using daily data for the period 13 February 2006 up to 21 July 2009. To model volatility, we employed the GARCH, EGARCH, and GJR models and various Markov regime-switching models using the

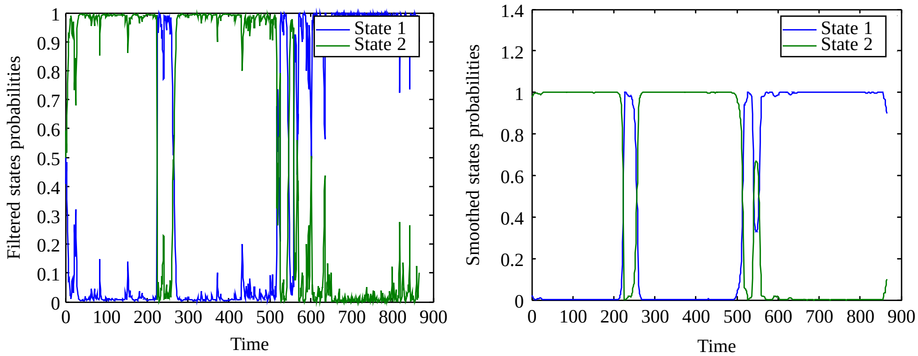

Figure 6.
Transitional probabilities in Markov regime-switching with GED

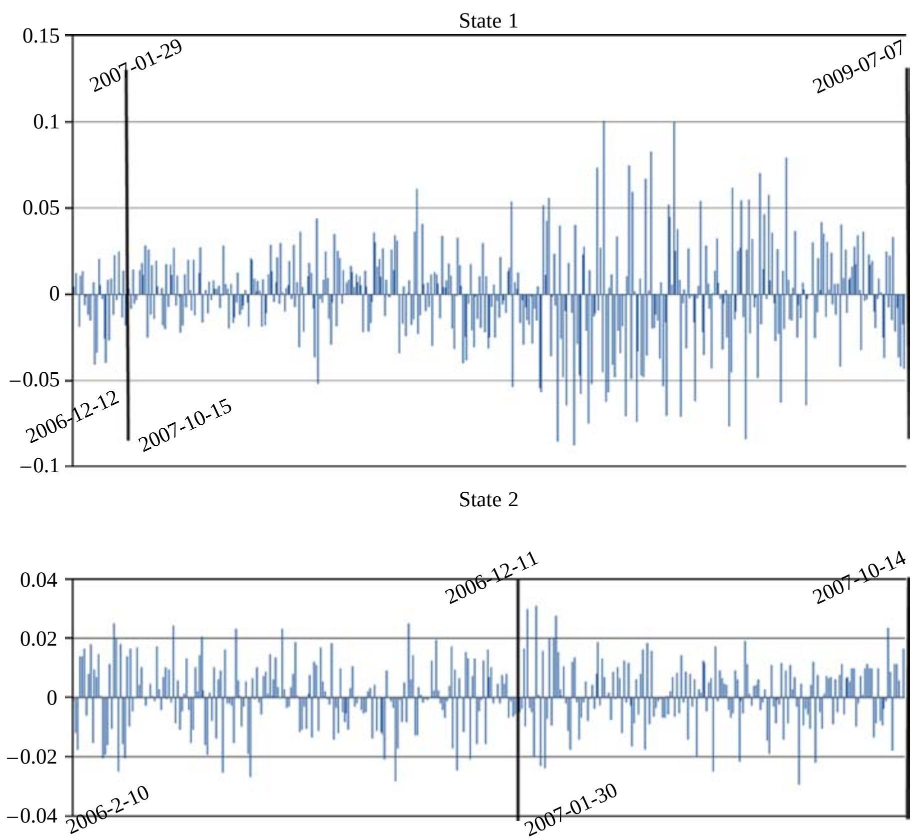

Figure 7.
Return of two regimes in historical time series

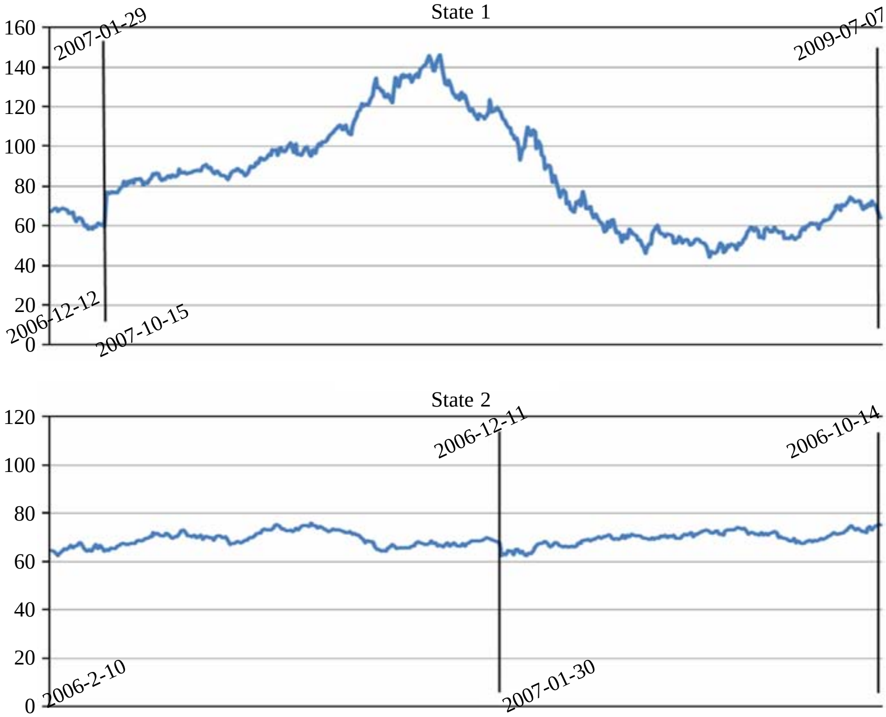

Figure 8.
Price of two regimes in historical time series

maximum-likelihood estimation technique. Codes are written in Matlab language. We have compared several parameter settings in all models. In GARCH models, the ARMAX (1,1,0)/GJR(2,1) yielded the best fitted result with maximum LLC value of 2,292.32 when assuming that our data follow a t-distribution. Markov regime-switching models generate similar fitted result but with a bit lower LLC value. Markov regime-switching modeling show interesting results by classifying historical data into two states: a normal one and a turmoil one. This can account for some market stories in financial crisis.

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Further reading

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