

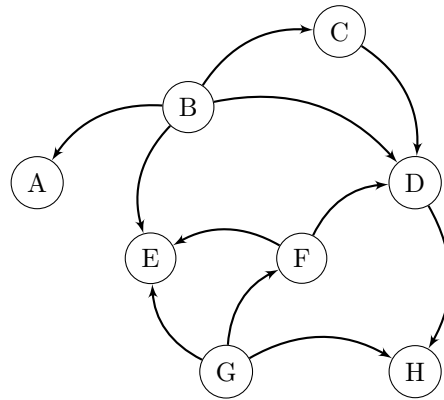
Problem set 11

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Week 11. Problem set

1. Write down all possible topological sortings for the nodes of the following directed graph:



Solution

1. B-A-C-G-F-D-E-H
2. B-A-C-G-F-D-H-E
3. B-A-C-G-F-E-D-H
4. B-A-G-C-F-D-E-H
5. B-A-G-C-F-D-H-E
6. B-A-G-C-F-E-D-H
7. B-A-G-F-C-D-E-H
8. B-A-G-F-C-D-H-E
9. B-A-G-F-C-E-D-H
10. B-A-G-F-E-C-D-H
11. B-C-A-G-F-D-E-H
12. B-C-A-G-F-D-H-E
13. B-C-A-G-F-E-D-H
14. B-C-G-A-F-D-E-H

15. B-C-G-A-F-D-H-E
16. B-C-G-A-F-E-D-H
17. B-C-G-F-A-D-E-H
18. B-C-G-F-A-D-H-E
19. B-C-G-F-A-E-D-H
20. B-C-G-F-D-A-E-H
21. B-C-G-F-D-A-H-E
22. B-C-G-F-D-E-A-H
23. B-C-G-F-D-E-H-A
24. B-C-G-F-D-H-A-E
25. B-C-G-F-D-H-E-A
26. B-C-G-F-E-A-D-H
27. B-C-G-F-E-D-A-H
28. B-C-G-F-E-D-H-A
29. B-G-A-C-F-D-E-H
30. B-G-A-C-F-D-H-E
31. B-G-A-C-F-E-D-H
32. B-G-A-F-C-D-E-H
33. B-G-A-F-C-D-H-E
34. B-G-A-F-C-E-D-H
35. B-G-A-F-E-C-D-H
36. B-G-C-A-F-D-E-H
37. B-G-C-A-F-D-H-E
38. B-G-C-A-F-E-D-H
39. B-G-C-F-A-D-E-H
40. B-G-C-F-A-D-H-E
41. B-G-C-F-A-E-D-H
42. B-G-C-F-D-A-E-H
43. B-G-C-F-D-A-H-E
44. B-G-C-F-D-E-A-H
45. B-G-C-F-D-E-H-A
46. B-G-C-F-D-H-A-E
47. B-G-C-F-D-H-E-A
48. B-G-C-F-E-A-D-H
49. B-G-C-F-E-D-A-H
50. B-G-C-F-E-D-H-A
51. B-G-F-A-C-D-E-H
52. B-G-F-A-C-D-H-E
53. B-G-F-A-C-E-D-H
54. B-G-F-A-E-C-D-H
55. B-G-F-C-A-D-E-H
56. B-G-F-C-A-D-H-E
57. B-G-F-C-A-E-D-H
58. B-G-F-C-D-A-E-H
59. B-G-F-C-D-A-H-E
60. B-G-F-C-D-E-A-H

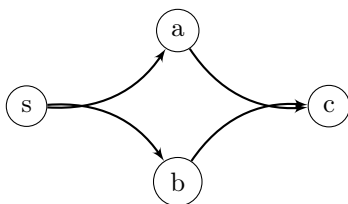
61. B-G-F-C-D-E-H-A
62. B-G-F-C-D-H-A-E
63. B-G-F-C-D-H-E-A
64. B-G-F-C-E-A-D-H
65. B-G-F-C-E-D-A-H
66. B-G-F-C-E-D-H-A
67. B-G-F-E-A-C-D-H
68. B-G-F-E-C-A-D-H
69. B-G-F-E-C-D-A-H
70. B-G-F-E-C-D-H-A
71. G-B-A-C-F-D-E-H
72. G-B-A-C-F-D-H-E
73. G-B-A-C-F-E-D-H
74. G-B-A-F-C-D-E-H
75. G-B-A-F-C-D-H-E
76. G-B-A-F-C-E-D-H
77. G-B-A-F-E-C-D-H
78. G-B-C-A-F-D-E-H
79. G-B-C-A-F-D-H-E
80. G-B-C-A-F-E-D-H
81. G-B-C-F-A-D-E-H
82. G-B-C-F-A-D-H-E
83. G-B-C-F-A-E-D-H
84. G-B-C-F-D-A-E-H
85. G-B-C-F-D-A-H-E
86. G-B-C-F-D-E-A-H
87. G-B-C-F-D-E-H-A
88. G-B-C-F-D-H-A-E
89. G-B-C-F-D-H-E-A
90. G-B-C-F-E-A-D-H
91. G-B-C-F-E-D-A-H
92. G-B-C-F-E-D-H-A
93. G-B-F-A-C-D-E-H
94. G-B-F-A-C-D-H-E
95. G-B-F-A-C-E-D-H
96. G-B-F-A-E-C-D-H
97. G-B-F-C-A-D-E-H
98. G-B-F-C-A-D-H-E
99. G-B-F-C-A-E-D-H
100. G-B-F-C-D-A-E-H
101. G-B-F-C-D-A-H-E
102. G-B-F-C-D-E-A-H
103. G-B-F-C-D-E-H-A
104. G-B-F-C-D-H-A-E
105. G-B-F-C-D-H-E-A
106. G-B-F-C-E-A-D-H

- 107. G-B-F-C-E-D-A-H
- 108. G-B-F-C-E-D-H-A
- 109. G-B-F-E-A-C-D-H
- 110. G-B-F-E-C-A-D-H
- 111. G-B-F-E-C-D-A-H
- 112. G-B-F-E-C-D-H-A
- 113. G-F-B-A-C-D-E-H
- 114. G-F-B-A-C-D-H-E
- 115. G-F-B-A-C-E-D-H
- 116. G-F-B-A-E-C-D-H
- 117. G-F-B-C-A-D-E-H
- 118. G-F-B-C-A-D-H-E
- 119. G-F-B-C-A-E-D-H
- 120. G-F-B-C-D-A-E-H
- 121. G-F-B-C-D-A-H-E
- 122. G-F-B-C-D-E-A-H
- 123. G-F-B-C-D-E-H-A
- 124. G-F-B-C-D-H-A-E
- 125. G-F-B-C-D-H-E-A
- 126. G-F-B-C-E-A-D-H
- 127. G-F-B-C-E-D-A-H
- 128. G-F-B-C-E-D-H-A
- 129. G-F-B-E-A-C-D-H
- 130. G-F-B-E-C-A-D-H
- 131. G-F-B-E-C-D-A-H
- 132. G-F-B-E-C-D-H-A

2. Give an example of a directed graph $G = (V, E)$, a source vertex s , and a set of edges $T \subseteq E$ such that
- T forms a tree and
 - for each vertex $v \in V$, the unique simple path in the graph (V, T) from s to v is a shortest path in G , yet
 - the set of edges T cannot be produced by running BFS on G , no matter how the vertices are ordered in the adjacency lists.

Solution

Consider the following directed graph $G = (V, E)$, where $V = \{s, a, b, c\}$ and $E = \{(s, a), (s, b), (a, c), (b, c)\}$.



The source vertex is s . The set of edges $T = \{(s, a), (a, c)\}$ forms a tree. For each vertex $v \in V$, the unique simple path in the graph (V, T) from s to v is a shortest path in G . However, the set of edges T cannot be produced by running BFS on G , no matter how the vertices are ordered in the adjacency lists. This is because BFS would always include the edge (s, b) in T before it includes (a, c) , since b is a direct neighbor of s and c is not.