Deep Dive Integration (LBA)

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1 **'Round and 'round we go**

a) If we take a vertical slice at x = 1, a circle is obtained. The area of this slice will be:

Area = πr^2 The radius can be obtained from f(x). Thus, we have: f(1) = 3, thus r = 3.

$$A = \pi r^2 \to A = \pi * 9 = 28.27 units^2$$

If we take a vertical slice at x = 0, we will again obtain a circle, but this time with a radius of 4, since $f(0) = 4 - 0 = 4$. Its area will then be:

$$A = \pi r^2 \Rightarrow A = \pi * 16 = 50.265 units^2$$

b) A vertical slice at any value of x will result give us a circle. Therefore, the general formula for the area will be $A = \pi * r^2$ where $r = f(x)$. Therefore, a general formula will be: $A = \pi * \left(f(x)\right)^2$

c)

i. An individual slice at any value of x:

Volume = area * height = $\pi\left(f(x)\right)^2 * \Delta x = \pi(4 - x^2)^2 * \Delta x$

ii. The sum of all slices if we have a total of n slices:

$$\sum_{i=1}^{n} \pi\left(f(x_i)\right)^2 * \Delta x$$

$$\sum_{i=1}^{n} \pi(4 - x_i{}^2)^2 * \Delta x$$

d)

$$\Delta x = \frac{b-a}{n} = \frac{2}{n}$$

$$x_i = a + i * \Delta x$$

We can use the formula we obtained for the volume of the sum of all slices if we have n slices and replace Δx and x_i.

$$\sum_{i=1}^{n} \pi(4 - x_i{}^2)^2 * \Delta x \ \Rightarrow$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \pi(4 - (i\Delta x)^2)^2 * \Delta x \ \Rightarrow$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \pi\left(4 - \left(\frac{2i}{n}\right)^2\right)^2 * \frac{2}{n}$$

This limit expression would give us the exact volume of the solid.

e)

We know that the area of the circle = $\pi * r^2$. From the problem statement, we have the boundaries [0,2]. Thus, the integral will be: $\int_0^2 \pi(4 - x^2)^2 dx$

f)

$$\int_0^2 \pi(4 - x^2)^2 dx \ \Rightarrow$$

$$\pi \int_0^2 16 - 8x^2 + x^4 \, dx \;\Rightarrow$$

$$\pi \left[16x - \frac{8x^3}{3} + \frac{x^5}{5}\right]_0^2 \;=$$

$$\pi * \frac{256}{15} =$$

$$53.50 units^3$$

g)

If we had revolved around the y-axis instead, the formula for the radius would have changed. Thus, the formula would be: $f(y) = \sqrt{(4-y)}$. Also, the volume of the solid object would have changed. The volume obtained by revolving f(x) around the y-axis would be:

From the figure, we can see that the boundaries are [0,4].

Thus, the volume would be:

$$Area = \pi * \left(\sqrt{(4-y)}\right)^2 = \pi * (4-y)$$

$$\int_0^4 Area \;\Rightarrow$$

$$\int_0^4 \pi * (4-y)dy =$$

$$\pi * \left[4y - \frac{y^2}{2}\right]_0^4 \;=$$

$$\pi * \; 8 \;=$$

$$25.132 units^3$$

Therefore, the volume of the solid of revolution obtained by revolving f(x) around the y-axis will be smaller than the one from part f.

h)

Volume for revolving around x-axis:

$V = \int_a^b \pi x^2 dx$ where $x = g(x)$

Volume for revolving around y-axis:

$V = \int_c^d \pi y^2 dy$ where $y = g(x)$

2. **Cookie-Cutting**

a)

The shape will be a hollow cylinder where height will vary along the x-axis.

The surface area of this shape is: $2 * \pi * (r_1 + r_2)(r_2 - r_1 + h)$

b)

The height of the inner and outer area depends on the radius therefore we can use the radius to find the height.

$f(x) = height$ for inner area

$f(x + \Delta x) = height$ for outer area

The inner area will be: $Area = 2\pi r_1 h = 2\pi x f(x)$

The outer area will be: $Area = 2\pi r_2 h = 2\pi(x + \Delta x) f(x + \Delta x)$

c)

For individual slices, the volume will be:

$$Volume = 2\pi\Delta x f(x)$$

For the sum of n total slices, the volume will be:

$$Volume = \sum_{i=1}^{n} 2\,\pi\Delta x f(x_i)$$

d)

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$$

The limit expression would be:

$$\lim_{n\to\infty}\sum_{i=1}^{n} 2\,\pi\Delta x f(i * \Delta x)\ \Rightarrow$$

$$\lim_{n\to\infty}\sum_{i=1}^{n} 2\,\pi\,\frac{2}{n} f\left(\frac{2i}{n}\right)$$

This limit expression would give us the exact volume of the solid.

e)

The boundaries of the integral will be [0,2]. The integral will be $\int_0^2 2\,\pi x f(x)dx =$

$2\pi \int_0^2 x(4 - x^2)dx = 2\pi \left[-\frac{(4-x^2)^2}{4}\right]_0^2 = 8\pi = 25.132units^3$

f)

Yes, the formula would change, because instead of f(x) we would have to do f(y) instead, and that will change the formula.

g)

The formula of the volume of the solid of revolution obtained by revolving g(x) around the y-axis:

$$Volume = \int_a^b 2\,\pi x g(x)dx$$

The formula of the volume of the solid of revolution obtained by revolving g(x) around the x-axis:

$$Volume = \int_c^d 2\,\pi y g(y)dy$$

h)

I would say it depends honestly. When we have the revolution happening around the x-axis, I felt like the disk method is easier, whereas when the revolution was happening around the y-axis, I felt like the shells method was easier.

i) This could be used to test the capacity of different products, with varying heights for example, or whatever other features.

3. **LBA**

a) I have chosen a fishing boat. The reason for that is because the city where I come from, is known for its production of fishing boats, and I felt that it's a good representative of my hometown where I currently am in Kosovo.

b)

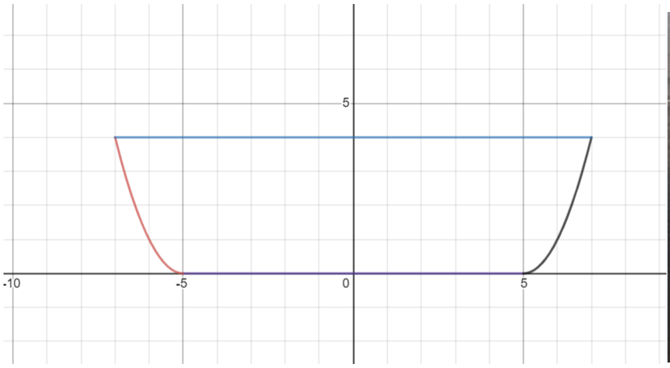
$f(x) = 0x + 4$ for $-7 \leq x \leq 7$

$g(x) = (x+5)^2$ for $-7 \leq x \leq -5$

$h(x) = (x-5)^2$ for $5 \leq x \leq 7$

$k(x) = 0x$ for $-5 \leq x \leq 5$

c)



e)

We can use the Method of Shells. Using this method we can easily obtain the volume of one half, and since the boat is symmetrical, the volume of the boat will be twice the volume of the computed half.

The volume under f(x) $= 2\pi \int_{-7}^{7} x * 0x + 4dx = 351.858cm^3$

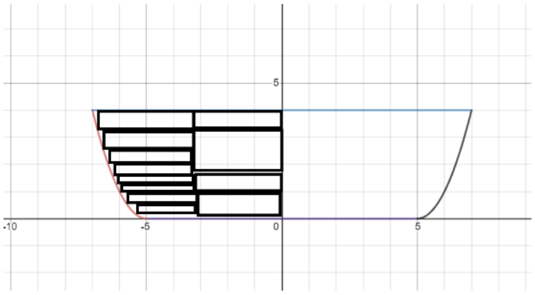
The volume under g(x) = $2\pi \int_{-7}^{-5} x * (x+5)^2 dx = -108.9cm^3$

The volume under h(x) = $2\pi \int_{5}^{7} x * (x-5)^2 dx = 108.9cm^3$

The volume under k(x) = $2\pi \int_{-5}^{5} x * 0xdx = 0cm^3$

The volume of the boat will be: $2 * (351.858cm^3 - 108.9cm^3 + 108.9cm^3) = 2 * 351.858cm^3 = 703.7cm^3$

f)



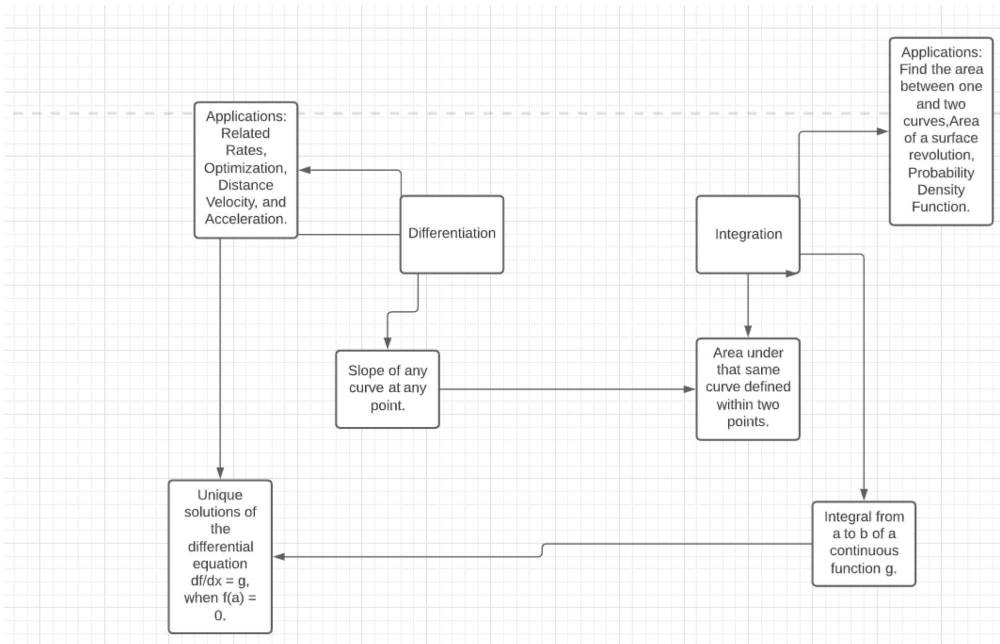
g)

Estimated volume: $536.85cm^3$; Integrated volume: $703.7cm^3$

As we can see these two volumes are different, because of the methods we have used to compute the volume of the boat. The estimated volume, which we computed by finding the volumes of solid shape parts, produced a lower volume compared to the integrated volume.

Reflection questions

1.



2.

One example of applying HCs was #breakitdown. This was particularly helpful on the second problem, where we first had to determine the shape that would be created by slicing the graph, and then find its surface area formula. To solve this problem, I first needed to draw by hand the slice, and see what's happening with the figure, and then after determining the shape, I started looking at the relationship between the two cylindrical shapes, and finally compute its area.

Another example of applying HCs I believe was algorithms. This was particularly applicable because in this assignments, lots of definite integrals had to be calculated, and at each step, I had to use the rules we've learned in class, step-by-step, in order to come up with the result.

3. I think I have applied science of learning through space repetition mainly. This is because for some concepts asked in the assignments that I found confusing, I went back to class readings, and listen to the definition and examples even more, which I believe helped me understand and engage with the material. I believe I could improve my approach by applying other techniques of science of learning, such as using flashcards for some more theoretical definitions and concepts.