Problem set 6

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1 Task 1

1.1 Statement

Consider a modification of the randomized QUICK-SORT algorithm [Cormen, §7.3] that stops recursion when the size of subarray becomes less than or equal to $k (k \leq n)$. For arrays of size $\leq k$, the modified algorithm performs BUBBLE-SORT. Answer the following questions about the modified algorithm:

(a) What is the worst case time complexity in terms of n and k ?
(b) What is the best case time complexity in terms of n and k ?
(c) What is the average[1] case time complexity in terms of n and k ?
The answer should be given using Θ-notation.
Provide a brief justification for each case (1-2 sentences).

1.2 Solution

(a) QUICK-SORT worst case time complexity for $n - k$ is $\Theta(n^2 - k^2)$. BUBBLE-SORT worst case time complexity for k is $\Theta(k^2)$. Hence, **the answer:** $\Theta(n^2 - k^2 + k^2) = \Theta(n^2)$.
(b) QUICK-SORT best case time complexity for $n-k$ is $\Theta(n(\log_2 n - \log_2 k)) = \Theta(n \log_2 \frac{n}{k})$. BUBBLE-SORT best case time complexity for k is $\Theta(k)$. Hence, **the answer:** $\Theta(n \log_2 \frac{n}{k} + k)$.
(c) QUICK-SORT average case time complexity for $n - k$ is $\Theta(n(\log_2 n - \log_2 k)) = \Theta(n \log_2 \frac{n}{k})$. BUBBLE-SORT average case time complexity for k is $\Theta(k^2)$. Hence, **the answer:** $\Theta(n \log_2 \frac{n}{k} + k^2)$.

2 Task 2

2.1 Statement

Apply Counting-Sort to the following input array where each column corresponds to one item with its numeric key and single-character satellite data:

2	8	2	6	7	6	3	4	1	6	2	0
D	T	O	G	A	R	N	G	R	E	I	U

You must demonstrate the final state of the auxiliary arrays used in the algorithm, as well as the output of the array.

2.2 Solution

counts (array of counts of elements in main array): 1 1 3 1 1 0 3 1 1
accum (partial sums for array of counts): 0 1 2 5 6 7 7 10 11
resultMainData (output integers): 0 1 2 2 2 3 4 6 6 6 7 8
resultSatelliteData (output chars): U R D O I N G G R E A T
Answer:

0	1	2	2	2	3	4	6	6	6	7	8
U	R	D	O	I	N	G	G	R	E	A	T

3 References

[Cormen] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. Introduction to Algorithms, Fourth Edition. The MIT Press 2022

[1] assuming all elements in the input array are distinct and any initial order in the array is equally likely