Problem Set 8

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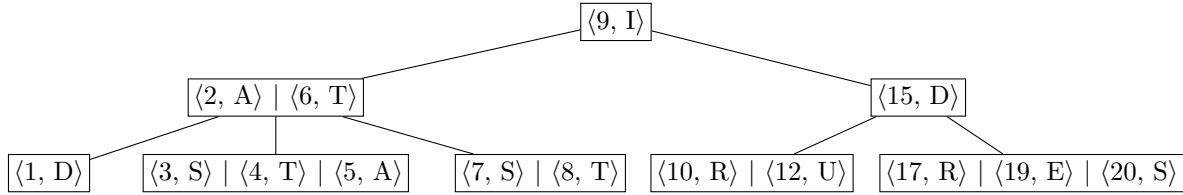
1 Task 1

1.1 Statement

Insert the ⟨key, value⟩ items into an empty B-tree [Cormen, §18] with minimum degree $t = 2$:

(a) ⟨33, U⟩, ⟨10, T⟩, ⟨17, I⟩, ⟨12, N⟩, ⟨23, U⟩

(b) ⟨1, A⟩, ⟨29, D⟩, ⟨36, Y ⟩, ⟨3, S⟩, ⟨5, T⟩

(c) ⟨19, P⟩, ⟨14, O⟩, ⟨7, I⟩, ⟨8, N⟩, ⟨39, I⟩

(d) ⟨27, I⟩, ⟨35, N⟩, ⟨20, O⟩, ⟨25, L⟩, ⟨31, S⟩

Show the state of the tree after every 5 insertions. Depict each tree as a sequence of arrays for each layer. For example, consider this B-tree:

The tree above must be depicted as follows:

(layer 1) ⟨9, I⟩

(layer 2) ⟨2, A⟩ ⟨6, T⟩   ⟨15, D⟩

(layer 3) ⟨1, D⟩   ⟨3, S⟩ ⟨4, T⟩ ⟨5, A⟩   ⟨7, S⟩ ⟨8, T⟩   ⟨10, R⟩ ⟨12, U⟩   ⟨17, R⟩ ⟨19, E⟩ ⟨20, S⟩

1.2 Answer

1. Insertion 1

(layer 1) ⟨17, I⟩

(layer 2) ⟨10, T⟩ ⟨12, N⟩   ⟨23, U⟩ ⟨33, U⟩

2. Insertion 2

(layer 1) ⟨10, T⟩ ⟨17, I⟩ ⟨29, D⟩

(layer 2) ⟨1, A⟩ ⟨3, S⟩ ⟨5, T⟩   ⟨12, N⟩   ⟨23, U⟩   ⟨33, U⟩ ⟨36, Y⟩

3. Insertion 3

(layer 1) ⟨17, I⟩

(layer 2) ⟨3, S⟩ ⟨10, T⟩   ⟨29, D⟩

(layer 3) $\boxed{\langle 1, A \rangle}$ $\boxed{\langle 5, T \rangle | \langle 7, I \rangle | \langle 8, N \rangle}$ $\boxed{\langle 12, N \rangle | \langle 14, O \rangle}$ $\boxed{\langle 19, P \rangle | \langle 23, U \rangle}$ $\boxed{\langle 33, U \rangle | \langle 36, Y \rangle | \langle 39, I \rangle}$

4. Insertion 4

(layer 1) $\boxed{\langle 17, I \rangle}$

(layer 2) $\boxed{\langle 3, S \rangle | \langle 10, T \rangle}$ $\boxed{\langle 23, U \rangle | \langle 29, D \rangle | \langle 36, Y \rangle}$

(layer 3) $\boxed{\langle 1, A \rangle}$ $\boxed{\langle 5, T \rangle | \langle 7, I \rangle | \langle 8, N \rangle}$ $\boxed{\langle 12, N \rangle | \langle 14, O \rangle}$ $\boxed{\langle 19, P \rangle | \langle 20, O \rangle}$ $\boxed{\langle 25, L \rangle | \langle 27, I \rangle}$ $\boxed{\langle 31, S \rangle | \langle 33, U \rangle | \langle 35, N \rangle}$ $\boxed{\langle 39, I \rangle}$

2 Task 2

2.1 Statement

Perform Heap-Sort [Cormen, §6.4] on the following input array:

1	3	7	8	0	2	5	4	6

Show the state of the array after each call to Max-Heapify (solution must have 12 arrays).

2.2 Answer

Call 1	1	3	7	8	0	2	5	4	6
Call 2	1	3	7	8	0	2	5	4	6
Call 3	1	8	7	6	0	2	5	4	3
Call 4	8	6	7	4	0	2	5	1	3
Call 5	7	6	5	4	0	2	3	1	8
Call 6	6	4	5	1	0	2	3	7	8
Call 7	5	4	3	1	0	2	6	7	8
Call 8	4	2	3	1	0	5	6	7	8
Call 9	3	2	0	1	4	5	6	7	8
Call 10	2	1	0	3	4	5	6	7	8
Call 11	1	0	2	3	4	5	6	7	8
Call 12	0	1	2	3	4	5	6	7	8

References

[Cormen] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. Introduction to Algorithms, Fourth Edition. The MIT Press 2022

[Goodrich] M. T. Goodrich, R. Tamassia, and M. H. Goldwasser. Data Structures and Algorithms in Java. WILEY 2014.