Problem set 11

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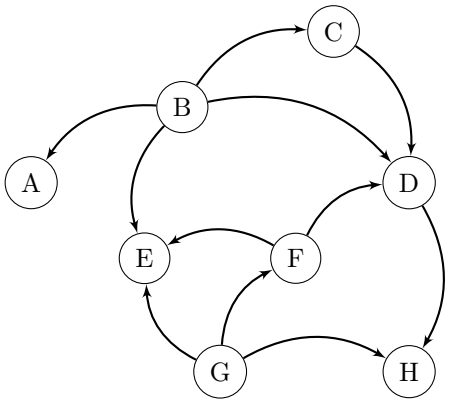
Week 11. Problem set

1. Write down all possible topological sortings for the nodes of the following directed graph:



Solution

1. B-A-C-G-F-D-E-H
2. B-A-C-G-F-D-H-E
3. B-A-C-G-F-E-D-H
4. B-A-G-C-F-D-E-H
5. B-A-G-C-F-D-H-E
6. B-A-G-C-F-E-D-H
7. B-A-G-F-C-D-E-H
8. B-A-G-F-C-D-H-E
9. B-A-G-F-C-E-D-H
10. B-A-G-F-E-C-D-H
11. B-C-A-G-F-D-E-H
12. B-C-A-G-F-D-H-E
13. B-C-A-G-F-E-D-H
14. B-C-G-A-F-D-E-H

15. B-C-G-A-F-D-H-E
16. B-C-G-A-F-E-D-H
17. B-C-G-F-A-D-E-H
18. B-C-G-F-A-D-H-E
19. B-C-G-F-A-E-D-H
20. B-C-G-F-D-A-E-H
21. B-C-G-F-D-A-H-E
22. B-C-G-F-D-E-A-H
23. B-C-G-F-D-E-H-A
24. B-C-G-F-D-H-A-E
25. B-C-G-F-D-H-E-A
26. B-C-G-F-E-A-D-H
27. B-C-G-F-E-D-A-H
28. B-C-G-F-E-D-H-A
29. B-G-A-C-F-D-E-H
30. B-G-A-C-F-D-H-E
31. B-G-A-C-F-E-D-H
32. B-G-A-F-C-D-E-H
33. B-G-A-F-C-D-H-E
34. B-G-A-F-C-E-D-H
35. B-G-A-F-E-C-D-H
36. B-G-C-A-F-D-E-H
37. B-G-C-A-F-D-H-E
38. B-G-C-A-F-E-D-H
39. B-G-C-F-A-D-E-H
40. B-G-C-F-A-D-H-E
41. B-G-C-F-A-E-D-H
42. B-G-C-F-D-A-E-H
43. B-G-C-F-D-A-H-E
44. B-G-C-F-D-E-A-H
45. B-G-C-F-D-E-H-A
46. B-G-C-F-D-H-A-E
47. B-G-C-F-D-H-E-A
48. B-G-C-F-E-A-D-H
49. B-G-C-F-E-D-A-H
50. B-G-C-F-E-D-H-A
51. B-G-F-A-C-D-E-H
52. B-G-F-A-C-D-H-E
53. B-G-F-A-C-E-D-H
54. B-G-F-A-E-C-D-H
55. B-G-F-C-A-D-E-H
56. B-G-F-C-A-D-H-E
57. B-G-F-C-A-E-D-H
58. B-G-F-C-D-A-E-H
59. B-G-F-C-D-A-H-E
60. B-G-F-C-D-E-A-H

61. B-G-F-C-D-E-H-A
62. B-G-F-C-D-H-A-E
63. B-G-F-C-D-H-E-A
64. B-G-F-C-E-A-D-H
65. B-G-F-C-E-D-A-H
66. B-G-F-C-E-D-H-A
67. B-G-F-E-A-C-D-H
68. B-G-F-E-C-A-D-H
69. B-G-F-E-C-D-A-H
70. B-G-F-E-C-D-H-A
71. G-B-A-C-F-D-E-H
72. G-B-A-C-F-D-H-E
73. G-B-A-C-F-E-D-H
74. G-B-A-F-C-D-E-H
75. G-B-A-F-C-D-H-E
76. G-B-A-F-C-E-D-H
77. G-B-A-F-E-C-D-H
78. G-B-C-A-F-D-E-H
79. G-B-C-A-F-D-H-E
80. G-B-C-A-F-E-D-H
81. G-B-C-F-A-D-E-H
82. G-B-C-F-A-D-H-E
83. G-B-C-F-A-E-D-H
84. G-B-C-F-D-A-E-H
85. G-B-C-F-D-A-H-E
86. G-B-C-F-D-E-A-H
87. G-B-C-F-D-E-H-A
88. G-B-C-F-D-H-A-E
89. G-B-C-F-D-H-E-A
90. G-B-C-F-E-A-D-H
91. G-B-C-F-E-D-A-H
92. G-B-C-F-E-D-H-A
93. G-B-F-A-C-D-E-H
94. G-B-F-A-C-D-H-E
95. G-B-F-A-C-E-D-H
96. G-B-F-A-E-C-D-H
97. G-B-F-C-A-D-E-H
98. G-B-F-C-A-D-H-E
99. G-B-F-C-A-E-D-H
100. G-B-F-C-D-A-E-H
101. G-B-F-C-D-A-H-E
102. G-B-F-C-D-E-A-H
103. G-B-F-C-D-E-H-A
104. G-B-F-C-D-H-A-E
105. G-B-F-C-D-H-E-A
106. G-B-F-C-E-A-D-H

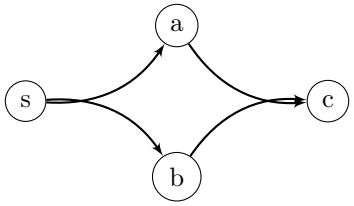
107. G-B-F-C-E-D-A-H
108. G-B-F-C-E-D-H-A
109. G-B-F-E-A-C-D-H
110. G-B-F-E-C-A-D-H
111. G-B-F-E-C-D-A-H
112. G-B-F-E-C-D-H-A
113. G-F-B-A-C-D-E-H
114. G-F-B-A-C-D-H-E
115. G-F-B-A-C-E-D-H
116. G-F-B-A-E-C-D-H
117. G-F-B-C-A-D-E-H
118. G-F-B-C-A-D-H-E
119. G-F-B-C-A-E-D-H
120. G-F-B-C-D-A-E-H
121. G-F-B-C-D-A-H-E
122. G-F-B-C-D-E-A-H
123. G-F-B-C-D-E-H-A
124. G-F-B-C-D-H-A-E
125. G-F-B-C-D-H-E-A
126. G-F-B-C-E-A-D-H
127. G-F-B-C-E-D-A-H
128. G-F-B-C-E-D-H-A
129. G-F-B-E-A-C-D-H
130. G-F-B-E-C-A-D-H
131. G-F-B-E-C-D-A-H
132. G-F-B-E-C-D-H-A

2. Give an example of a directed graph $G = (V, E)$, a source vertex s, and a set of edges $T \subseteq E$ such that

- T forms a tree and
- for each vertex $v \in V$, the unique simple path in the graph (V, T) from s to v is a shortest path in G, yet
- the set of edges T cannot be produced by running BFS on G, no matter how the vertices are ordered in the adjacency lists.

Solution

Consider the following directed graph $G = (V, E)$, where $V = \{s, a, b, c\}$ and $E = \{(s, a), (s, b), (a, c), (b, c)\}$.



The source vertex is s. The set of edges $T = \{(s, a), (a, c)\}$ forms a tree. For each vertex $v \in V$, the unique simple path in the graph (V, T) from s to v is a shortest path in G. However, the set of edges T cannot be produced by running BFS on G, no matter how the vertices are ordered in the adjacency lists. This is because BFS would always include the edge (s, b) in T before it includes (a, c), since b is a direct neighbor of s and c is not.