Data Structures and Algorithms Spring 2024 — Problem Sets

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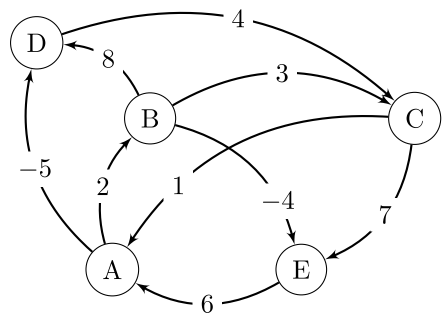
Week 13. Problem set

1. Run the Floyd-Warshall algorithm [Cormen, §23.2] on the following graph. Use the alphabetic order of vertices. Show the state of distance matrix D after each iteration of outer loop in the algorithm. Since the graph has 5 vertices, you must provide five 5×5 matrices in your answer. No justification is required.



2. Provide a graph with exactly 4 vertices (A, B, C, D) and 4 weighted edges, such that Dijkstra's algorithm [Cormen, §22.3] does **not** give a correct shortest distance for at least one vertex:
   (a) Provide the graph (the graph must be rendered in a clear way, text representation is not enough); Weights must be (small) integers.
   (b) Provide the result of Dijkstra's algorithm for each vertex: any shortest path **and** corresponding total weight;
   (c) Provide the correct shortest path and corresponding total weight for each vertex;
   (d) Explain why Dijkstra's algorithm did not provide the correct answer (specifically for your example, generic justification is not accepted).

3. Since Dijkstra's shortest paths algorithm [Cormen, §22.3] does not work with negative edges in general, consider an algorithm that, for a given graph G, if it has a negative edge, finds the minimum edge in G with the weight $(-W)$ and adds $(+W)$ to all edges in the original graph, resulting in a new graph G^{+W}. Then the modified algorithm runs Dijkstra's algorithm on G^{+W}. Are the resulting shortest paths in G^{+W} also shortest in G? If yes, prove it. If no, provide a concrete counterexample and a justification.

References

[Cormen] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. *Introduction to Algorithms, Fourth Edition.* The MIT Press 2022