Matrices, Vectors Review

- **A matrix is a rectangular array of numbers or functions which we will enclose in brackets. For example,**
- Rows, columns
- Entries: elements in matrices.

$$\begin{bmatrix} 0.3 & 1 & -5 \\ 0 & -0.2 & 16 \end{bmatrix}, \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{bmatrix} e^{-x} & 2x^2 \\ e^{6x} & 4x \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0.5 & 5 \end{bmatrix}, \qquad \begin{bmatrix} 1 \\ -5 \end{bmatrix}$$

- General Concepts:
- We denote matrices by capital letters A, B, C, , or by writing the general entry in brackets $A=\left[a_{ij}\right]$
- Matrix size : By an $m\times n$ matrix, we mean a matrix with m rows and n columns.
- **If m = n, then the matrix is called a square matrix**
- Then , Main diagonal: a_{11}, a_{22} , …, a_{nn}

Vector: is a matrix with one column or one row.

- Its entries are called the components

$$A=\left[a_{ij}\right]=\begin{bmatrix} a_{11} & a_{12}\cdots & a_{1n} \\ a_{21} & a_{22}\cdots & a_{2n} \\ \cdots\cdots & \cdots & \cdots\cdots \\ a_{m1} & a_{m2}\cdots & a_{mn} \end{bmatrix}$$

- Example:

$$\begin{bmatrix} 0.3 & 1 & -5 \\ 0 & -0.2 & 16 \end{bmatrix}, \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad \begin{bmatrix} e^{-x} & 2x^2 \\ e^{6x} & 4x \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0.5 & 5 \end{bmatrix}, \qquad \begin{bmatrix} 1 \\ -5 \end{bmatrix}$$

- **Addition and Scalar Multiplication of Matrices and Vectors:**
- Equality of Matrices:
- Two matrices A and B are equal A=B, if and only if they have the same size and the corresponding entries are equal.
- Example: Let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -2 \\ 4 & 0 \end{bmatrix}$$

Then when does A = B?

Same size and all entries in A = corresponding entries in B

- **Addition and Scalar Multiplication of Matrices and Vectors:**
- Addition of Matrices:

The sum of two matrices A and B of the same size is written A + B and has the entries obtained by adding the corresponding entries of A and B.

Example: 1) If $A=\begin{bmatrix} -4 & 6 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 5 & -1 & 0 \\ 3 & 1 & 0 \end{bmatrix}$, find A + B?

$$A+B=\begin{bmatrix} -4 & 6 & 3 \\ 0 & 1 & 2 \end{bmatrix}+\begin{bmatrix} 5 & -1 & 0 \\ 3 & 1 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 5 & 3 \\ 3 & 2 & 2 \end{bmatrix}$$

2) $C=\begin{bmatrix} 5 & 7 & 2 \end{bmatrix}$ and $D=\begin{bmatrix} -6 & 2 & 0 \end{bmatrix}$

$$C+D=\begin{bmatrix} 5 & 7 & 2 \end{bmatrix}+\begin{bmatrix} -6 & 2 & 0 \end{bmatrix}=\begin{bmatrix} -1 & 9 & 2 \end{bmatrix}$$

- **Addition and Scalar Multiplication of Matrices and Vectors:**
- Scalar Multiplication:

The product of any matrix A and a scaler c is cA where c is multiplied by all entries in the matrix.

Example: If $A=\begin{bmatrix} -4 & 6 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 5 & -1 & 0 \\ 3 & 1 & 0 \end{bmatrix}$ find:

1) 2A? 2) 2A-B.

$$2A=\begin{bmatrix} -8 & 12 & 6 \\ 0 & 2 & 4 \end{bmatrix} \quad 2A-B=\begin{bmatrix} -8 & 12 & 6 \\ 0 & 2 & 4 \end{bmatrix}-\begin{bmatrix} 5 & -1 & 0 \\ 3 & 1 & 0 \end{bmatrix}=\begin{bmatrix} -13 & 13 & 6 \\ -3 & 1 & 4 \end{bmatrix}$$

- **Addition and Scalar Multiplication of Matrices and Vectors:**
- **Rules**:
- 1) $A + B = B + A$ (commutative)
- 2) $(A + B) + C = A + (B+C)$ (associative)
- 3) $A + 0 = A$ (0 is a zero matrix here)
- 4) $A + (- A) = 0$ (0 is a zero matrix here)

For c and k scalars :

- 5) $c(A + B) = cA + cB$
- 6) $(c+k) A = cA + kA$
- 7) $c(kA) = (ck) A$