Data Structures and Algorithms (Spring 2024) — Problem Sets

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Week 1. Problem set

1. Compute asymptotic worst case time complexity of the following algorithm (see pseudocode conventions in [Cormen, Section 2.1]). You **must** use Θ-notation. For justification, provide execution cost and frequency count for each line in the body of the `secret` procedure. Optionally, you may provide the details for the computation of the running time $T(n)$ for worst case scenario. Proof for the asymptotic bound is not required for this exercise.

```
1        /* A is a 0-indexed array,
2         * n is the number of items in A */
3        secret(A, n):
4          k := 0
5          for i = 1 to n-1
6            k := k + 1
7            j := i
8            while j < n and A[j-1] ≥ A[j]
9              j := 2 * j
10           exchange A[i] with A[min(j, n - 1)]
```

2. Indicate, for each pair of expressions (A, B) in the table below whether $A = O(B)$, $A = o(B)$, $A = \Omega(B)$, $A = \omega(B)$, or $A = \Theta(B)$. Write your answer in the form of the table with *yes* or *no* written in each box:

A	B	$A = O(B)$	$A = o(B)$	$A = \Omega(B)$	$A = \omega(B)$	$A = \Theta(B)$
1.0001^n	n^{1000}					
3	$(1 + 1/n)^n$					
$n^{\sin n}$	$\log_2 n$					
$\log_2^3 n$	$\sqrt[6]{n}$					

3. Let f and g be functions from positive integers to positive reals. Assume $g(n) > n$ for $n > 0$. Using definition of asymptotic notation, prove formally that

$$\max(f(n) + \sqrt{n}, g(n) - n) = O(f(n) + g(n))$$

References

[Cormen] Cormen, T.H., Leiserson, C.E., Rivest, R.L. and Stein, C., 2022. *Introduction to algorithms, Fourth Edition*. MIT press.