HW2: Q1:

$$p(w|t) = N(w|m_n, S_n)$$

$$\propto$$

$$= \frac{p(t|X, w, \beta) \times p(w)}{p(t|X, \beta)}$$ slide 32

$$\Rightarrow \propto \underbrace{p(t|x, w, \beta)}_{\text{likelihood}} \times \underbrace{p(w)}_{\text{prior}}$$

$$\prod_{n=1}^{N} \underbrace{N(t_n | w^T \phi X_n, \beta^{-1})} \times \underbrace{p(w)}$$

$$\ln \left(= \prod_{n=1}^{N} \exp\left(-\frac{\beta}{2}(t-\phi w)^T(t-\phi w)\right) \times \exp\left(-\frac{1}{2}(w-m_0)^T S_0^{-1}(w-m_0)\right) \right)$$

$$\sum^{N} -\frac{\beta}{2}(t-\phi w)^T(t-\phi w) + -\frac{1}{2}(w-m_0)^T S_0^{-1}(w-m_0)$$

$$\sum_{1}^{n} -\frac{\beta}{2}(t-\phi(x_i)w)^T(t-\phi(x_i)w) - \frac{1}{2}(w-m_0)^T S_0^{-1}(w-m_0)$$

$$= -\frac{\beta}{2}(t-\phi(x_n)w)^T(t-\phi(x_n)w) - \frac{1}{2}(w-m_0)^T S_0^{-1}(w-m_0)$$

now $\frac{d}{dw} \Rightarrow$ $$-\frac{\beta}{2} \cdot 2(\phi(x_n))(t-\phi(x_n)w) - \frac{1}{2} \cdot 2 \cdot (w-m_0) \cdot S_0^{-1}$$

$$= -\beta\, \phi(x_n)(t-\phi(x_n)w) - (w-m_0) \cdot S_0^{-1}$$

expand $$= -\beta\phi(x_n)t + \beta\phi(x_n)\phi(x_n)w - wS_0^{-1} + m_0 S_0^{-1}$$

$$\Rightarrow \beta\phi(x_n)\phi(x_n)w - \beta\phi(x_n)t - wS_0^{-1} + m_0 S_0^{-1} = 0$$

$$\beta\phi(x_n)\phi(x_n)w - wS_0^{-1} = \beta\phi(x_n)t - m_0 S_0^{-1}$$

(• inverse) $$w(\beta\phi(x_n)\phi(x_n) - S_0^{-1}) = \beta\phi(x_n)t - m_0 S_0^{-1}$$

$$w = \boxed{(\beta\phi(x_n)t - m_0 S_0^{-1})}\boxed{(\beta\phi(x_n)\phi(x_n) - S_0^{-1})^{-1}}$$