Problem set 12

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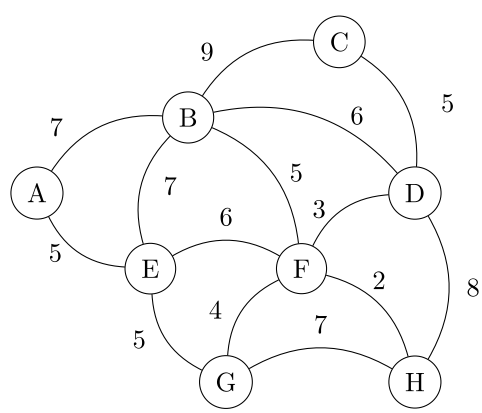
Week 12. Problem set

1 Task 1

1.1 Statement

Run Prim-Jarník algorithm [Cormen, §21.2] on the following graph, starting at vertex C. Assuming that the algorithm is using Binary heap implementation [Cormen, §6] of a priority queue, show the state of the Binary heap after each iteration of the algorithm (i.e. after adding each new vertex to the MST). The graph contains 8 vertices, which means that your solution must provide 8 states of the Binary heap. Each heap state must be represented as an array. No justification is required.



1.2 Solution

Number	Vertices	Edges	Weights
1.	-	-	-
2.	B	CB	9
3.	B	DB	6
	H	DH	8
4.	B	FB	5
	G	FG	4
	E	FE	6
5.	E	GE	5
6.	A	BA	7
7.	A	EA	5
8.	-	-	-

2 Task 2

2.1 Statement

Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How fast can you make Kruskal's algorithm run by modifying it somehow? What if the edge weights are integers in the range from 1 to W for some constant W? Justify your answer in at most two paragraphs.

2.2 Solution

If all edge weights in a graph are integers in the range from 1 to $|V|$, we can use a linear time sorting algorithm such as counting sort instead of a comparison-based sorting algorithm. This would make the sorting step of Kruskal's algorithm run in $O(V + E)$ time, and since the disjoint-set operations run in nearly constant time, the overall time complexity of the algorithm would be $O(V + E)$.

If the edge weights are integers in the range from 1 to W for some constant W, we can still use counting sort or radix sort to sort the edges in linear time. However, if W is significantly larger than $|V|$ and $|E|$, the time complexity of the sorting step could be dominated by W. In this case, the overall time complexity of the algorithm would be $O(W + V + E)$.

References

[Cormen] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. Introduction to Algorithms, Fourth Edition. The MIT Press 2022