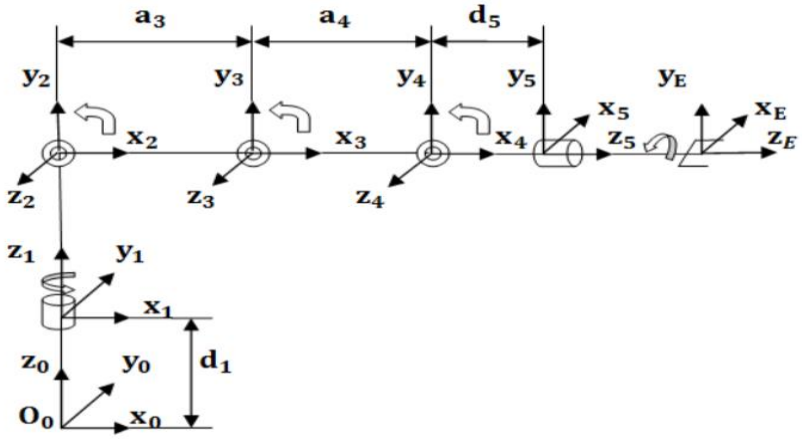
6 DOF explanation

1. **Robot coordinate frame:**



2. **In order to find the position and orientation parameters, the following composition of coordinate transformation in the frame represents:**

   a_i : The length distance from z_i to z_{i+1} measured along z_i

   α_i: The twist angle between z_i and z_{i+1} measured about x_i

   d_i: The offset distance from x_i to x_{i+1} measured along z_i

   θ_i: The angle between x_i and x_{i+1}measured about z_i

3. **Transformation matrix:**

$$Ai = \begin{bmatrix} C\theta i & -S\theta i C\alpha i & S\theta i S\alpha i & aiC\theta i \\ S\theta i & C\theta i C\alpha i & -C\theta i S\alpha i & aiS\theta i \\ 0 & S\alpha i & C\alpha i & di \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Forward kinematics A1 and A6:

Link	a	α	d	θ
1	0	0	d1	θ1
2	0	90	0	θ2
3	a3	0	0	θ3
4	a4	0	0	θ4
5	0	90	d5	θ5
6	0	0	0	θ6

$$A_1=\begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_4=\begin{bmatrix} C_4 & -S_4 & 0 & a_4C_4 \\ S_4 & C_4 & 0 & a_4S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2=\begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_5=\begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3=\begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_6=\begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Total matrix of transformation is the multiplication of matrices A1-A6. Where [p(x,y,z) represent the position, while n(x,y,z), o(x,y,z), and a(x,y,z) represent the orientation]:

$$A_6^0 = A_1^0 * A_2^1 * A_3^2 * A_4^3 * A_5^4 * A_6^5$$

$$= \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. **Finally, to calculate position parameters, we used the following equations:**

px = (a4* (cos (t1)* cos(t2)- sin(t1)*sin(t2))* cos(t3)* cos(t4))- (a4* (cos (t1)*cos(t2)- sin(t1)*sin(t2))* sin(t3)* sin(t4))+ ((cos(t1)*sin(t2)+sin(t1)*cos(t2))*d5)+(a3*(cos(t1)* cos(t2)- sin(t1)*sin(t2))*cos(t3));

py = (a4* (cos (t1)* sin(t2)+ sin(t1)*cos(t2))* cos(t3)* cos(t4))- (a4* (cos (t1)*sin(t2)+ sin(t1)*cos(t2))* sin(t3)* sin(t4))- ((cos(t1)*cos(t2)- sin(t1)*sin(t2))*d5)+(a3*(cos (t1)* sin(t2)+ sin(t1)*cos(t2))*cos(t3));

pz = (a4*sin(t3)*cos(t4)) - (a4* cos(t3)*sin(t4)) + (a3*sin(t3))+d1;

6 DOF algorithm