1. Prove that
$$[\hat{p}_x, f(\hat{x})] = -i\hbar\frac{\partial f}{\partial x},$$
$$[\hat{x}, f(\hat{p}_x)] = i\hbar\frac{\partial f}{\partial p_x},$$
where f is a smooth differentiable function.

2. Calculate the commutators: $[\hat{x}, \hat{p}_x^2]$, $[\hat{x}^2, \hat{p}_x]$, $[\hat{x}^2, \hat{p}_x^2]$. Adapt and use the results of the exercise above. To calculate the last commutator, also the following general property of commutators might be useful (to be proven in the next lesson):
$$[\hat{A}, \hat{B}\hat{C}] = \hat{B}[\hat{A}, \hat{C}] + [\hat{A}, \hat{B}]\hat{C}.$$

3. Find the explicit form of the operator $\exp\left(\frac{i}{\hbar}\hat{p}_x a\right)$, where a is a real number.