Problem Set 8

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1 Task 1

1.1 Statement

Insert the ⟨key, value⟩ items into an empty B-tree [Cormen, §18] with minimum degree $t = 2$:

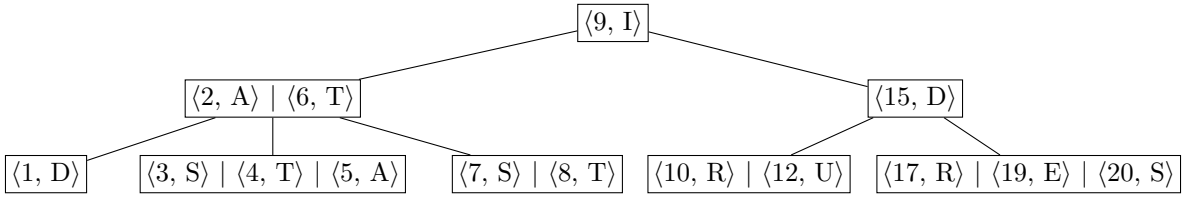
(a) ⟨33, U⟩, ⟨10, T⟩, ⟨17, I⟩, ⟨12, N⟩, ⟨23, U⟩

(b) ⟨1, A⟩, ⟨29, D⟩, ⟨36, Y ⟩, ⟨3, S⟩, ⟨5, T⟩

(c) ⟨19, P⟩, ⟨14, O⟩, ⟨7, I⟩, ⟨8, N⟩, ⟨39, I⟩

(d) ⟨27, I⟩, ⟨35, N⟩, ⟨20, O⟩, ⟨25, L⟩, ⟨31, S⟩

Show the state of the tree after every 5 insertions. Depict each tree as a sequence of arrays for each layer. For example, consider this B-tree:



The tree above must be depicted as follows:

(layer 1) [⟨9, I⟩]

(layer 2) [⟨2, A⟩|⟨6, T⟩] [⟨15, D⟩]

(layer 3) [⟨1, D⟩] [⟨3, S⟩|⟨4, T⟩|⟨5, A⟩] [⟨7, S⟩|⟨8, T⟩] [⟨10, R⟩|⟨12, U⟩] [⟨17, R⟩|⟨19, E⟩|⟨20, S⟩]

1.2 Answer

1. Insertion 1

(layer 1) [⟨17, I⟩]

(layer 2) [⟨10, T⟩|⟨12, N⟩] [⟨23, U⟩|⟨33, U⟩]

2. Insertion 2

(layer 1) [⟨10, T⟩|⟨17, I⟩|⟨29, D⟩]

(layer 2) [⟨1, A⟩|⟨3, S⟩|⟨5, T⟩] [⟨12, N⟩] [⟨23, U⟩] [⟨33, U⟩|⟨36, Y⟩]

3. Insertion 3

(layer 1) [⟨17, I⟩]

(layer 2) [⟨3, S⟩|⟨10, T⟩] [⟨29, D⟩]

(layer 3) | ⟨1, A⟩ | | ⟨5, T⟩ | ⟨7, I⟩ | ⟨8, N⟩ | | ⟨12, N⟩ | ⟨14, O⟩ | | ⟨19, P⟩ | ⟨23, U⟩ | | ⟨33, U⟩ | ⟨36, Y⟩ | ⟨39, I⟩ |

4. Insertion 4

(layer 1) | ⟨17, I⟩ |

(layer 2) | ⟨3, S⟩ | ⟨10, T⟩ | | ⟨23, U⟩ | ⟨29, D⟩ | ⟨36, Y⟩ |

(layer 3) | ⟨1, A⟩ | | ⟨5, T⟩ | ⟨7, I⟩ | ⟨8, N⟩ | | ⟨12, N⟩ | ⟨14, O⟩ | | ⟨19, P⟩ | ⟨20, O⟩ | | ⟨25, L⟩ | ⟨27, I⟩ |
| ⟨31, S⟩ | ⟨33, U⟩ | ⟨35, N⟩ | | ⟨39, I⟩ |

2 Task 2

2.1 Statement

Perform Heap-Sort [Cormen, §6.4] on the following input array:

1	3	7	8	0	2	5	4	6

Show the state of the array after each call to Max-Heapify (solution must have 12 arrays).

2.2 Answer

Call 1	1	3	7	8	0	2	5	4	6
Call 2	1	3	7	8	0	2	5	4	6
Call 3	1	8	7	6	0	2	5	4	3
Call 4	8	6	7	4	0	2	5	1	3
Call 5	7	6	5	4	0	2	3	1	8
Call 6	6	4	5	1	0	2	3	7	8
Call 7	5	4	3	1	0	2	6	7	8
Call 8	4	2	3	1	0	5	6	7	8
Call 9	3	2	0	1	4	5	6	7	8
Call 10	2	1	0	3	4	5	6	7	8
Call 11	1	0	2	3	4	5	6	7	8
Call 12	0	1	2	3	4	5	6	7	8

References

[Cormen] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. Introduction to Algorithms, Fourth Edition. The MIT Press 2022

[Goodrich] M. T. Goodrich, R. Tamassia, and M. H. Goldwasser. Data Structures and Algorithms in Java. WILEY 2014.