INNOPOLIS UNIVERSITY

Physics

Homework 2

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1 Problem 1

Solution:

Newton's Second Law in Vector Form:

$$\vec{F}_{\text{net}} = m\vec{a}$$

For an object in free-fall with air resistance, at terminal velocity, the net force is zero ($\vec{F}_{\text{net}} = 0$), meaning the gravitational force is balanced by the drag force:

$$m\vec{g} + \frac{1}{2}C_d\rho A\vec{v}^2 = \vec{0}$$

At terminal velocity ($v = v_t$), the forces are balanced:

$$mg = \frac{1}{2}C_d\rho A v_t^2$$

Solving for the terminal velocity:

$$v_t = \sqrt{\frac{2mg}{C_d\rho A}}$$

Given Data: - Air density: $\rho = 1.007\,\text{kg/m}^3$ - Gravity: $g = 9.8\,\text{m/s}^2$
For the Jeep Rubicon:

$$C_d = 0.5, \quad A = 2.58\,\text{m}^2, \quad m = 2000\,\text{kg}$$

$$v_t = \sqrt{\frac{2 \times 2000 \times 9.8}{0.5 \times 1.007 \times 2.58}} = 173.7\,\text{m/s}$$

For the Dodge Challenger SRT (2015):

$$C_d = 0.38, \quad A = 2.41\,\text{m}^2, \quad m = 2450\,\text{kg}$$

$$v_t = \sqrt{\frac{2 \times 2450 \times 9.8}{0.38 \times 1.007 \times 2.41}} = 228.2\,\text{m/s}$$

For the Subaru WRX STi:

$$C_d = 0.33, \quad A = 2.225\,\text{m}^2, \quad m = 1550\,\text{kg}$$

$$v_t = \sqrt{\frac{2 \times 1550 \times 9.8}{0.33 \times 1.007 \times 2.225}} = 202.7\,\text{m/s}$$

Final Terminal Velocities:

$$\begin{aligned} \text{Jeep Rubicon} &: \boxed{173.7\,\text{m/s}} \\ \text{Dodge Challenger SRT (2015)} &: \boxed{228.2\,\text{m/s}} \\ \text{Subaru WRX STi} &: \boxed{202.7\,\text{m/s}} \end{aligned}$$

2 Problem 2

Solution:

2.1. Position of maximum tension

In vertical circular motion, the tension in the string is due to both the centripetal force and the gravitational force. The forces acting on the object are:

- **Centripetal force**: which is directed towards the center of the circle. - **Gravitational force**: always acting downward.

At the **top** of the circular trajectory, both the gravitational force and the centripetal force act in the same direction, towards the center. At the **bottom**, they act in opposite directions. Therefore, the **maximum tension** occurs at the **bottom of the circle**, where the gravitational force opposes the centripetal force.

Thus, the position of maximum tension is at the bottom of the circle.

2.2. Maximum speed before the string breaks

At the bottom of the circular path, the tension in the string is given by:

$$T = \frac{mv^2}{R} + mg$$

where: - T is the tension in the string, - $m = 0.25\,\text{kg}$ is the mass of the object, - v is the speed of the object, - $R = 0.7\,\text{m}$ is the radius of the circle, - $g = 9.8\,\text{m/s}^2$ is the acceleration due to gravity.

To find the maximum speed v_{max} when the string does not break, we rearrange the equation:

$$T_{\text{max}} = \frac{mv_{\text{max}}^2}{R} + mg$$

Solving for v_{max}:

$$v_{\text{max}} = \sqrt{\frac{(T_{\text{max}} - mg)R}{m}}$$

Substituting the known values:

$$v_{\text{max}} = \sqrt{\frac{(30 - 0.25 \times 9.8) \times 0.7}{0.25}}$$

$$v_{\text{max}} = \sqrt{\frac{(30 - 2.45) \times 0.7}{0.25}} = \sqrt{\frac{27.55 \times 0.7}{0.25}} = \sqrt{\frac{19.285}{0.25}} = \sqrt{77.14}$$

$$v_{\text{max}} \approx 8.78\,\text{m/s}$$

Thus, the maximum speed is $\boxed{8.8\,\text{m/s}}$.

3 Problem 3

Step 1: Applying Newton's second law

The equation of motion for the boat is:

$$m\frac{dv}{dt} = F_{\text{engine}} - kv$$

where:

- $m = 3000\,\text{kg}$ (mass of the boat),
- $k = 100\,\text{kg/s}$ (drag coefficient),
- F_{engine} is the constant engine force,
- $v(t)$ is the velocity as a function of time.

Rearranging the equation:

$$\frac{dv}{dt} + \frac{k}{m}v = \frac{F_{\text{engine}}}{m}$$

This is a first-order linear differential equation.

Step 2: Solving the differential equation

The solution to this equation is of the form:

$$v(t) = V_{\max}\left(1 - e^{-\frac{k}{m}t}\right)$$

where $V_{\max} = \frac{F_{\text{engine}}}{k}$ is the maximum velocity the boat can reach when the drag force equals the engine force.

Step 3: Determining the engine force

We know that after $t = 33$ seconds, the velocity is $v(33) = 2\,\text{m/s}$. Substituting this into the velocity equation:

$$2 = V_{\max}\left(1 - e^{-\frac{100}{3000}\times 33}\right)$$

Simplifying the exponent:

$$2 = V_{\max}\left(1 - e^{-1.1}\right)$$

Using $e^{-1.1} \approx 0.3329$:

$$2 = V_{\max} \times (1 - 0.3329)$$

$$2 = V_{\max} \times 0.6671$$

Thus,

$$V_{\max} = \frac{2}{0.6671} \approx \boxed{2.999\,\text{m/s}}$$

4 Problem 4

Step 1: Forces acting on Block A For Block A, the forces are:

$$W_A = m_A \cdot g$$

where W_A is the weight and T_A is the tension in the rope. Since the system is stationary, from Newton's Second Law:

$$T_A = m_A \cdot g \tag{1}$$

Step 2: Forces acting on Block B For Block B, the horizontal tension T_B is resisted by the frictional force F_f:

$$T_B = F_f$$

The maximum static friction force is:

$$F_f = \mu_s \cdot N_B = \mu_s \cdot m_B \cdot g \tag{2}$$

Substituting the known values $\mu_s = 0.25$, $m_B = 50\,\text{kg}$, and $g = 9.8\,\text{m/s}^2$, we get:

$$T_B = 0.25 \cdot 50 \cdot 9.8 = 122.5\,\text{N} \tag{3}$$

Step 3: Forces at the Knot At the knot, we have the following equilibrium conditions: - Horizontal equilibrium:

$$T_B = T\cos\theta \tag{4}$$

- Vertical equilibrium:

$$T_A = T\sin\theta \tag{5}$$

Step 4: Solving for T From equation (4), we can express T as:

$$T = \frac{T_B}{\cos\theta} \tag{6}$$

Substituting equation (3) into equation (6), we get:

$$T = \frac{122.5}{\cos 30^\circ} = \frac{122.5}{0.866} \approx 141.4\,\text{N} \tag{7}$$

Step 5: Substitute T into the Vertical Equilibrium Equation From equation (5):

$$T_A = T\sin\theta$$

Substituting equation (7):

$$T_A = 141.4 \cdot \sin 30^\circ = 141.4 \cdot 0.5 = 70.7\,\text{N} \tag{8}$$

Using equation (1), we can now solve for m_A:

$$m_A \cdot g = 70.7$$

$$m_A = \frac{70.7}{9.8} \approx 7.2\,\text{kg} \tag{9}$$

Final Answer: The maximum mass of Block A for which the system remains stationary is $\boxed{7.2\,\text{kg}}$.