

$$1. (a) P(\text{toothache}) = \underbrace{0.108 + 0.012}_{w/\text{ cavity}} + \underbrace{0.016 + 0.064}_{w/o \text{ cavity}}$$

$$P(\text{toothache}) = \underline{\underline{0.20}}$$

$$(b) \vec{P}(\text{cavity}) = \langle P(\text{cavity}), P(\neg \text{cavity}) \rangle$$

$$P(\text{cavity}) = 0.108 + 0.012 + 0.072 + 0.008 \\ = 0.20$$

$$\alpha = \frac{1}{0.2 + 0.8} = 1$$

$$P(\neg \text{cavity}) = 0.016 + 0.064 + 0.144 + 0.376 \\ = 0.80$$

$$\alpha \langle 0.20, 0.80 \rangle \\ = \langle 0.20, 0.80 \rangle$$

$$\boxed{\vec{P}(\text{cavity}) = \langle 0.20, 0.80 \rangle}$$

$$(c) \vec{P}(\text{toothache} | \text{cavity}) = \langle P(\text{toothache} | \text{cavity}), P(\neg \text{toothache} | \text{cavity}) \rangle$$

$$= \alpha \langle 0.108 + 0.012, 0.072 + 0.008 \rangle$$

$$\alpha = \frac{1}{0.12 + 0.08} = 5$$

$$= \alpha \langle 0.12, 0.08 \rangle$$

$$= \boxed{\langle 0.6, 0.4 \rangle}$$

$$2. (a) P(b, i, \neg m, g, i)$$

$$= P(b) \cdot P(\neg m) \cdot P(i | b, \neg m) \cdot P(g | b, i, \neg m) \cdot P(j | g)$$

$$= (0.9) \cdot (0.8) \cdot (0.5) \cdot (0.8) \cdot (0.8)$$

$$= \boxed{0.23}$$

$$(b) \vec{P}(J | b, i, m) = \alpha \cdot \vec{P}(J, b, i, m)$$

$$= \alpha \cdot \sum_{g'} \vec{P}(J | g') \cdot P(b) \cdot P(i | b, m) \cdot P(m) \cdot P(g' | i, b, m)$$

$$= \beta \cdot \sum_{g'} \vec{P}(J | g') \cdot P(g' | i, b, m)$$

$$= \beta \cdot \left[ \begin{array}{l} P(j | g) \cdot P(g | i, b, m) + P(j | \neg g) \cdot P(\neg g | i, b, m), \\ P(\neg j | g) \cdot P(g | i, b, m) + P(\neg j | \neg g) \cdot P(\neg g | i, b, m) \end{array} \right]$$

$$= \beta \cdot [(0.8) \cdot (0.9) + (0.1) \cdot (0.1), (0.2) \cdot (0.9) + (0.9) \cdot (0.1), (0.2) \cdot (0.9) + (0.9) \cdot (0.1)]$$

$$= \beta \cdot [.73, .27]$$

$$= \boxed{[0.73, 0.27]}$$

$$(c). \vec{P}(J | \neg b, \neg i, m) = \alpha \vec{P}(J, \neg b, \neg i, m) = \alpha \vec{P}(J, \neg b, \neg i, m, g)$$

$$= \alpha \vec{P}(J, g') P(\neg b) P(\neg i | \neg b, m) P(m) P(g' | \neg b, \neg i, m)$$

$$\propto \beta \cdot \sum_{g'} \vec{P}(J | g') \cdot P(g' | \neg b, \neg i, m)$$

$$= \beta \cdot \left[ \begin{array}{l} P(j | g) \cdot P(g | \neg b, \neg i, m) + P(j | \neg g) \cdot P(\neg g | \neg b, \neg i, m) \\ P(\neg j | g) \cdot P(g | \neg b, \neg i, m) + P(\neg j | \neg g) \cdot P(\neg g | \neg b, \neg i, m) \end{array} \right]$$

$$= \beta \cdot [(0.8)(0) + (0.1)(0.1), (0.2)(0) + (0.9)(0.1)]$$

$$= \beta \cdot [0.1, 0.9]$$

$$= \boxed{[0.1, 0.9]}$$