1. Using the operators $\hat{a}$ and $\hat{a}^+$, calculate expectation values for the operators $\hat{x}^2, \hat{x}^4, \hat{x}^{2k+1}$ and $\hat{p}^2$ in the n-th stationary state of 1D quantum harmonic oscillator. Compute the variances of x and p for the n-th state. What is the minimal value that their product $\Delta x \cdot \Delta p$ can take?

2. Calculate the wave function $\psi_2(x)$ of the state $|2\rangle$ of 1D quantum harmonic oscillator acting by the operator a^+ on the ground-state wave function

$$\psi_0(x) = \frac{1}{\pi^{1/4}\sqrt{x_0}} e^{-\frac{x^2}{2x_0^2}}.$$

Write down $\psi_2(x)$ also using the general solution form

$$\psi_n(x) = \frac{1}{\sqrt{2^n \cdot n! \cdot x_0 \sqrt{\pi}}} H_n\left(\frac{x}{x_0}\right) e^{-\frac{x^2}{2x_0^2}}$$

and make sure you get the identical results.

3. Imagine QHO in the ground state. Characteristic length of the QHO $x_0 = \sqrt{\frac{\hbar}{m\omega}}$ has the meaning of the classical turning point, i.e. the point beyond which the classical particle cannot go (because in classical turning point the potential energy $U(x_0) = E_0 = \hbar\omega/2$)[1]. However, the behavior of quantum particle is different. Calculate the probability to find QHO outside the *classically allowed region* $-x_0 < x < x_0$.

[1] In other words, x_0 is the amplitude of classical oscillator with the energy $E = \hbar\omega/2$.