1. Find the eigenvalues of the Hermitian operator $\hat{F}$ such that

$$\hat{F}^3 = c^2\hat{F}, \quad c \in \mathbb{R}.$$

2. Find the eigenfunctions and eigenvalues of $f(\hat{F})$, where $\hat{F}$ is the Hermitian operator and f is a smooth function. Consider spectrum F_n and eigenfunctions ψ_n of $\hat{F}$ to be known.

3. Find the eigenvalues and eigenfunctions of the operator

$$\hat{F} = \sin\frac{d}{d\varphi},$$

where φ is a polar angle.

4. Stationary states (eigenstates) of a particle in 1D infinite well of the width a are described by the wave functions:

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}}\sin\left(\frac{\pi n}{a}x\right), & 0 < x < a \\ 0, & 0 \geq x \geq a \end{cases}$$

which correspond to the energy eigenvalues

$$E_n = \frac{\pi^2\hbar^2 n^2}{2ma^2}, \quad n \in \mathbb{N},$$

where m is the particle mass. Consider the particle in the state with the wave function

$$\Psi(x) = \frac{4}{\sqrt{5a}}\sin^3\left(\frac{\pi x}{a}\right).$$

Find the observed values of energy in this state, their probabilities, and the expectation value of the energy in this state.