

Parallel Computing Lab Assignment 1

In this lab you will write MPI code to generate the **prime numbers between 2 and N (inclusive)** and test scalability and performance.

General notes:

- The name of the source code file is: `genprimes.c`
- Write your program in such a way that to execute it I type: `./genprimes N`
Where `N` is a positive number bigger than 2 and less than or equal to 10,000,000 (10Millions).
- The output of your program is a text file `N.txt` (`N` is the number entered as argument to your program).
For example, if I type: `./genprimes 10`
The output must be a text file with the name `10.txt` and that file contains: `2 3 5 7`
You need to put a single space between each two prime numbers in the file.
- You can assume that we will not do any tricks with the input (i.e. We will not deliberately test your program with wrong values of `N`).
- As a way to help you, the following table contains the number of prime numbers below `x`.

<u>x</u>	<u>number of primes</u>
10	4
100	25
1,000	168
10,000	1,229
100,000	9,592
1,000,000	78,498
10,000,000	664,579

Even though there are several algorithms for generating prime numbers, some are even more efficient than this one, you will implement the following algorithm for generating prime numbers:

There are many algorithms for generating prime numbers and for primality testing. Some are more efficient than others. For this lab, we will implement the following algorithm. This is the sequential version.

Input: `N`

1. Generate all numbers from 2 to `N`.
2. First number is 2, so remove all numbers that are multiple of 2 (i.e. 4, 6, 8, ... `N`). Do not remove the 2 itself.
3. Following number is 3, so remove all multiple of 3 that have not been removed from the previous step. That will be: 9, 15, ... till you reach `N`.
4. The next number that has not been crossed so far is 5. So, remove all multiple of 5 that have not been crossed before, till you reach `N`.
5. Continue like this till $\text{floor}((N+1)/2)$.
6. The remaining numbers are the prime numbers.

Example:

Suppose $N = 20$

floor of $(20+1)/2 = 10$ ← where we stop.

Initially we have:

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Let's cross all multiple of 2 (but leave 2):

2, 3, 4, 5, 6, 7, 8, 9, ~~10~~, 11, ~~12~~, 13, ~~14~~, 15, ~~16~~, 17, ~~18~~, 19, ~~20~~

Next number is 3, so we cross all multiple of 3 that have not been crossed:

2, 3, 4, 5, 6, 7, 8, 9, ~~10~~, 11, ~~12~~, 13, ~~14~~, ~~15~~, ~~16~~, 17, ~~18~~, 19, ~~20~~

Next number that has not been crossed is 5, so we will cross multiple of 5 (i.e. 10, 15, and 20).
As you see below, they are all already crossed.

2, 3, 4, 5, 6, 7, 8, 9, ~~10~~, 11, ~~12~~, 13, ~~14~~, ~~15~~, ~~16~~, 17, ~~18~~, 19, ~~20~~

Next number that has not been crossed is 7, so we will cross multiple of 7 (i.e. 14). As you see below, they are all already crossed.

2, 3, 4, 5, 6, 7, 8, 9, ~~10~~, 11, ~~12~~, 13, ~~14~~, ~~15~~, ~~16~~, 17, ~~18~~, 19, ~~20~~

The next number that has not been crossed is 11. This is bigger than 10, so we stop here.

The numbers that have not been crossed are the prime numbers:

2, 3, 5, 7, 11, 13, 17, 19

The file that your program generates is 20.txt and looks like (only spaces between each two numbers, no commas):

2 3 5 7 11 13 17 19

Your MPI Implementation

- You need to design your program in such a way as to distribute equal amount of work, as much as possible, among the processes. If we have two processes, for example, then the first one will take care the first $(N-1)/2$ numbers. That is, to find the prime numbers in them. And the second process will take care of the second half. If we have P processes, then process 0 will take care of the first $(N-1)/P$, process 1 will take the following $(N-1)/P$, etc. Of course you need to take care of the case when $(N-1)$ is not divisible by P .
- Once each process gets its assigned numbers, it will apply the algorithms described above to get the prime numbers in its range. This is a bit tricky. So, read the next point.
- Suppose we have three processes and $N = 100$. Then we need to check for primality by check the remainder of the divisions by 2, then by 3, ... till $\text{floor}((100+1)/2) = 50$ as explained earlier. Each process will be responsible for $\text{ceil}((100+1)/3) = 34$ numbers. Remember that $34 \times 3 = 102$. So, the last process must do one less number (i.e. 33 instead of 34). Each process will check the remainder of the divisions by 2 to 50. You can optimize a little bit here because all the numbers below the divisor do not need to be divided. For example, any number below 30, for instance, does not need to be divided by 30. And all the numbers that have been crossed and removed from an earlier iteration do not need to be divided.
- Process 0 will be responsible of generating the file N.txt and writing the prime numbers there. So, you need to think a bit about how the processes will communicate to get their share of the problem and how process 0 will get the results.
- The prime numbers must be listed in N.txt in order.

How to measure the performance and scalability of your code?

To see how efficient your implementation is, you need to compare the performance with different number of processes.

Therefore, do the following:

- Prepare three tables similar to slides 35, 36, and 37 from lecture MPI-III
- So, one table for times (in seconds), one for speedup (relative to execution with one process), and the third is for efficiency (speedup/ #processes).
- In these tables, make $N = 1000, 10000, 100000, 1000000$, and 10000000
- The number of processes (i.e. `MPI_COMM_WORLD` size) = 1, 2, 5, 10, and 100

What do you have to submit:

A single zip file. The file name is your lastname.firstname.zip

Inside that zip file you need to have:

- genprimes.c
- pdf file containing the two tables

Submit the zip file through NYU classes.

Note: As a way to help you, we provide, on the course website, a list of the first 1 million prime numbers for you to check your implementation for correctness. This file is for checking your results, but it is not formatted in the correct format required for the file N.txt

Enjoy!