INNOPOLIS UNIVERSITY

Physics

Homework 1

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1 Problem 1

Step 1: Acceleration Phase (0 to t_1)

- Initial velocity, $v_0 = 0$ m/s
- Final velocity at t_1, $v_1 = 4$ m/s
- Time interval, $t_1 = 4$ s

The acceleration a_1 is calculated as:

$$a_1 = \frac{v_1 - v_0}{t_1 - 0} = \frac{4-0}{4} = 1 \text{ m/s}^2$$

The displacement during this phase x_1 is:

$$x_1 = \frac{1}{2}a_1 t_1^2 = \frac{1}{2} \times 1 \times 4^2 = 8 \text{ m}$$

Step 2: Constant Velocity Phase (t_1 to t_2)

- Velocity $v = 4$ m/s
- Time interval $t_2 - t_1 = 10 - 4 = 6$ s

The displacement during this phase x_2 is:

$$x_2 = v \cdot (t_2 - t_1) = 4 \times 6 = 24 \text{ m}$$

Step 3: Deceleration Phase (t_2 to t_3)

- Initial velocity $v_2 = 4$ m/s
- Final velocity $v_3 = 0$ m/s
- Time interval $t_3 - t_2 = 18 - 10 = 8$ s

The deceleration a_3 is:

$$a_3 = \frac{v_3 - v_2}{t_3 - t_2} = \frac{0-4}{8} = -0.5 \text{ m/s}^2$$

The displacement during this phase x_3 is:

$$x_3 = v_2 \cdot (t_3 - t_2) + \frac{1}{2}a_3(t_3 - t_2)^2 = 4 \times 8 + \frac{1}{2} \times (-0.5) \times 8^2 = 16 \text{ m}$$

Step 4: Total Displacement and Average Velocity

The total displacement from t_1 to t_3 is:

$$\text{Total Displacement} = x_2 + x_3 = 24 + 16 = 40 \text{ m}$$

The average velocity over the interval $[t_1, t_3]$ is:

$$v_{\text{avg}} = \frac{\text{Total Displacement}}{t_3 - t_1} = \frac{40}{18-4} = \frac{40}{14} \approx 2.86 \text{ m/s}$$

Final Answer

- Total displacement from t_1 to t_3: 40 meters - Average velocity over the time interval $[t_1, t_3]$: 2.86 m/s

Plots

Below are the plots for acceleration and displacement as functions of time:

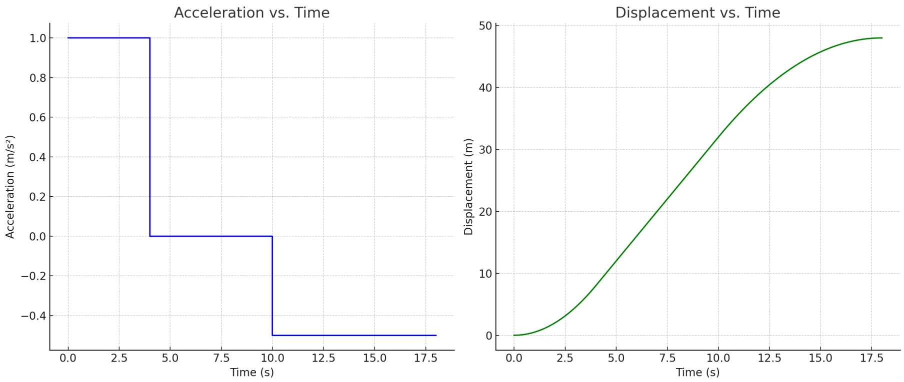


Figure 1: Acceleration and Displacement vs. Time

2 Problem 2

To determine the initial speed of a ball thrown horizontally from a height of 20 meters, where the ball hits the ground with a speed three times its initial speed, we will use the principles of projectile motion. Since the horizontal motion is uniform and unaffected by gravity, the initial horizontal velocity v_0 remains constant. The vertical motion is governed by the acceleration due to gravity. By relating the final speed to the initial speed using the Pythagorean theorem, we can solve for v_0.

- The horizontal motion of the ball is described by:

$$v_x = v_0$$

- The vertical motion of the ball under gravity can be described by the following equations:

$$\text{Displacement: } y = \frac{1}{2}gt^2$$

$$\text{Vertical velocity: } v_y = gt$$

The total speed v_f of the ball when it hits the ground is given by the vector sum of the horizontal and vertical velocities:

$$v_f = \sqrt{v_x^2 + v_y^2}$$

We are given that v_f is three times the initial horizontal speed:

$$v_f = 3v_0$$

Substituting this into the speed equation:

$$3v_0 = \sqrt{v_0^2 + v_y^2}$$

Since the vertical velocity just before impact is:

$$v_y = \sqrt{2gh}$$

we can substitute v_y into the equation:

$$3v_0 = \sqrt{v_0^2 + 2gh}$$

Squaring both sides to remove the square root:

$$9v_0^2 = v_0^2 + 2gh$$

Rearranging and solving for v_0:

$$8v_0^2 = 2gh$$

$$v_0^2 = \frac{2gh}{8}$$

$$v_0 = \sqrt{\frac{2gh}{8}} = \frac{\sqrt{gh}}{2}$$

Substituting $g = 9.8\,\mathrm{m/s^2}$ and $h = 20\,\mathrm{m}$:

$$v_0 = \frac{\sqrt{9.8 \times 20}}{2} = \frac{\sqrt{196}}{2} = \frac{14}{2} = 7\,\mathrm{m/s}$$

The initial speed of the ball is $\boxed{7\,\mathrm{m/s}}$.

3 Problem 3

To determine the speed of the river relative to the ground, we need to use concepts of relative motion. The boat travels upstream and drops a bottle, which then moves downstream with the river current. The fisherman, upon returning downstream, finds the bottle 5 km from the bridge. We will set up equations based on the relative speeds of the boat and the river, and solve for the river's speed.

We denote v_r as the speed of the river relative to the ground and v_b as the speed of the boat relative to the ground. The distance traveled by the bottle downstream in half an hour is $0.5 \times v_r$ km. The fisherman goes up the river for the following distance:

$$S_{up} = (v_b - v_r) \times 0.5$$

The time t_1 for the fisherman to reach the bridge again is:

$$t_1 = \frac{(v_b - v_r) \times 0.5}{v_b + v_r}$$

The time t_2 for the fisherman to reach the bottle from the bridge is:

$$t_2 = \frac{5}{v_b + v_r}$$

The overall time $t_1 + t_2$ is equal to:

$$t = t_1 + t_2 = \frac{(v_b - v_r) \times 0.5 + 5}{v_b + v_r} = \frac{5}{v_r}$$

Thus, solving this equation:

$$\frac{(v_b - v_r) \times 0.5 + 5}{v_b + v_r} = \frac{5}{v_r}$$

Here is the derived quadratic equation:

$$v_r^2 + v_r v_b - 10 v_b = 0$$

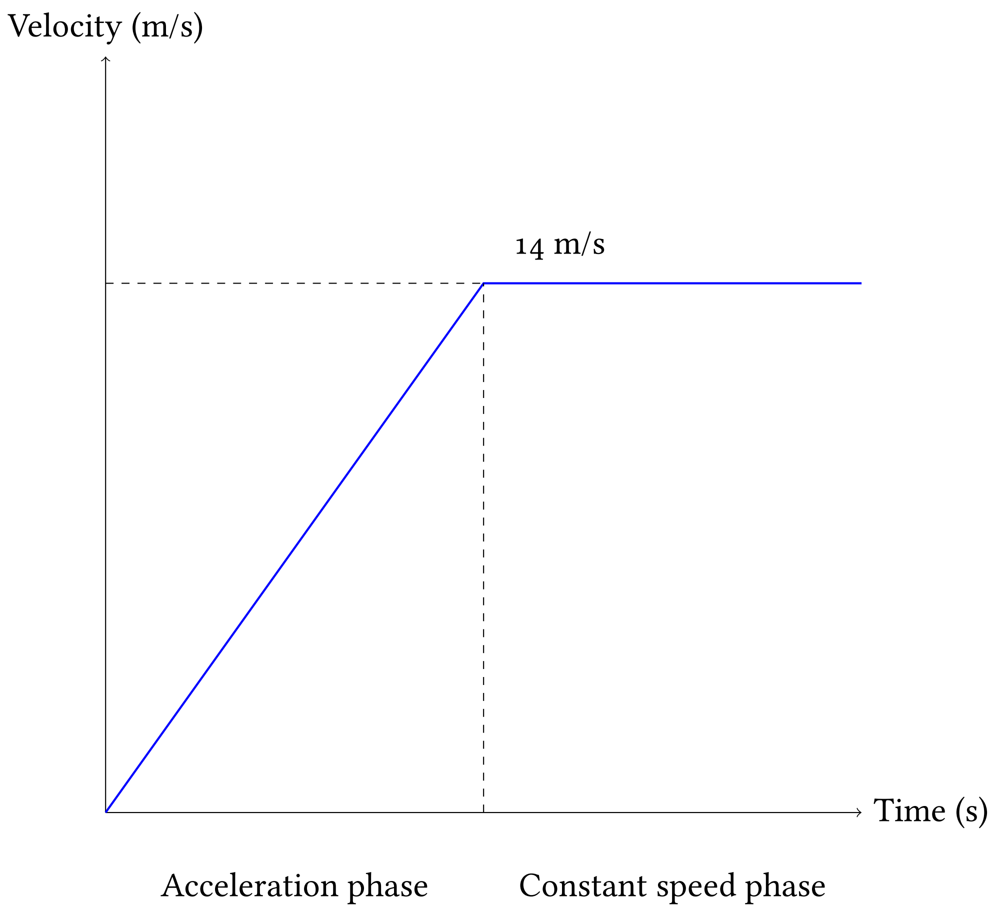
Let's solve it for v_r:

$$D = v_b^2 - 40 v_b$$

$$v_r = \frac{v_b \pm \sqrt{v_b^2 - 40 v_b}}{2}$$

4 Problem 4

To find the acceleration of the athlete, we will first determine the distance covered during the acceleration phase and then find the time spent accelerating. Given the total distance and time, as well as the maximum speed, we can use the equations of motion to solve for acceleration. We will solve for the acceleration a by considering the equations for uniformly accelerated motion and the total distance covered in the race.



We start with the following known values:

- Maximum speed, $v = 14\,\mathrm{m/s}$
- Total distance, $d = 100\,\mathrm{m}$
- Total time, $t = 11\,\mathrm{s}$

Let's denote:

- Time spent accelerating as t_1

- Time spent at maximum speed as t_2
- Acceleration as a

The time t_2 during which the athlete runs at maximum speed is given by $t_2 = 11 - t_1$.

The distance covered during acceleration (d_1) can be calculated using:

$$d_1 = \frac{1}{2}at_1^2$$

The distance covered at constant speed (d_2) is:

$$d_2 = vt_2 = 14(11 - t_1)$$

The total distance covered is:

$$d_1 + d_2 = 100$$

$$\frac{1}{2}at_1^2 + 14(11 - t_1) = 100$$

The athlete reaches maximum speed v after time t_1:

$$v = at_1 \implies t_1 = \frac{v}{a} = \frac{14}{a}$$

Substitute $t_1 = \frac{14}{a}$ into the distance equation:

$$\frac{1}{2}a\left(\frac{14}{a}\right)^2 + 14\left(11 - \frac{14}{a}\right) = 100$$

$$\frac{1}{2} \cdot \frac{196}{a} + 154 - \frac{196}{a} = 100$$

$$\frac{196}{2a} - \frac{196}{a} = 100 - 154$$

$$\frac{196}{2a} - \frac{196}{a} = -54$$

$$\frac{196 - 392}{2a} = -54$$

$$\frac{-196}{2a} = -54$$

$$\frac{196}{2a} = 54$$

$$\frac{196}{108} = a$$

$$a \approx 1.81\,\mathrm{m/s^2}$$

The acceleration of the athlete is approximately $\boxed{1.81\,\mathrm{m/s^2}}$.