Data Structures and Algorithms Spring 2024 — Problem Sets

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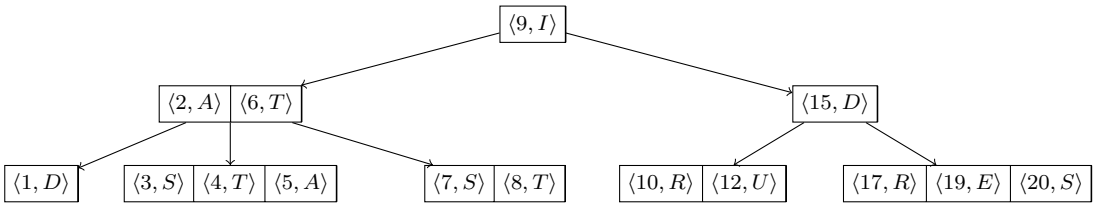
Week 8. Problem set

1. Insert the $\langle \mathsf{key}, \mathsf{value} \rangle$ items into an empty B-tree [Cormen, §18] with minimum degree $t = 2$:

   (a) $\langle 33, U \rangle$, $\langle 10, T \rangle$, $\langle 17, I \rangle$, $\langle 12, N \rangle$, $\langle 23, U \rangle$
   (b) $\langle 1, A \rangle$, $\langle 29, D \rangle$, $\langle 36, Y \rangle$, $\langle 3, S \rangle$, $\langle 5, T \rangle$
   (c) $\langle 19, P \rangle$, $\langle 14, O \rangle$, $\langle 7, I \rangle$, $\langle 8, N \rangle$, $\langle 39, I \rangle$
   (d) $\langle 27, I \rangle$, $\langle 35, N \rangle$, $\langle 20, O \rangle$, $\langle 25, L \rangle$, $\langle 31, S \rangle$

   Show the state of the tree after every 5 insertions. Depict each tree as a sequence of arrays for each layer. For example, consider this B-tree:



   The tree above must be depicted as follows:

(layer 1) $[\langle 9, I \rangle]$
(layer 2) $[\langle 2, A \rangle | \langle 6, T \rangle]$ $[\langle 15, D \rangle]$
(layer 3) $[\langle 1, D \rangle]$ $[\langle 3, S \rangle | \langle 4, T \rangle | \langle 5, A \rangle]$ $[\langle 7, S \rangle | \langle 8, T \rangle]$ $[\langle 10, R \rangle | \langle 12, U \rangle]$ $[\langle 17, R \rangle | \langle 19, E \rangle | \langle 20, S \rangle]$

2. Perform Heap-Sort [Cormen, §6.4] on the following input array:

1	3	7	8	0	2	5	4	6

   Show the state of the array after each call to Max-Heapify (solution must have 12 arrays).

3. **(+1% extra credit)** A d-ary heap is similar to a binary heap, except non-leaf nodes have d children instead of 2 children (except the last non-leaf node, which is allowed to have fewer children). Adjust the array representation and the efficient implementations of Max-Heapify and Build-Max-Heap. Perform Heap-Sort [Cormen, §6.4] but using a 3-ary heap on the following input array:

1	3	7	8	0	2	5	4	6

   Show the state of the array after each call to Max-Heapify (solution must have 11 arrays).

References

[Cormen] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. *Introduction to Algorithms, Fourth Edition.* The MIT Press 2022

[Goodrich] M. T. Goodrich, R. Tamassia, and M. H. Goldwasser. *Data Structures and Algorithms in Java.* WILEY 2014.