GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LOGISTIC REGRESSION

RECAP

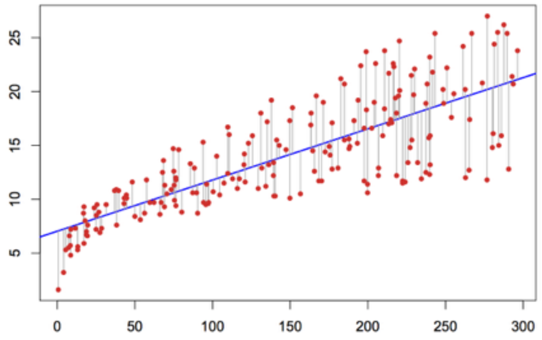
LAST TIME:

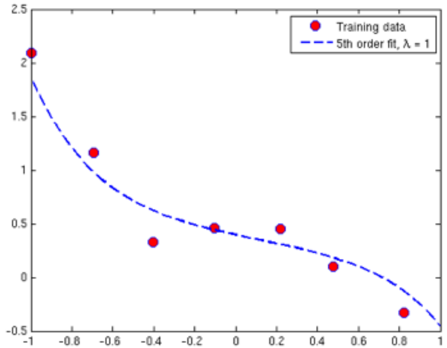
I. LINEAR REGRESSION (INCL. MULTIPLE REGRESSION)
II. POLYNOMIAL REGRESSION
III. REGULARIZATION

LAB:

IV. IMPLEMENTING MULTIPLE REGRESSION & POLYNOMIAL REGRESSION IN PYTHON

QUESTIONS?





QUESTIONS?

WHAT WAS THE MOST INTERESTING THING YOU LEARNT?

WHAT WAS THE HARDEST TO GRASP?

AGENDA

I. LOGISTIC REGRESSION
II. OUTCOME VARIABLES
III. ERROR TERMS

LAB: IMPLEMENTING LOGISTIC REGRESSION IN PYTHON

IV. INTERPRETING RESULTS

KEY OBJECTIVES

- WHAT IS LOGISTIC REGRESSION
- HOW IS LOGISTIC REGRESSION USED
- WHAT ARE THE ADVANTAGES OF USING LOGISTIC REGRESSION
- HOW TO IMPLEMENT LOGISTIC REGRESSION IN PYTHON

I. LOGISTIC REGRESSION

SO, WHERE ARE WE ON OUR 2X2?

	Continuous	Categorical
Supervised	???	???
Unsupervised	???	???

- *Name is somewhat misleading…*
- *Really a technique for classification, not regression*
- *"Regression" comes from fact that we fit a linear model to the feature space*

	Continuous	Categorical
Supervised	regression	classification
Unsupervised	dimension reduction	clustering

Q: What is **logistic regression***?*

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A: A generalization of the linear regression model to classification problems.

Q: Why is logistic regression so valuable to know?

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A: It addresses many commercially valuable classification problems, such as:

- *Fraud detection (payments, e-commerce)*
- *Churn prediction (marketing)*
- *Medical diagnoses (is the test positive or negative?)*
- *and many, many others...*

It's a binary classification technique

which means….

Two classes: $Y = \{0, 1\}$

Our goal is to learn to classify correctly two types of examples

- *Class 0 – labeled as 0*
- *Class 1 – labeled as 1*

We would like to learn ***f: X —> {0, 1}***

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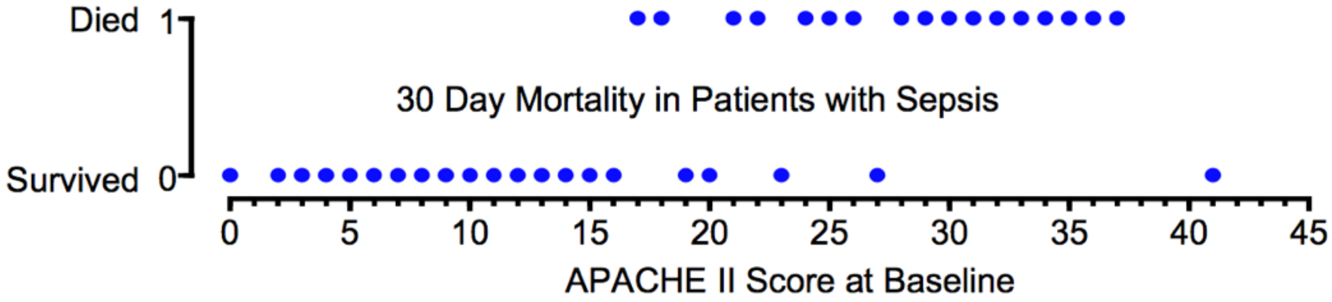
In logistic regression, we use a set of covariates to predict ***probabilities of (binary) class membership***

These probabilities are then mapped to class labels, thus solving the classification problem.

A motivating example:

The following figure shows 30 day mortality in a sample of septic patients as a function of their baseline APACHE II score. Patients are coded as 1 or 0 depending on whether they are dead or alive in 30 days, respectively.

How can we predict death from baseline APACHE II score in these patients?

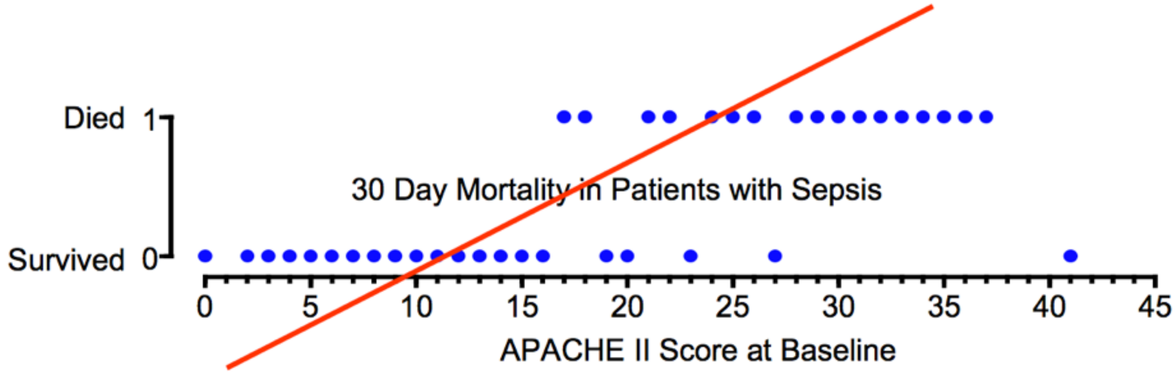


Source: http://www.mc.vanderbilt.edu/gcrc/workshop_files/2004-11-12.pdf

Q: How can we predict death from baseline APACHE II score in these patients?

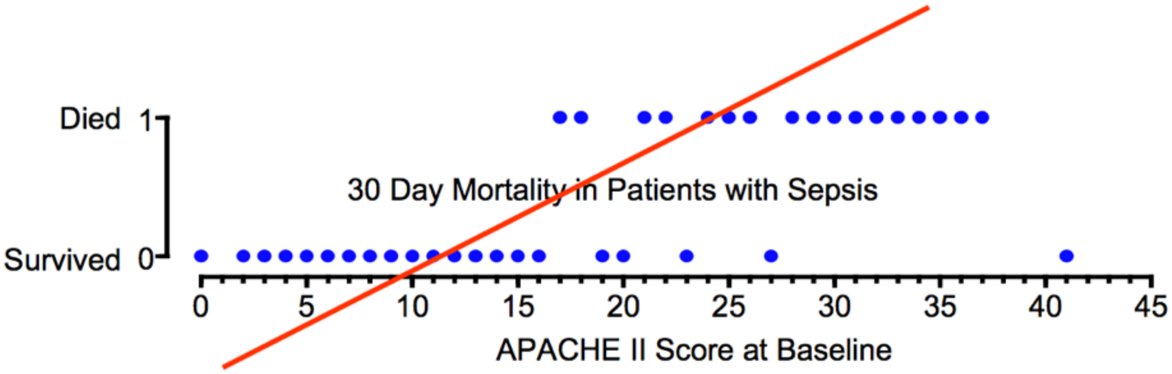
Let p(x) be the probability that a patient with score x will die within 30 days.

Well, linear regression would not work well here, because it could produce probabilities less than zero or greater than one. Also, one new value could greatly change our model…

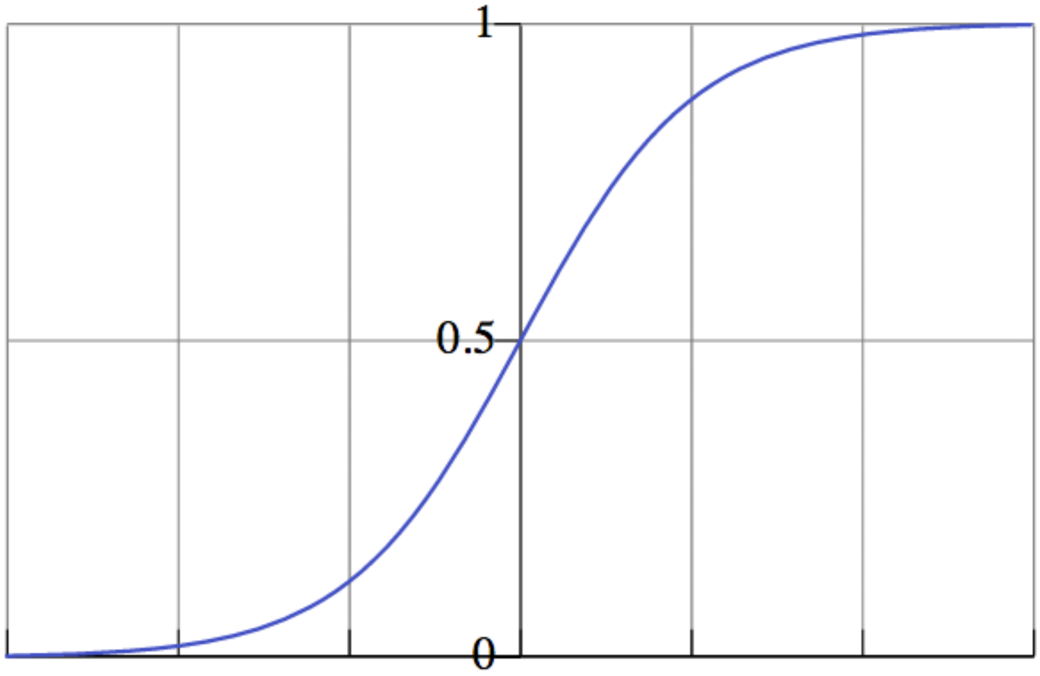


Source: http://www.mc.vanderbilt.edu/gcrc/workshop_files/2004-11-12.pdf

So, what can we do instead of linear regression?



probability of belonging to class

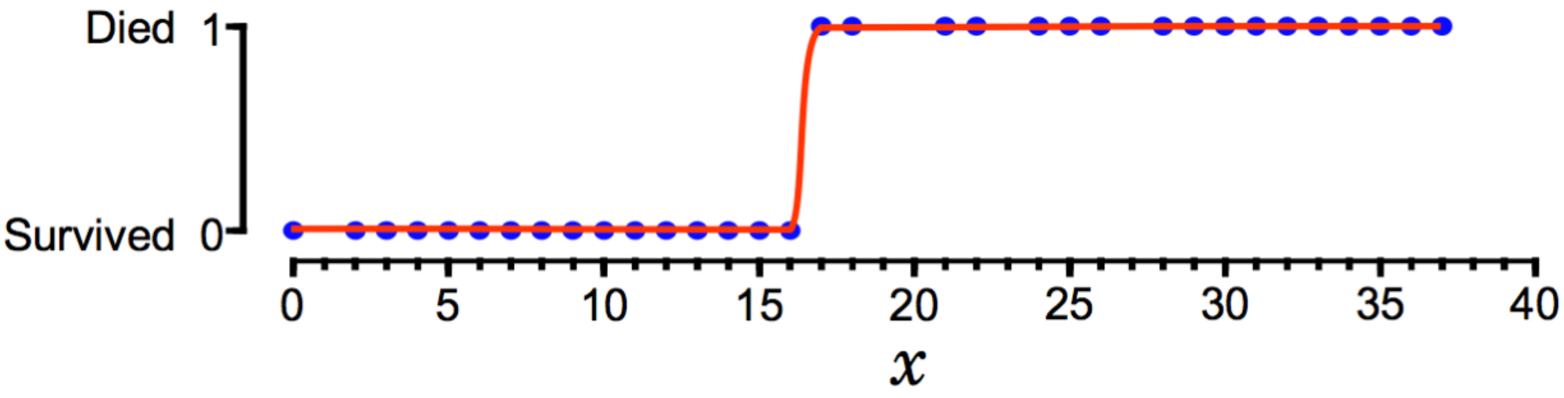


value of independent variable

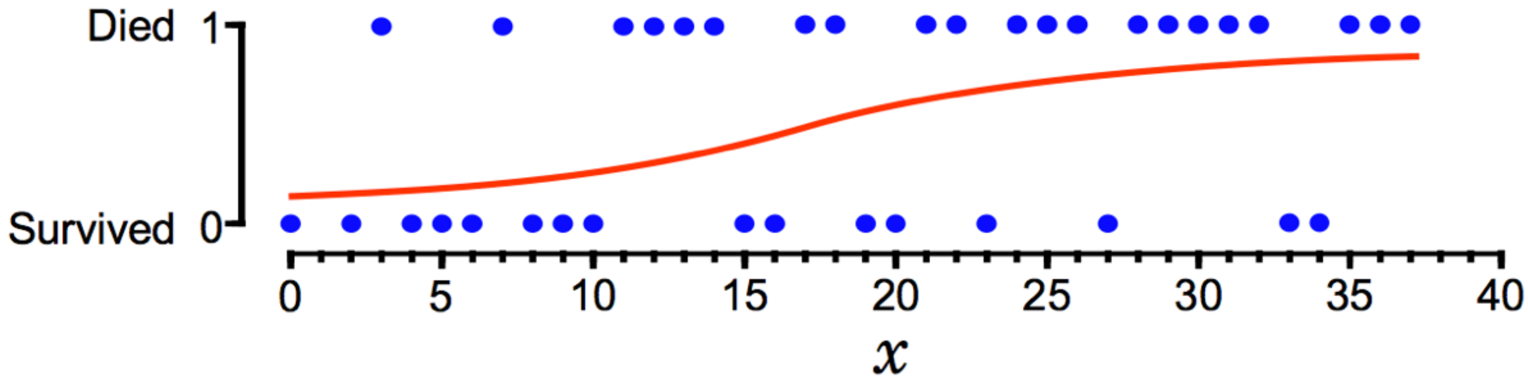
NOTE

Probability predictions look like this.

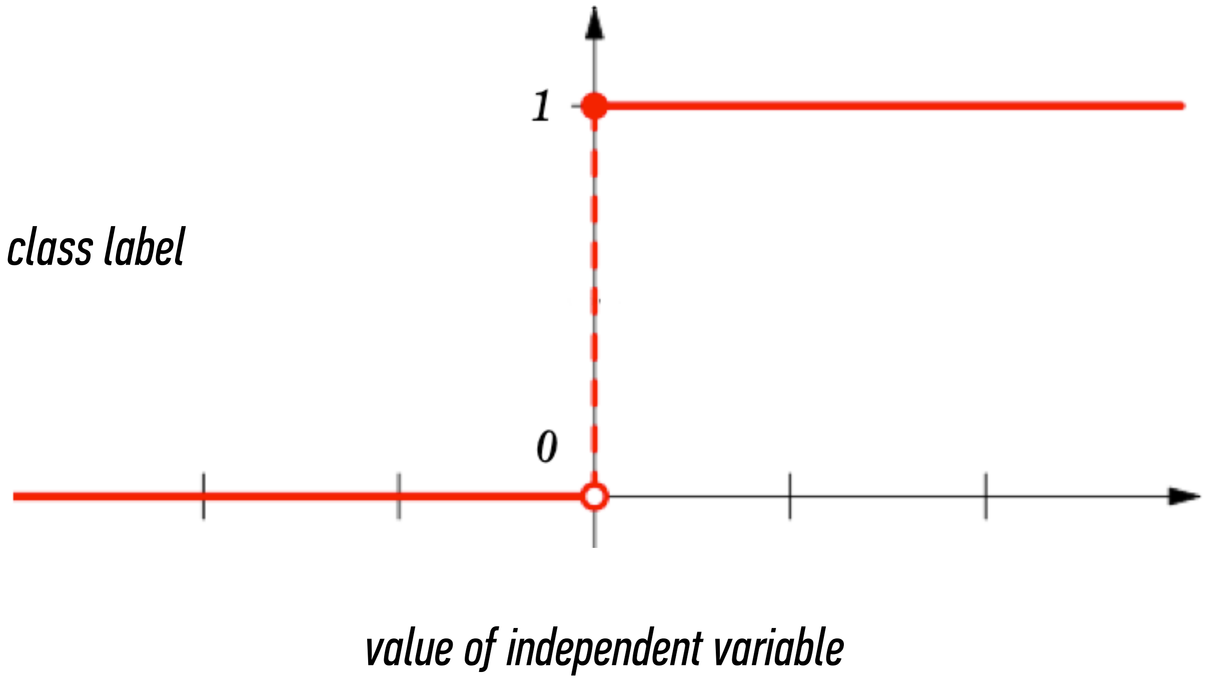
Going back to our example of patient survival given a sepsis test score:
Data that has a sharp cut off point between the two classes (living / dying) should have a large value of B_1.



Going back to our example of patient survival given a sepsis test score:
Data that has a lengthy transition between the two classes (living / dying) should have a small value of B_1.



CLASS LABELS



NOTE

Probabilities are "snapped" to class labels (e.g. by thresholding at 50%).

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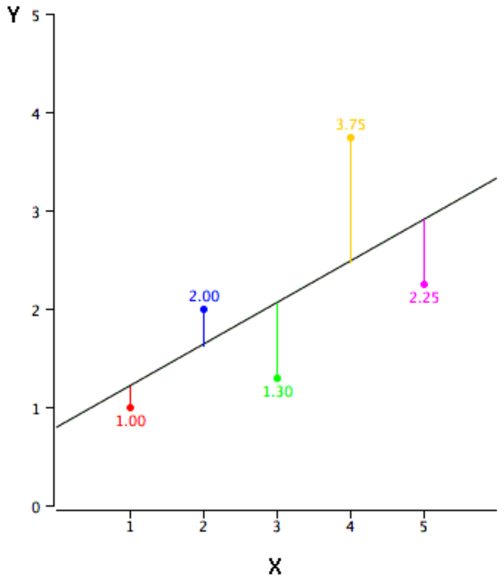
The first difference is in the outcome variable.

II. OUTCOME VARIABLES

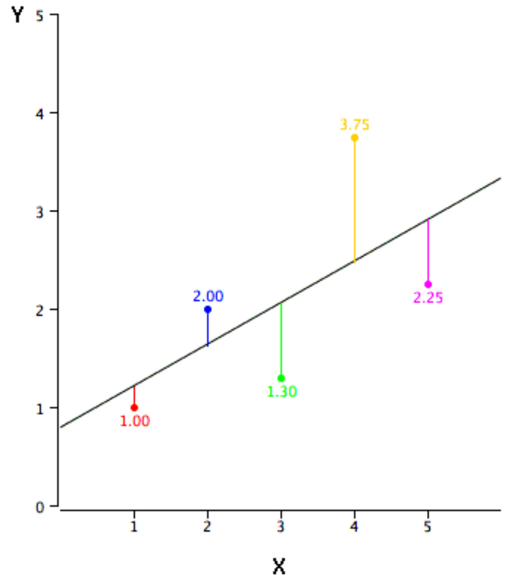
The key variable in any regression problem is the the outcome variable y *given the value of the covariate* x.

$$\mathrm{y} = \alpha + \beta_1 \mathrm{x}_1 + \ldots + \beta_n \mathrm{x}_n + \varepsilon$$



In linear regression, we assume that this outcome value is a linear function taking values in $(-\infty, +\infty)$*:*

$$y = \alpha + \beta_1 x_1 + \ldots + \beta_n x_n + \varepsilon$$



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Q: How do we do this?

A: By using a transformation called the **logistic function***:*

$$E(y|x) = \pi(x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

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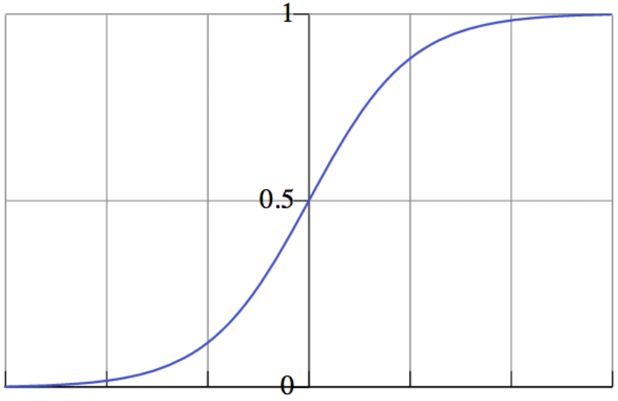
We've already seen what this looks like:

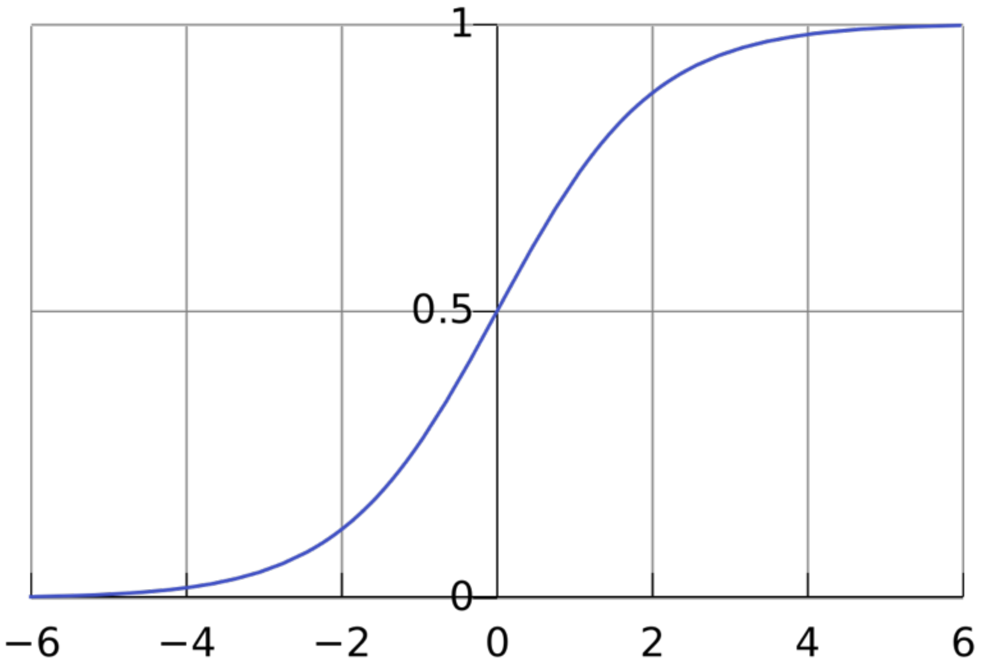


NOTE

For any value of x, y is in the interval [0, 1]

This is a nonlinear transformation!

probability of belonging to class



value of independent variable

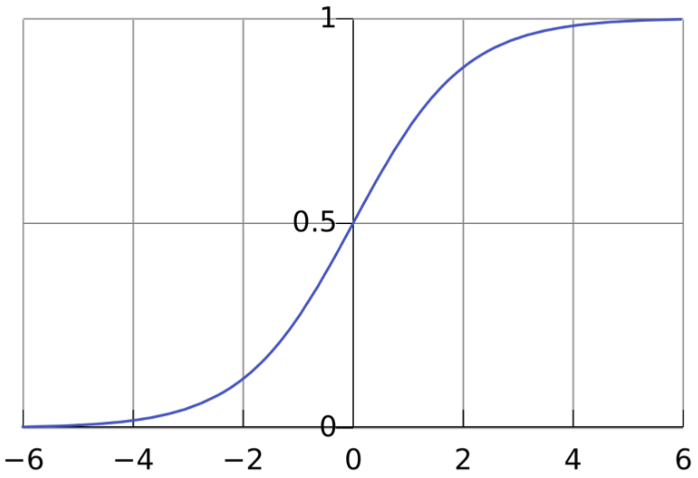
NOTE

Probability predictions look like this.

This function fits our problem much better:

$$0 \leq h_\theta(x) \leq 1$$

In other words, our classifier will output values between 0 and 1. It asymptotically approaches 0 and 1.



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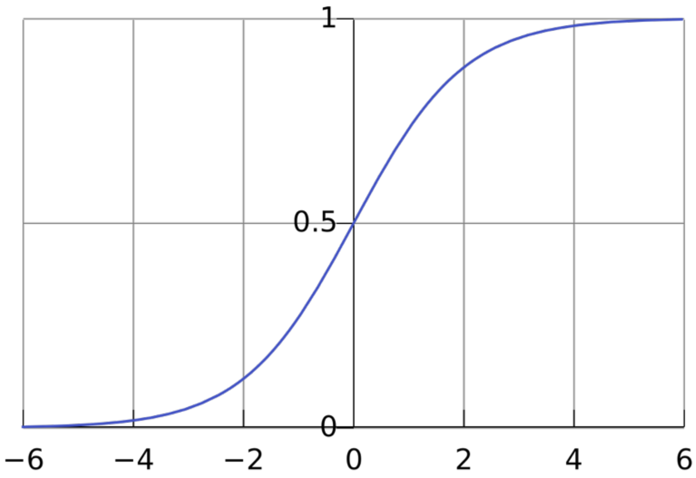

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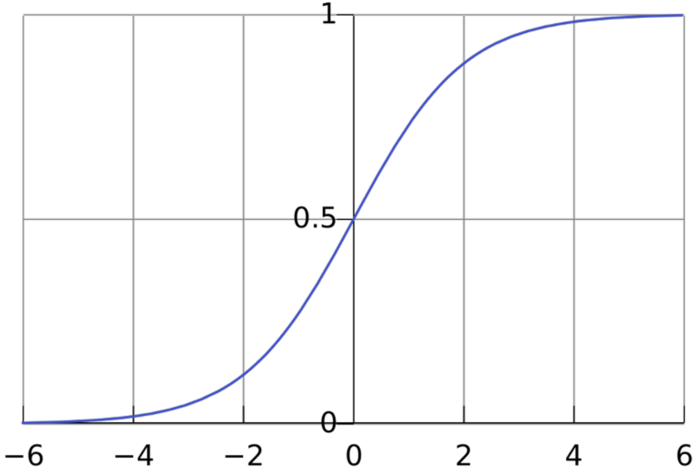
NOTE

This function gives Logistic Regression its name!

The logistic function:

$$F(t) = \frac{e^t}{e^t + 1} = \frac{1}{1 + e^{-t}}$$



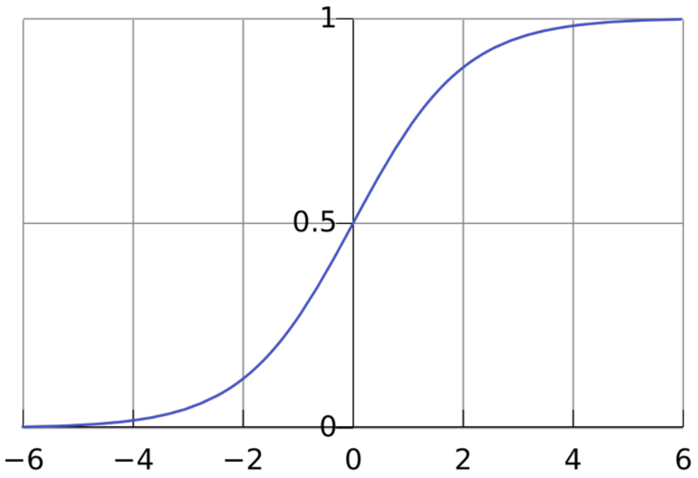
Notice that f(t) = 0.5 when t = 0

f(t) >= 0.5 when t > 0

f(t) <= 0.5 when t < 0

Suppose we predict class 1 when f(t) >= 0.5 and class 0 when f(t) < 0.5

$$F(t) = \frac{e^t}{e^t + 1} = \frac{1}{1 + e^{-t}}$$



So, if the t in the logistic function is a linear function of an explanatory variable x, or a linear combination of explanatory variables, the logistic function becomes:

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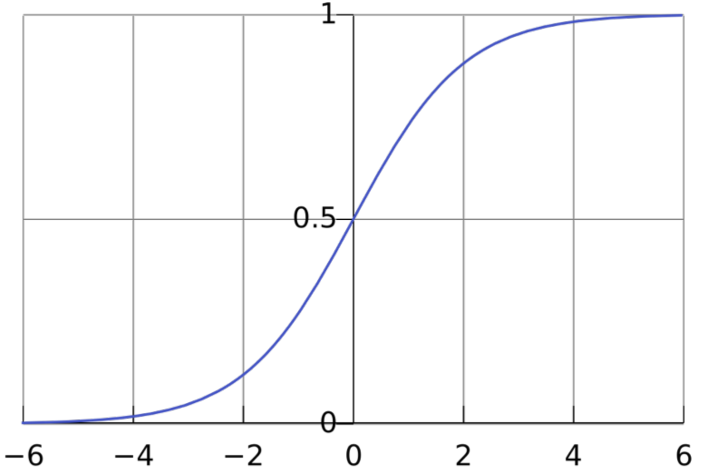

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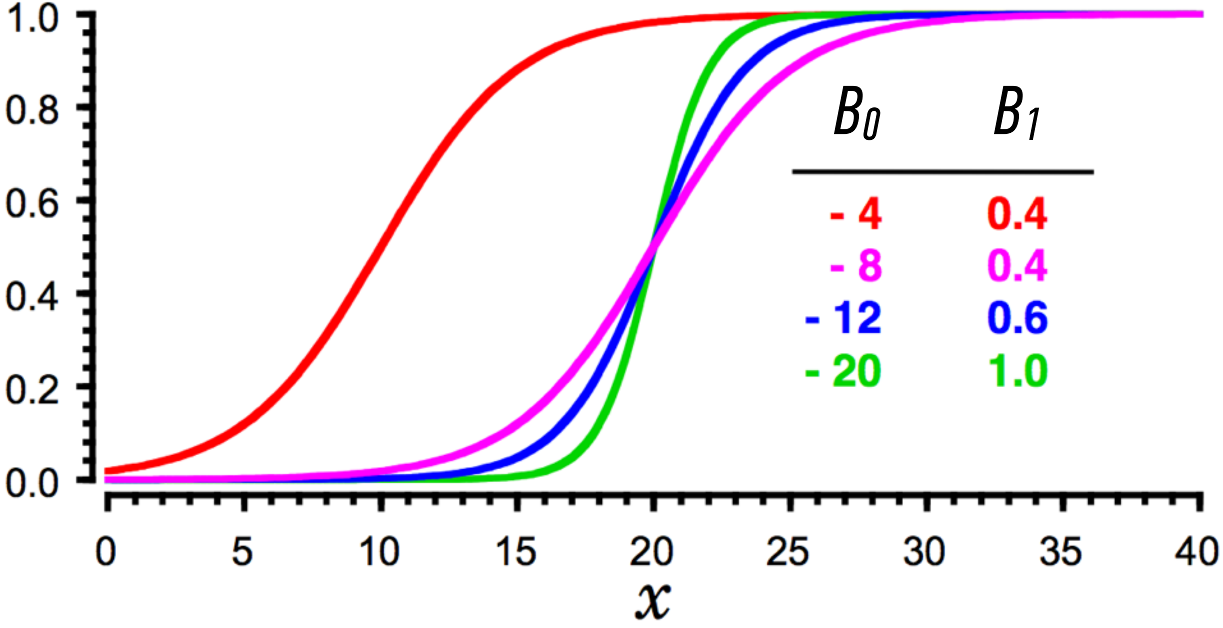

When $B_0 + B_1x = 0$, then $F(x) = 0.5$, which is the inflection point on all these curves.

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NOTE

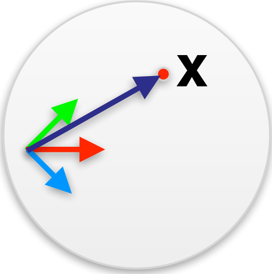
This name hints at its usefulness in interpreting our results.

We will see why shortly.

HOW LOGISTIC REGRESSION WORKS

1. Model consists of a vector **β** in n-dimensional feature space

$$\boldsymbol{\beta} = \beta_1 + \beta_2 + \ldots + \beta_n$$



source: http://courses.washington.edu/css490/2012.Winter/lecture_slides/05b_logistic_regression.pdf

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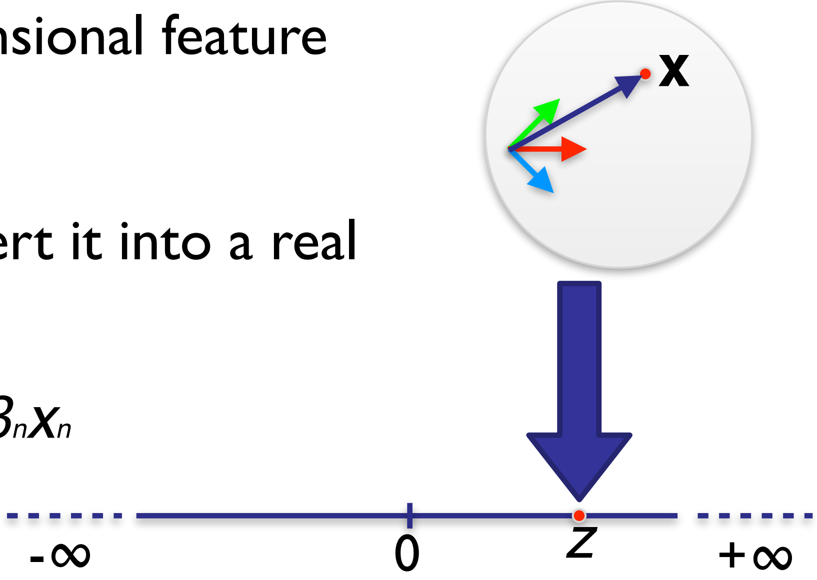
$$\boldsymbol{\beta} = \beta_1 + \beta_2 + \ldots + \beta_n$$

2. For a point **x**, project it onto β to convert it into a real number z in the range -∞ to +∞

$$z = \alpha + \boldsymbol{\beta}.\mathbf{x} = \alpha + \beta_1 x_1 + \beta_2 x_2 + \ \ldots + \beta_n x_n$$



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USING LOGISTIC FUNCTION

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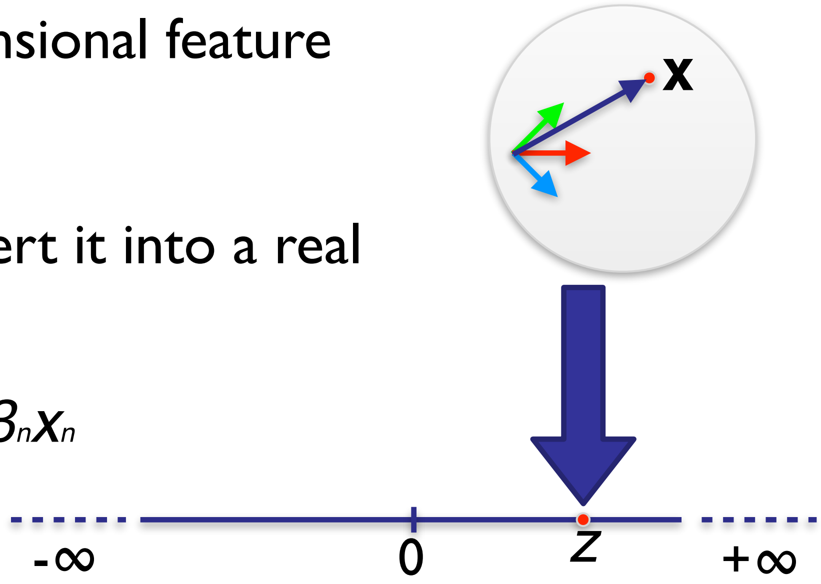
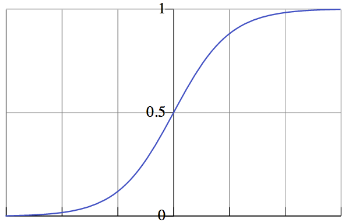


3. Map z to the range 0 to 1 using the logistic function

$$p = 1/(1 + e^{-z})$$



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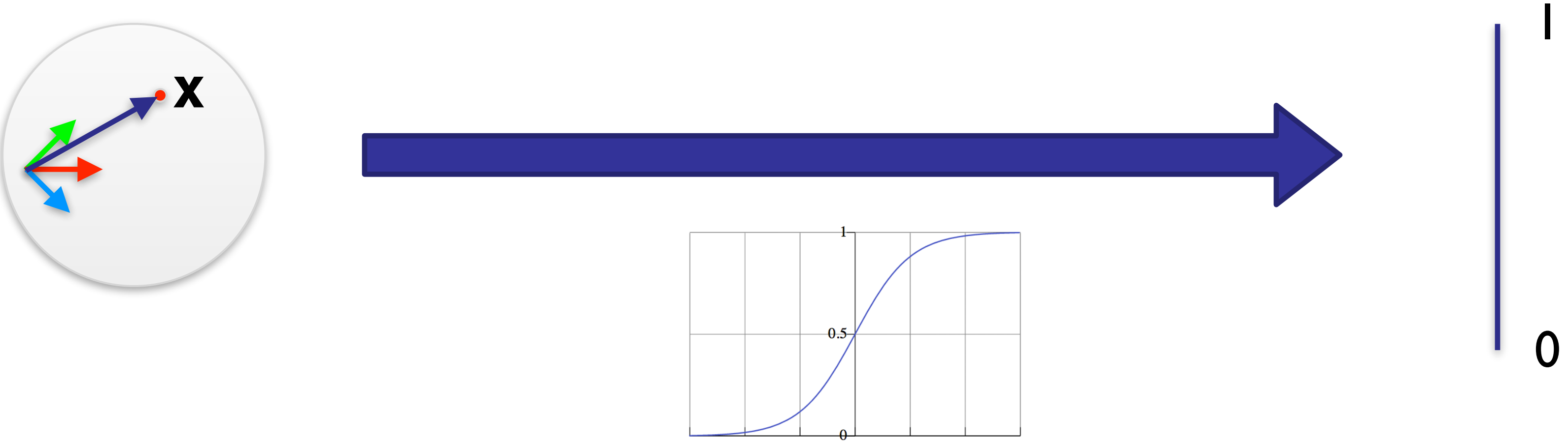

Overall, logistic regression maps a point x in n-dimensional feature space to a value in the range 0 to 1.

source: http://courses.washington.edu/css490/2012.Winter/lecture_slides/05b_logistic_regression.pdf

prediction from a logistic regression model as:

A probability of class membership

Need to optimize β so the model gives the best possible reproduction of training set labels

When we fit a logistic regression model in scikit-learn, this is what we are doing!

The logistic regression model is an extension of the linear regression model, with a couple of important differences.

The first difference is in the outcome variable.

The second difference is in the error terms.

III. ERROR TERMS

The second difference between linear regression and the logistic regression model is in the error term.

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One of the key assumptions of linear regression is that the error terms follow independent Gaussian distributions with zero mean and constant variance:

$$\epsilon \sim N(0, \sigma^2)$$

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It's easy to show from this that instead of following a Gaussian distribution, the error term in logistic regression follows a Bernoulli distribution:

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NOTE

This is the same distribution followed by a coin toss.

Think about why this makes sense!

These two key differences define the logistic regression model, and they also lead us to a kind of unification of regression techniques called **generalized linear models.**

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Briefly, GLMs generalize the distribution of the error term, and allow the conditional mean of the response variable to be related to the linear model by a **link function**.

In the present case, the error term follows a Bernoulli distribution, and the logit is the link function that connects us to the linear predictor.

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LAB: LOGISTIC REGRESSION IN PYTHON

IV. INTERPRETING RESULTS

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QUESTION

What is the range of the odds?

Quiz: *You're trying to determine whether a customer will convert or not. The customer conversion rate is 33.33%. what are the odds that a customer will convert?*

Take 2 minutes and work this out.

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$$Odds = \frac{\pi}{1-\pi} = \frac{.3333}{.6666} = \frac{1}{2}$$

NOTE

This means that for every customer that converts you will have two customers that do not convert

What would happen if we took the odds of the logistic function?

$$\frac{\pi}{1-\pi} = \frac{e^{\beta_0 + \beta_1 x} / (1 + e^{\beta_0 + \beta_1 x})}{1 - e^{\beta_0 + \beta_1 x} / (1 + e^{\beta_0 + \beta_1 x})}$$

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$$= \frac{e^{\beta_0+\beta_1 x}/(1+e^{\beta_0+\beta_1 x})}{(1+e^{\beta_0+\beta_1 x})/(1+e^{\beta_0+\beta_1 x}) - e^{\beta_0+\beta_1 x}/(1+e^{\beta_0+\beta_1 x})} = e^{\beta_0+\beta_1 x}$$

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$$= \frac{e^{\beta_0+\beta_1 x}/(1+e^{\beta_0+\beta_1 x})}{(1+e^{\beta_0+\beta_1 x})/(1+e^{\beta_0+\beta_1 x}) - e^{\beta_0+\beta_1 x}/(1+e^{\beta_0+\beta_1 x})} = e^{\beta_0+\beta_1 x}$$

Notice if we take the logarithm of the odds, we return a linear equation

$$\log\left(\frac{\pi}{1-\pi}\right) = \log\left(e^{\beta_0+\beta_1 x}\right) = \beta_0 + \beta_1 x$$

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NOTE

What is the range of the logit function?

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$$\log\left(\frac{\pi}{1-\pi}\right) = \log\left(e^{\beta_0 + \beta_1 x}\right) = \beta_0 + \beta_1 x$$

This simple relationship between the odds ratio and the parameter β is what makes logistic regression such a powerful tool.

In linear regression, the parameter β_1 represents the change in the ***response variable*** *for a unit change in x.*

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*This means that e^{β_1} gives us the change in the **odds** for a unit change in x.*

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Q: What does this mean?

Example: *Suppose we are interested in mobile purchase behavior. Let y be a class label denoting purchase/no purchase, and let x denote whether phone was an iPhone.*

We perform a logistic regression, and we get β_1 *= 0.693.*

In this case the odds ratio is exp(0.693) = 2, meaning the likelihood of purchase is twice as high if the phone is an iPhone.

Once we understand the basic form for logistic regression, we can easily extend the definition to include multiple input values.

Logit function

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Logistic function

$$\pi = \frac{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p}}{1 + e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p}}$$

*In linear regression, the parameter β_1 represents the change in the **response variable** for a unit change in x.*

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Logistic Regression