Data Structures and Algorithms Spring 2024 — Problem Sets

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Week 3. Problem set (solutions)

1. Consider a hash table with 15 slots and the hash function $h(k) = (k^2 - 2k + 7) \mod 13$. Show the state of the hash table after inserting the keys (in this order)

   $$5, 28, 19, 15, 20, 33, 12, 17, 10, 13, 3, 34$$

   with collisions resolved by linear probing [Cormen, Section 11.4].

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2. Consider the following algorithm (see pseudocode conventions in [Cormen, Section 2.1]). The inputs to this algorithm are a map M and a key k. Additionally, assume the following:

   (a) The map M uses the positive integers both for keys and for values.

   (b) The map M is not empty and contains n distinct keys.

   (c) The map M is represented as a hashtable with load factor α.

   (d) The map M resolves collisions via double hashing [Cormen, Section 11.4].

   (e) For all keys k, if a value in M at k exists, then it is smaller than k.

```
/* M is a map with a load factor α and size n,
 * k is a key that is present in M */
secret(M, k):
  total := 0
  v := M.get(k)
  for i=1 to v
    if M.hasKey(i)
      total := total + M.get(i)
  return total
```

   Compute the average case time complexity of `secret`. The answer **must** use O-notation and may depend on n, k, and α. For the average case analysis, use *independent uniform permutation hashing*. Assume worst case for the contents of the input map M.

   Briefly justify your answer (2–3 sentences). Detailed proof for the asymptotic complexity is not required for this exercise.

3. In your own words, explain how it is possible to implement deletion of a key-value pair from a hashtable with $O(1)$ **worst case** time complexity if collision resolution is implemented using chaining? Specify the data structures and methods involved.

References

[Cormen] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. *Introduction to Algorithms, Fourth Edition*. The MIT Press 2022