Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Lecture with Computer Exercises: Modelling and Simulating Social Systems

Project Report

Trade!t

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Contents

Abstract

The paper re-runs an agent based simulation done on artificial financial market by (Raberto & Cincotti, 2005). Agents are traders that supply stock in return for cash. The initial total amount is kept throughout the experiment without additional money creation. As a mechanism for price formation we considered the limit order book. The share price of a stock is determined through the ask and bid prices, which correspond to the lowest sell order and the highest buy order. Each order is stored in a limit order book as an entry. The order generation process is formed according to the parameters presented in the paper. Additionally, in this paper, a slight change has been made on how the price volatility influences the order placement of traders. This is performed through placing bounds on the standard deviation which characterizes how traders act. The findings reveal that, that model is robust enough to accommodate riskier assets and still is able to recreate important characteristics of financial markets.

Individual contributions

Alessandro Menichelli (A.M.), Vincent Wüst (V.W.) and Firehiwot Kedir (F.K.) conceptualized the problem and technical framework. A.M. V.W. and F.K. developed the model. A.M. and V.W. created and tested the algorithm. A.M., V.W., and F.K. wrote the manuscript.

Introduction and Motivations

The paper re-creates a real financial market using agents as per the research of (Raberto & Cincotti, 2005). Agents are given exact amounts of assets to trade. The total amount will be used throughout the trading process without additional money creation. This limits the decision making process of agents (Raberto, Cincotti, Focardi, & Marchesi, 2001). As stated by the authors, they have so far created artificial markets that resemble the real market situation.

Key feature presented in the research of (Raberto & Cincotti, 2005), comparing with other similar agent based artificial market simulation researches, is the formation of price. In previous researches price is formed used the concept of clearing house. In the clearing house mechanism buy and sell limit orders are accumulated through time and, using the intersection of demand and supply curves, the market is cleared. The equilibrium created by the intersection of demand and supply curves in economics satisfies both the supplier and consumer. The point of the intersection signifies the formation of an equilibrium price in which suppliers are willing to sell their goods and consumers are willing to buy.

On the contrary, the concept of limit order book fits the behavior of stock exchange for price formation (Raberto & Cincotti, 2005). The limit order book is a record of all unexecuted orders that did not meet the limit price. The limit order book has two entries; one for sell orders and the other one for buy orders. Traders announce their offers (sell and buy orders) and may accept other offers (sell and buy orders). Offers will be available as long as the market is open (end of the day), or until they're cancelled. A transaction happens when a trader manages to hit the bid or ask price.

The primary motivation of the paper is to mimic a real financial market using agents based simulation. Secondary motivation is to identify and indicate possible impediments in simulating the market. The paper addresses three main research questions of the authors. Firstly, what is the reproducibility potential of agent based financial market simulation executed in the research of (Raberto & Cincotti, 2005)? Secondly, what are the similarities and differences in the outputs of the simulations? Is improvement possible?

Description of the Model

In this section, the model used in the paper is explained. Agents represent traders that exchange stock for cash. Agents are given equal amounts of cash and stock at the very beginning of the model. Decisions made by agents are random and are restricted by the amount of cash and stock they possess. Such fundamental features make sure the experiment exhibits real trading features. To start the order creation, an i-th trader is randomly chosen in the model to issue a sell $s_i(t_h)$ or a buy $b_i(t_h)$ order with a probability of 50%. After each order the model waits an exponentially distributed waiting time $\Delta\tau_h$ with an average waiting time of τ_{avg} before choosing the next agent for the new order placement.

Limit Order Book (LOB):

The LOB is organized in a way that a registry of the agent ID, the sell/buy price, and a time stamp on when the order expires. Every unfulfilled sell and buy order is stored at time step t_h. The LOB can be empty in two instances: The first one is after the end of each trading day when the LOB is emptied. The second case is when all the orders have been matched. In such cases, a trader will use the last successful bid or ask price and the order will get stored in the book.

Table 1: Limit Order Book arrangement

Limit order Book					
ID of agent	Sell Price	Expiration time	Agent ID	Buy price	Expiration time
...	...	...	...	...	...

Price formation:

Steps taken by (Raberto & Cincotti, 2005) are duplicated in this paper as explained in the following text. Once the sell and buy orders are stored in the LOB, a limit price is needed to form a transaction. The LOB allows the sell and price formation. The decision whether a transaction occurs between traders is governed by the following set of rules. If the sell order $s_i(t_h)$ at time t_h is larger than the bid price $d(t_{h-1})$ at the previous time point t_{h-1}, no transaction occurs and the order gets stored in the LOB. Otherwise, a transaction occurs at the bid price $d(t_{h-1})$. If the buy order $b_i(t_h)$ at t_h is smaller than the ask price $a(t_{h-1})$ at t_{h-1}, no transaction occurs and again the order gets stored in the LOB. Otherwise, a transaction occurs at the ask price $a(t_{h-1})$.

Transaction occurrence:

The quantity that both buyers and sellers offer when creating an order is the amount of money they're willing to pay for one share (money per share ratio). A trade occurs, when the offer from the current trader matches to an order from the LOB. The actual amount of shares that the traders exchange is a random fraction of the minimum of the amount of shares that the seller can sell at most and the buyer can afford to buy at the money per share ratio. As shown in the figure below, the quoted spread represents the differences between ask price and bid price. It essentially represents the difference between the highest price a buyer is willing to purchase an asset and the lowest price a seller is willing to sell the asset for. The midprice is a simplistic form used to estimate a stock price in a financial market.

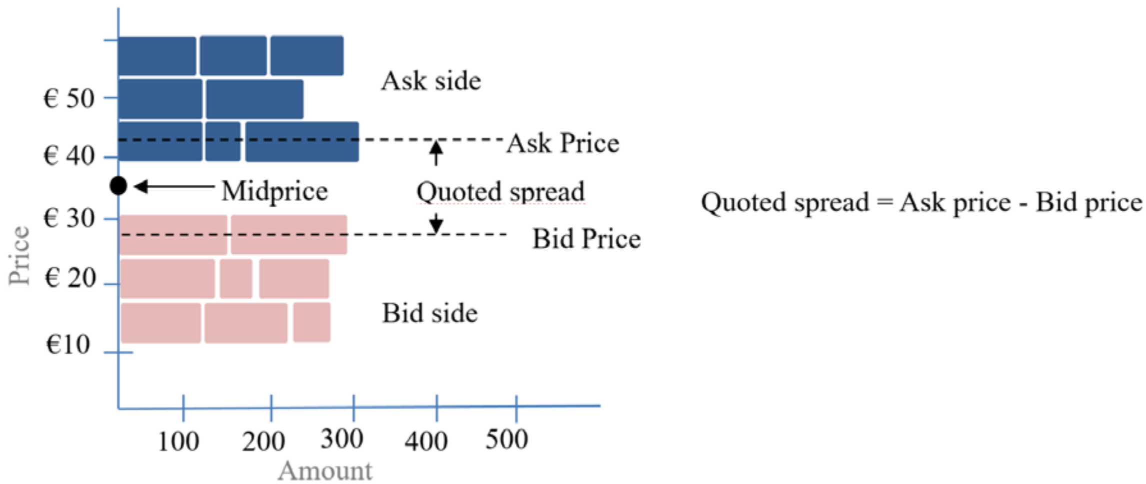

Figure 1: Quoted spread and midprice, adapted from (Pérez, n.d.)

The process used in modelling and simulating the financial market is summarized in figure 2. The concept used to produce the process diagram in figure 2 was taken from (Raberto & Cincotti, 2005). To experiment on the model presented in the previous section, time, agents, and stock parameters were used. Timing parameters: In the model adopted by (Raberto & Cincotti, 2005), an auction system where traders (i) – buyers and sellers – trade is divided into M daily sections and these sections in turn are further divided into T elementary time steps. The time between two consecutive orders placements is called an order waiting time expressed as $\Delta\tau_h = t_h - t_{h-1}$ and is exponentially distributed over time with mean λ^o.

Expiration time:

In the model, orders have a limited amount of time they can stay in the LOB. The limit is set to 600 seconds. Once an order enters the LOB, the exact time the order entered the book plus the 600 seconds limit amount make up its expiration time.

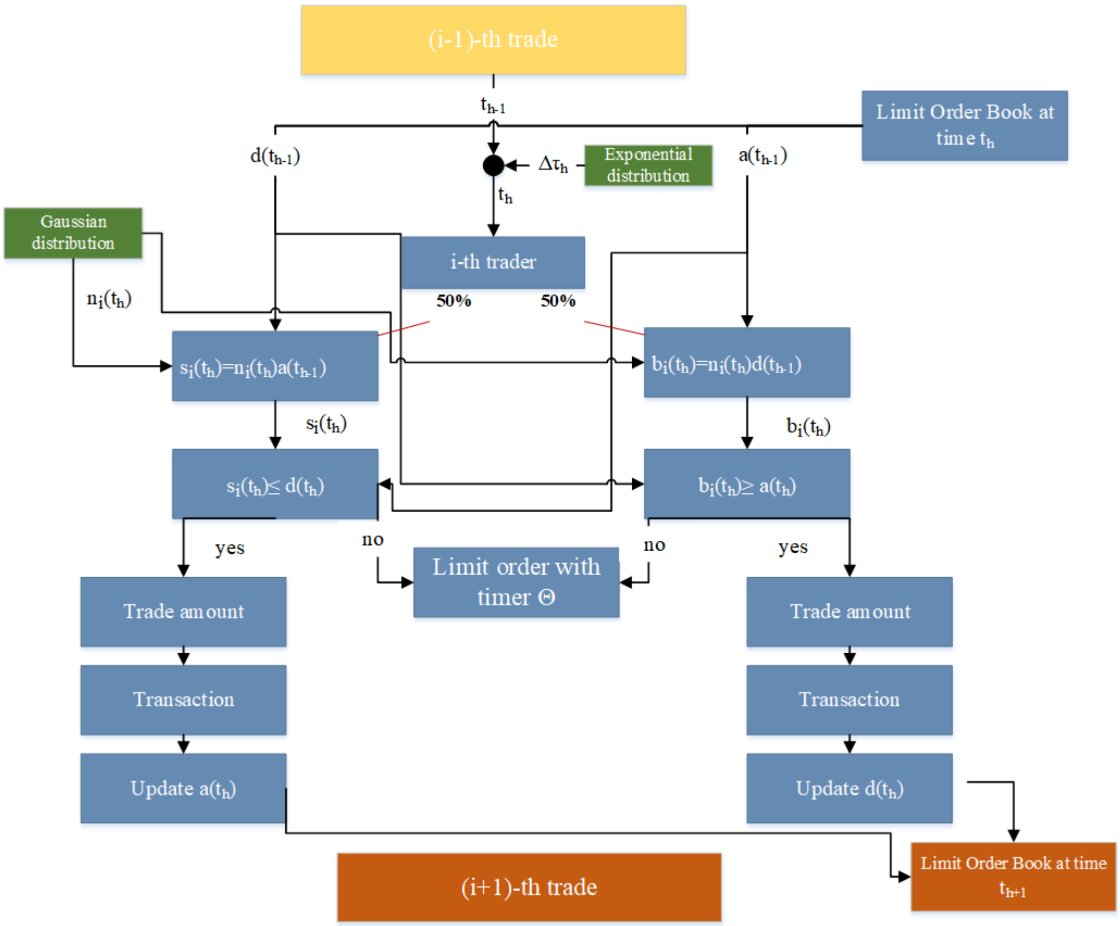

Figure 2: Process diagram of one order generation. One iteration starts after the (i-1)-th order generation shown in yellow. All processes influencing the current ith order generation step are represented by the color blue. The subsequent (i+1)-th order generation is represented by brown. Green represents functions that describe the type of distribution in time of the time steps they are applied to.

Implementation

For the implementation part we used similar parameters as in the paper (Raberto & Cincotti, 2005). We set the daily time steps to T = 25'200, the lifespan of an order in the LOB to Θ = 600, the average order waiting time to τ_{avg}= 20, the number of trading agents to 10'000 agents, the initial price per share to 100, the initial amount of cash of an agent to 100'000 and the initial amount of shares of an agent to 1000 shares. All this parameter values are the same as used in the paper (Raberto & Cincotti, 2005) and stay constant through all the simulations. Nevertheless, during the implementation of the model authors included additional contribution to the research of (Raberto & Cincotti, 2005). These include (1) adding limitation of the varying standard deviation. (2) Performing more experiments with different parameters to analyze the model and test its limits and behaviour.

The numbers of days has been set for certain simulations to M = 10 and for others to M = 100. The standard deviation has been computed in two different ways. The first type of simulations were done with a constant standard deviation of $\sigma = 0.005$ and $\sigma = 0.01$. Then it has been changed to a variating σ with a volatility feedback that makes the standard variation linearly dependent on the intra-day volatility of the stock. The equation for this relation is given as $\sigma = k \cdot \sigma_{T_i}$, where k is the proportionality constant and σ_{T_i} stands for the intra-day volatility of the stock. k will stay for most of the simulations at the constant value of 4.25. The standard deviation σ_{T_i} is the sample standard deviation of the log returns. The samples considered are all the ones within the last T_i time steps. The time T_i itself is a uniformly distributed random variable taking values between 600 and 6000 time steps. To assure a stable behaviour of the market the standard deviation has been constrained by $\sigma \in [0.001; 0.01]$ in most of the examples.

To better analyse the market behaviour in the next section "Simulation Results and Discussion" two graphs have been created. One plots the changing stock price, which is estimated by taking the average between the bid and ask price $p(t_h) = \frac{d(t_h) + a(t_h)}{2}$. Another visualizes the log returns $log\ return = ln\left(\frac{p(t_h)}{p(t_{h-1})}\right)$. The log return gives the amount of the change with respect to the past value. It's similar to the percentage but more precise for smaller changes due to the logarithmic behaviour, which makes it a good tool to visualize volatility. To deal with the inhomogeneous time windows for the order waiting times, similar to (Raberto & Cincotti, 2005), previous tick interpolation (with a time window of 60 seconds) has been used to generate homogeneous time intervals. This technique is suitable for the implementation of the model because the underlying time steps (seconds) are homogeneous.

To quantify how well our models describe the real world financial markets, we considered the distribution of the log returns. In the real world, commonly outliners are observed in the log return samples. This is referred to as the fat tails of the log return distribution. To get a feeling about the tails of the distribution one usually compares them to the tails of a normal distribution. This can be done with the Jarque-Bera test. If the null hypothesis of a normal distribution can be rejected at a significance level of 0.05, the behaviour of fat tails can be recovered.

To have an overview of the behaviour of the model different scenarios have been implemented. We can differ between simulation length and type of standard deviation (simple if constant, advanced if dependent). To check the stability of the system also simulations without limited standard deviation were implemented.

Simulation Results and Discussion

Results with constant standard deviation:

The influence of the standard deviation has been analyzed by plotting the results with the parameters M = 10 and in one case $\sigma = 0.005$ and in the other case $\sigma = 0.01$.

If the changes in stock price and log return of the two simulations (figures 3a, 3b and figures 4a, 4b) are directly compared it can been seen that the volatility of the price has a strong dependence on the standard deviation. Comparing the log-return graph it can be seen that the log return (and the volatility) in figure 4b is similar to double of the log-return of figure 3b.

Note: Plots with shorter simulation times (3a and 4a) or a smaller standard deviation (6a) are scaled differently than plots with longer simulation time (7a) or bigger standard deviation (5a and 8a) to visualize the behaviour better.

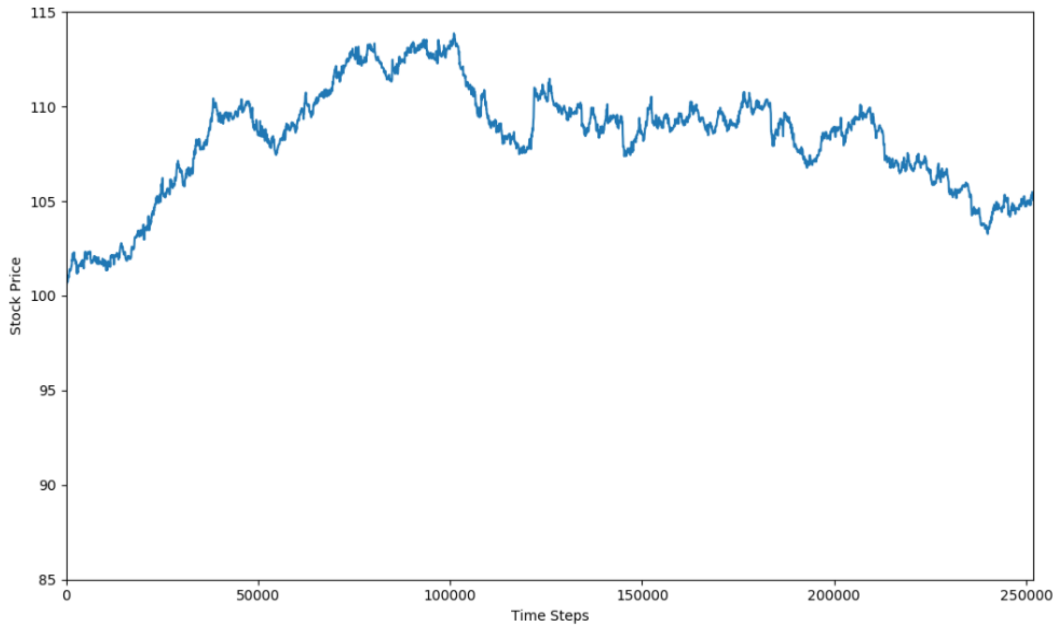

Figure 3a: Stock price variation during 10 days with $\sigma = 0.005$.

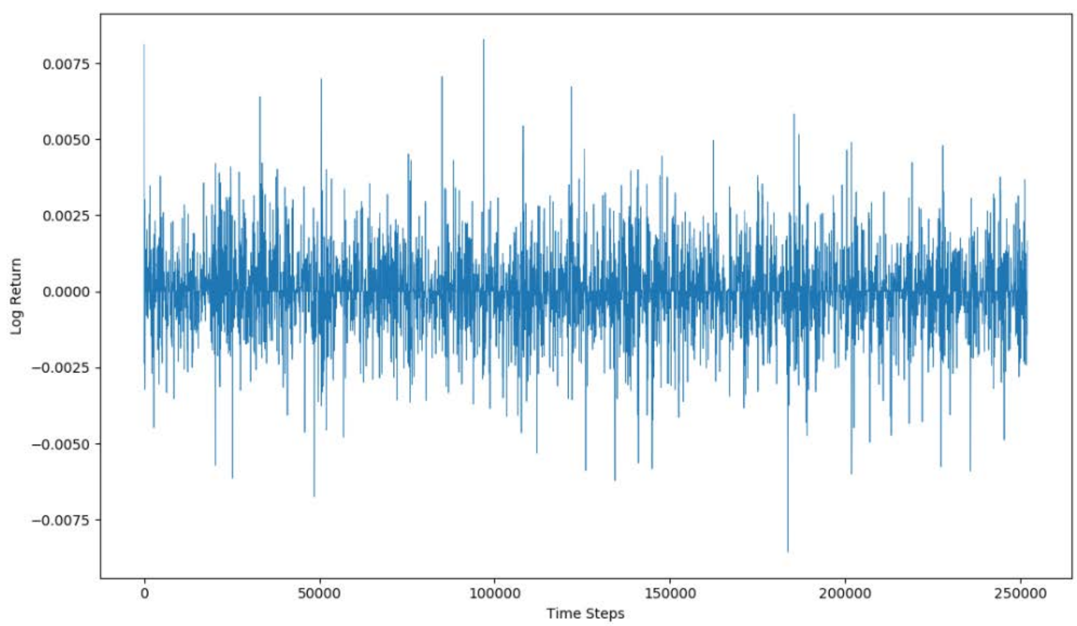

Figure 3b: Log return during 10 days with $\sigma = 0.005$.

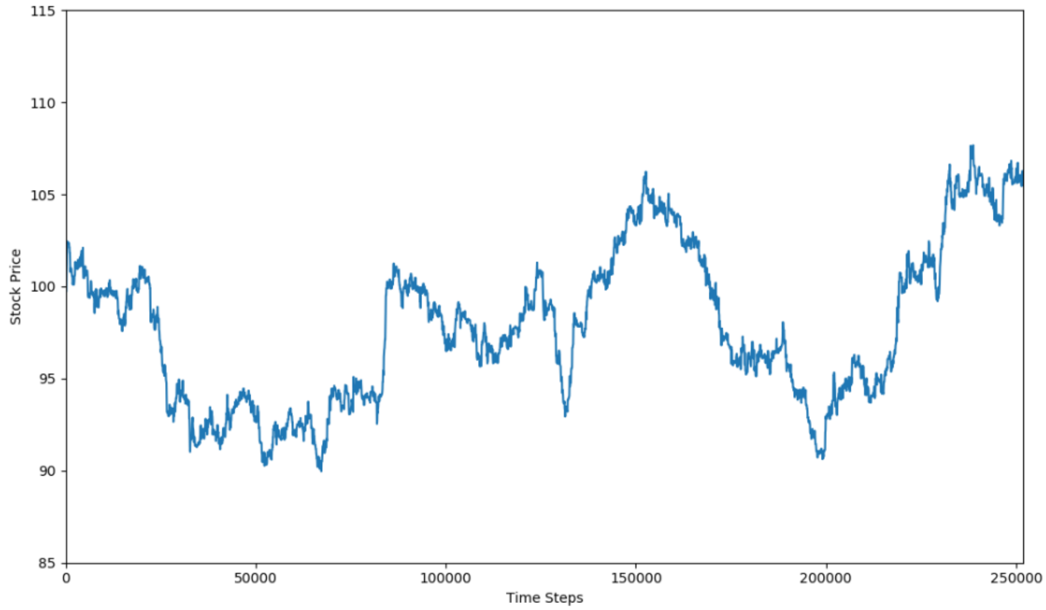

Figure 4a: Stock price variation during 10 days with $\sigma = 0.01$.

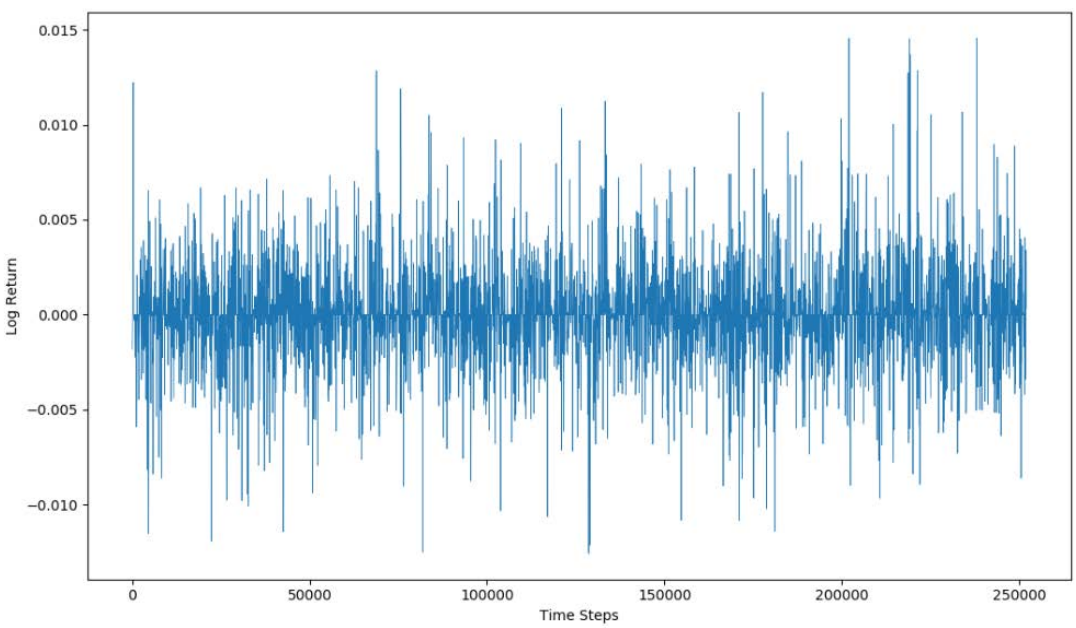

Figure 4b: Log return during 10 days with $\sigma = 0.01$.

To ensure that at high standard deviations the model stays stable, another simulation has been implemented with M = 100.

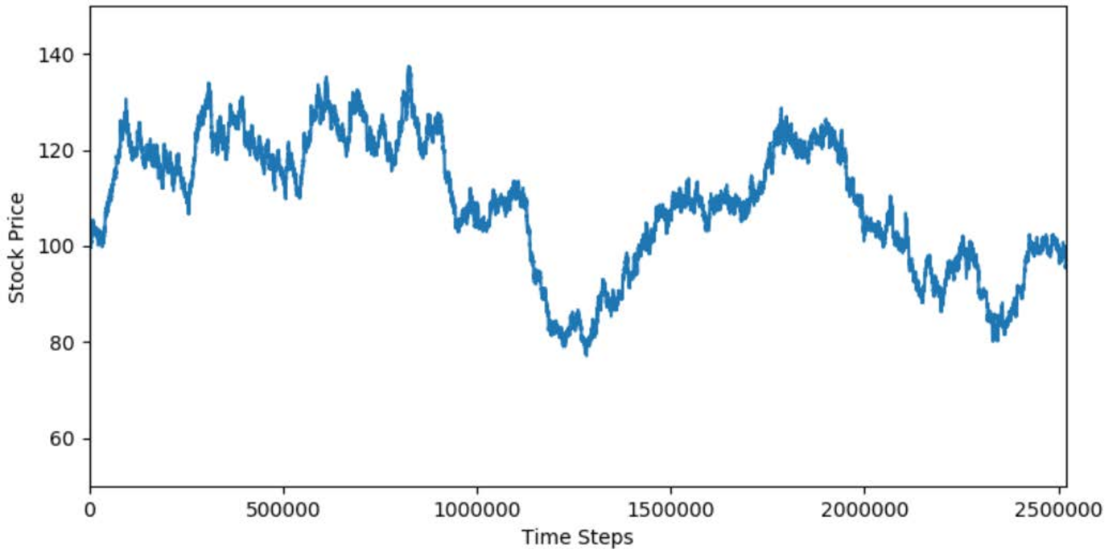

Figure 5a: Stock price variation during 100 days with $\sigma = 0.01$.

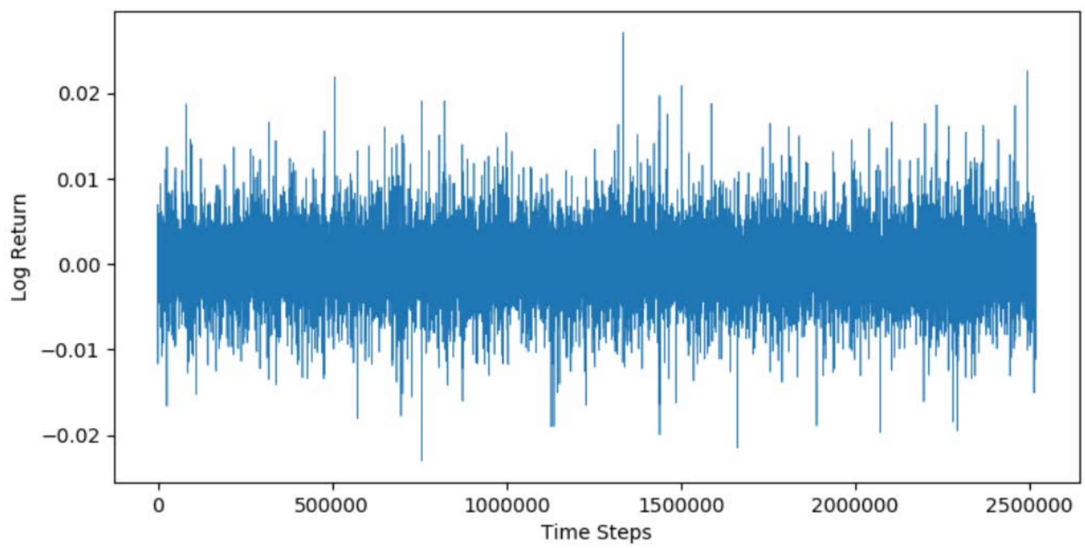

Figure 5b: Log return during 100 days with $\sigma = 0.01$.

Analyzing figure 5a one can see that although the volatility is high and the difference between the highest and lowest value is much bigger than in the past simulations, the system doesn’t reach a point where it goes to infinity or zero so it can be assumed as stable for at least the simulation period. In figure 5b the log return is high but follows a nearly straight line with little exceptions.

Regarding the distribution of the log returns for this model we used the Jarque-Bera test as explained in previous chapters. As in (Raberto & Cincotti, 2005) we can reject the null hypotheses of a normal distribution (due to the p-value being below 0.05) and recover with this the known fact of fat tails of the log return distribution in real world financial markets.

Results with variating standard deviation:

The first three simulations with the varying σ it is show that for a high constant k some limits for σ are needed to make sure that the system has a stable behaviour. Therefore the number of days is set to 10 and the value of k to 1.5 or 4.25.

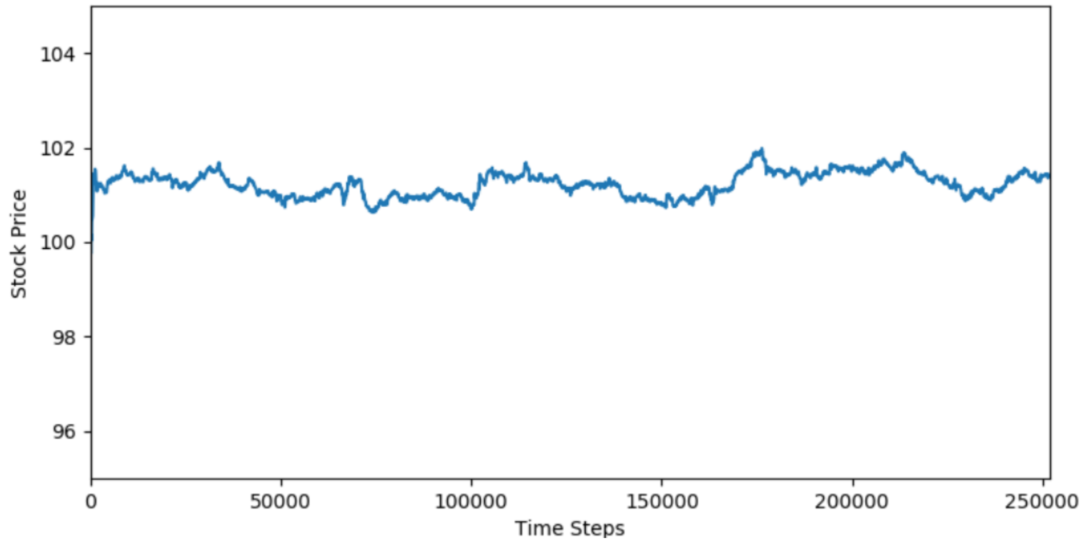

Figure 6a: Stock price variation during 10 days with $k = 1.5$ *and limitless* σ*. This plot is scaled smaller than the others because otherwise the fluctuations wouldn't be visible.*

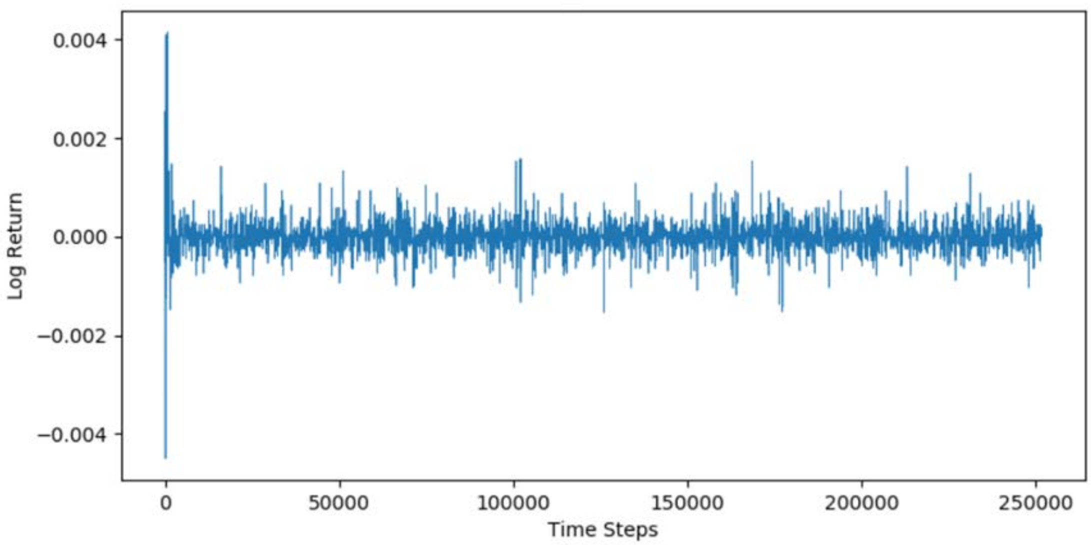

Figure 6b: Log return during 10 days with $k = 1.5$ *and limitless* σ*.*

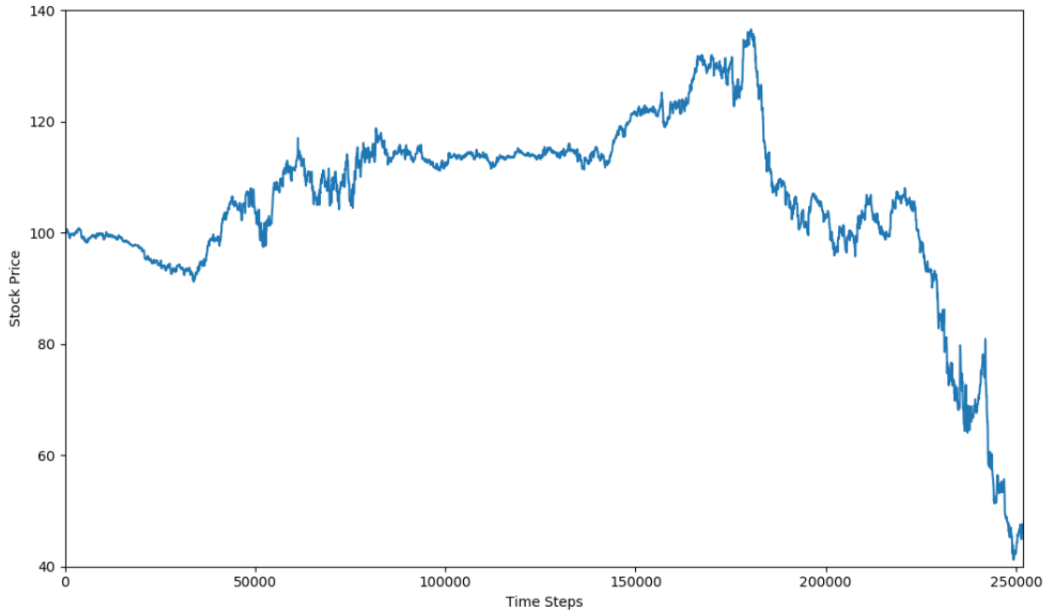

Figure 7a: Stock price variation during 10 days with $k = 4.25$ and limitless σ.

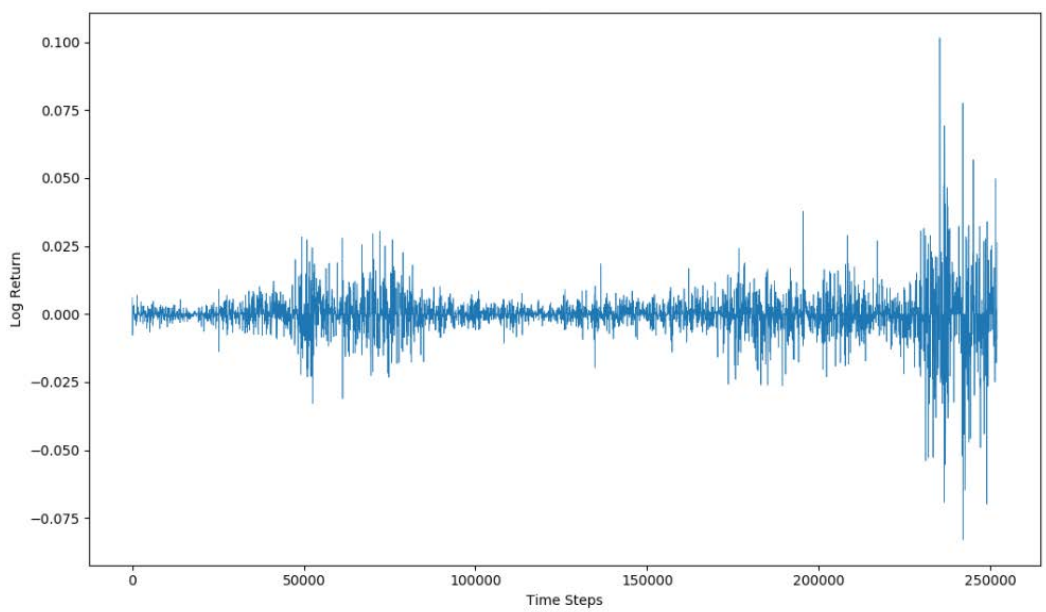

Figure 7b: Log return during 10 days with $k = 4.25$ and limitless σ.

Comparing figures 6a, 6b and figures 7a, 7b it is clear that a big constant brings a high price instability and therefore the need for boundaries. In figure 6b the volatility never gets that high to completely rapidly change the behaviour of the graph as one can see in figure 7b, where the volatility at the end of the simulation seems to “blow up” especially if one has a look at the values.

Plotting the price per share graph from a simulation with M = 100, $k = 4.25$ and $\sigma \in [0.001; 0.01]$ shows that the behaviour of the curve and the return has been stabilized by the limits.

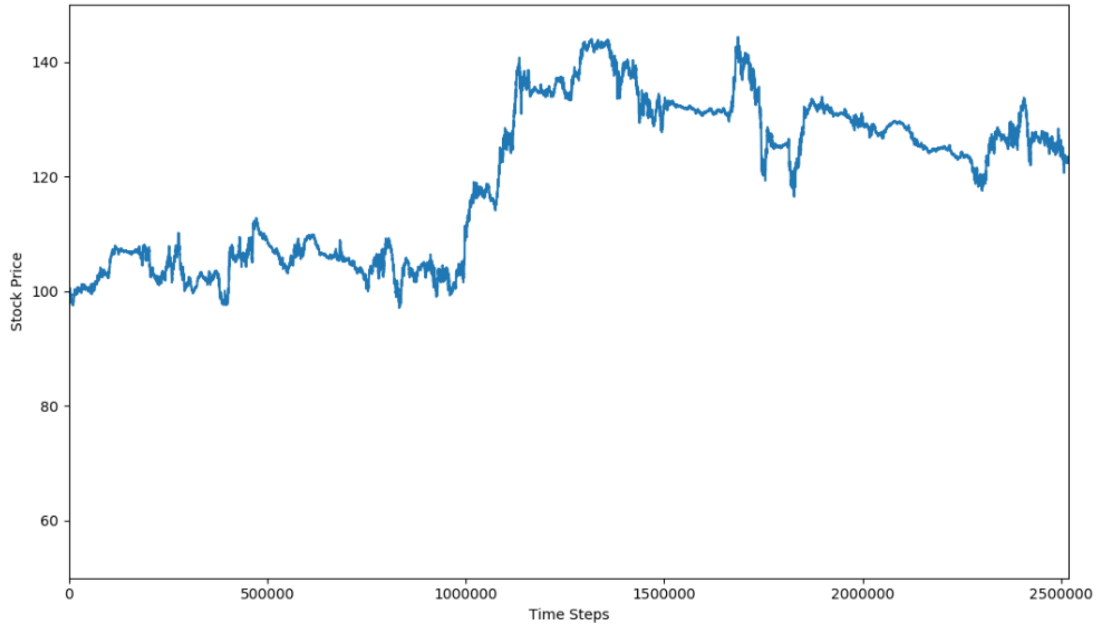

Figure 8a: Stock price variation during 100 days with $k = 4.25$ and $\sigma \in [0.001; 0.01]$.

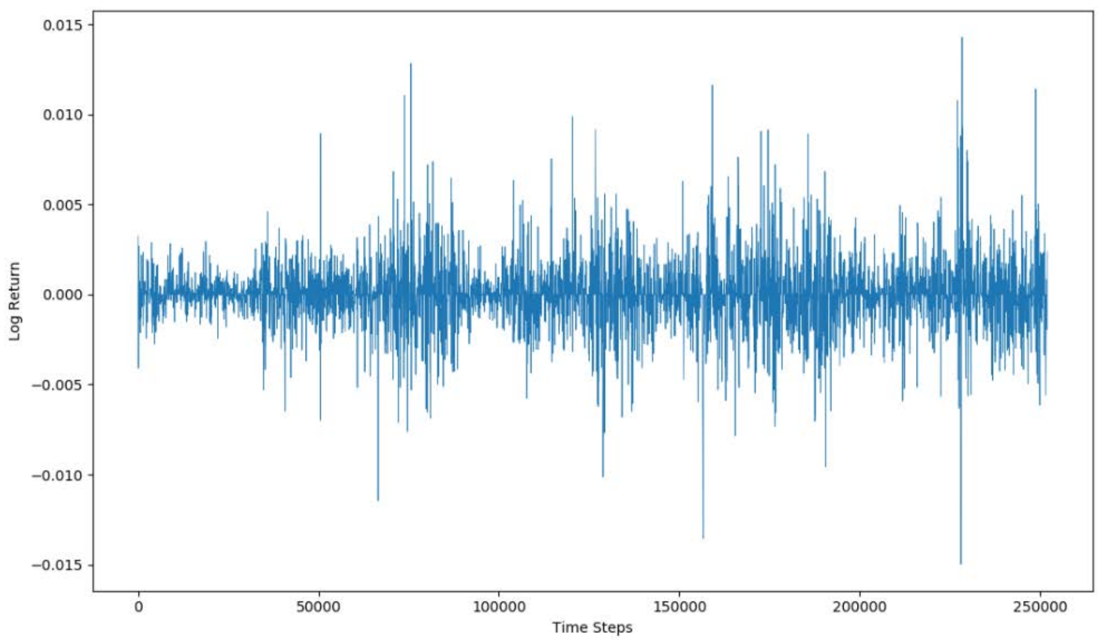

Figure 8b: Log return during 100 days with $k = 4.25$ and $\sigma \in [0.001; 0.01]$.

As for the first model we applied the Jarque-Bera test to see whether we can reject a null hypotheses of normal distribution. Similar to (Raberto & Cincotti, 2005) also for this model we can confirm that a normal distribution is very unlikely for the log returns due to a p-value lower than 0.05.

Summary and Outlook

Using the improved representation of the financial market simulated in (Raberto & Cincotti, 2005), the paper simulates an artificial financial market using agents in place of real traders and using the concept of limit order book to form transactions. After a section of simulations with different parameter it can be summarized that the standard deviation has a fundamental impact on the stock price behaviour and hence affects the financial market as whole. In the case of a constant σ the price stays stable but becomes volatile with a high constant. The results for a varying standard deviation indicate: for a small k the system stays bounded without significant uncertainties but for a higher k some boundaries for σ are needed in order to avoid big and rapid changes of the price path that lead to very high uncertainty and to an unstable behaviour of the system. Further computational experiments exhibit fat tails of the distribution of the log returns as as already stated and proven in the research of (Raberto & Cincotti, 2005). This shows that the model is a good approximation of the real financial market.

Although the real financial market are too complex to model, (Raberto & Cincotti, 2005) showed an improvement by including the LOB and price intraday volatility. In the case of the current research, we showed that the model performs well with riskier assets using further experiments. For further improvement of the model, (1) instead of taking the arithmetic average of the ask and bid price to produce the price path, taking a better method such as the micros price to estimate could help to quantify the stock price more accurately. (2) To make sure that longer simulations with a higher k and a limitless σ can be computed, the code written by the authors could be improved in a way that the bid price cannot reach the value zero.

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