# Competitive Programming Notebook

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## Contents



# 1 Graph

## 1.1 Prim MST

```cpp
// Status: tested (UVA10048)
// O(E log V) time, O(V+E) space

vector<vector<pair<int, int>> adj(M), mst(M);
vector<bool> taken(M, false);
int cost = 0;
using iii = pair<int, pair<int, int>>;
priority_queue<iii, vector<iii>, greater<iii> pq;

void process(int v) {
    taken[v] = true;
    for (auto &[w, u]: adj[v])
        if (!taken[u])
            pq.push({w, {v, u}});
}

void run(int n) {
    process(0);
    while (!pq.empty()) {
        int w = pq.top().first,
            v = pq.top().second.first,
            u = pq.top().second.second;
        pq.pop();
        if (!taken[u]) {
            mst_cost += w;
            mst[u].push_back({w, v});
            mst[v].push_back({v, w});
            process(u);
        }
    }
    for (int v = 1; v < n; ++v)
        if (!taken[v]) {
            process(v);
            run(n);
        }
}
```

## 1.2 Dijkstra SSSP

```cpp
// Status: tested (CF20C)
// O((V+E) log V) time, O(V^2) space

using ii = pair<int, int>;
const int inf = 0x3f3f3f3f;
vector<vector<ii>> adj(M);
vector<int> dist(M, inf), par(M, -1);

void dijkstra(int s) {
    dist[s] = 0;
    priority_queue<ii, vector<ii>,
        greater<pair<int, int>>> pq;
    pq.push(make_pair(0, s));
    while (!pq.empty()) {
        int w = pq.top().first;
```

```
        int v = pq.top().second;
        pq.pop();
        if (w > dist[v]) continue;
        for (auto &[d, u]: adj[v])
            if (dist[v] != inf && dist[v]+d <
                dist[u]) {
                par[u] = v;
                dist[u] = dist[v]+d;
                pq.push(make_pair(dist[u], u));
            }
    }
}
```

## 1.3 Graph Check

```
// Usage: graphCheck(firstVertex, -1) (p stands
    for parent)
// O(V+E) time & space

int UNVISITED = -1, EXPLORED = 0, VISITED = 1;
vector<vector<int>> adj(M);
vector<int> tin;

void graphCheck(int v, int p) { //vertex, parent
    tin[v] = EXPLORED;
    for (auto u: adj[v]) {
        if (tin[u] == UNVISITED) { //tree edge
            graphCheck(u, v);
        } else if (tin[u] == EXPLORED) {
            if (u == p)
                ; //two way edge u <-> v
            else
                ; //back edge v -> u
        } else if (tin[u] == VISITED) {
            ; //forward/cross edge u-v
        }
    }
    tin[v] = VISITED;
}
```

## 1.4 Articulations and Bridges

```
// Usage: dfs(source, -1)
// Status: not tested
// O(V+E) time & space

int tk = 0;
vector<int> tin(M, -1);
vector<vector<int>> adj(M);

void dfs(int v, int p) {
    tin[v] = low[v] = tk++;
    int children = 0;
    for (auto u: adj[v]) {
        if (u == p) continue;
        else if (tin[u] == -1) {
            ++children;
            dfs(u, v);
            if (low[u] >= tin[v] && p != v)
                ; //articulation point
            if (low[u] > tin[v])
                ; //bridge u-v
            low[v] = min(low[v], low[u]);
        } else {
            low[v] = min(low[v], tin[u]);
        }
    }
}
```

## 1.5 Euler Tour

```
// Usage: tour(cyc.begin(), start\_vertex)
// Status: not tested
// Source: CP3 (pg. 205)
// O(E^2) time

list<int> cyc;
vector<vector<int>> adj(M);
vector<vector<bool>> traversed(M, vector<bool>(M,
    false));

//euler tour (list for fast insertion)
void tour(list<int>::iterator i, int v) {
    for (auto u: adj[v]) {
        if (!traversed[v][u]) {
            traversed[v][u] = true;
            for (auto t: adj[u])
                if (t == v && !traversed[u][t]) {
                    traversed[u][t] = true;
                    break;
                }
            tour(cyc.insert(i, v), u);
        }
    }
}
```

## 1.6 Kahn's topological sort

```
// Status: tested (UVA11060)
// O(VE) time, O(V+E) space

vector<vector<int>> adj(M);
vector<int> sorted;

void kahn(int n) {
    vector<int> indeg(n, 0);
    vector<bool> valid(n, true);
    priority_queue<int> pq;

    for (int v = 0; v < n; ++v)
        for (auto u: adj[v])
            indeg[u]++;
    for (int v = 0; v < n; ++v)
        if (!indeg[v]) pq.push(v);

    while (!pq.empty()) {
        int v = pq.top(); pq.pop();
        sorted.push_back(v);
        valid[v] = false;
        for (auto u: adj[v])
            if (valid[u] && !(--indeg[u]))
                pq.push(u);
    }
}
```

## 1.7 Max Cardinality Bipartite Matching

```
// Status: not tested
// Source: CP3 (pg. 209)
// O(VE) time

vector<vector<int>> adj(M);
vector<int> match(M, -1);
vector<bool> visited(M);

bool augment(int left) { //match one on the left
    with one on the right
    if (visited[left]) return false;
    visited[left] = true;
    for (auto right: adj[left])
        if (match[right] == -1 ||
            augment(match[right])) {
            match[right] = left;
            return true;
        }
    return false;
}

//usage
//(mcbm = V iff there's at least one way to
    completely match both sides)
```

```
int mcbm = 0; //number of matched vertices
match.assign(M, -1);
for (int v = 0; v < ls; ++v) {//ls = size of the left set
    visited.assign(ls, false);
    mcbm += augment(v);
}
```

## 1.8 Lowest Common Ancestor

```
// Status: not tested
// O(N log N) time, O(N log N) space

//--- binary lifting
int n, l = ceil(log2(n));
vector<vector<int>> adj;
int tk = 0;
vector<int> tin(n), tout(n);
vector<vector<int>> up(n, vector<int>(l+1)); // ancestor

void dfs(int v, int p) { // run dfs(root, root) to initialize
    tin[v] = ++tk;
    up[v][0] = p;
    for (int i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];
    for (int u : adj[v])
        if (u != p)
            dfs(u, v);
    tout[v] = ++tk;
}

bool ancestor(int v, int u) { // v is ancestor of u
    return tin[v] <= tin[u] && tout[v] >= tout[u];
}

int lca(int v, int u) {
    if (ancestor(v, u)) return v;
    if (ancestor(u, v)) return u;
    for (int i = l; i >= 0; --i)
        if (!ancestor(up[v][i], u))
            v = up[v][i];
    return up[v][0];
}

//--- euler path
using ii = pair<int, int>;
vector<ii> t;
vector<int> idx(n);
int tk = 1;

void dfs(int v, int d) { // call with dfs(root, 0);
    for (auto u : adj[v]) {
        st.update(tk, {d, v});
        tk++;
        dfs(u, d+1);
    }
    idx[v] = tk;
    st.update(tk, {d, v});
    tk++;
}

int lca(int v, int u) {
    int l = idx[v], r = idx[u];
    return st.minquery(l, r).second; // .first is depth
}
```

## 1.9 Tarjan Strongly Connected Component

```
// Usage: Tarjan(N, adj)
// Status: tested (UVA247, UVA11838)
// O(V+E) time & space

vector<int> tin(M, -1), low(M, -1);
vector<vector<int> adj(M);
stack<int> S;
int tk = 0;

void dfs(int v) {
    low[v] = tin[v] = tk++;
    S.push(v);
    visited[v] = true;
    for (auto u: adj[v]) {
        if (tin[u] == -1)
            dfs(u);
        if (visited[u])
            low[v] = min(low[v], low[u]);
    }
    if (low[v] == tin[v])
        while (true) {
            int u = S.top(); S.pop(); visited[u] = false;
            if (u == v) break;
        }
}
```

## 1.10 Kosaraju SCC

```
// run kosaraju()
// tested:  cf103931M
// source: cp-algorithms
// O(V+E) time & space (2 dfs calls)

int n; // number of vertices
vector<vector<int>> adj(n), adj_rev(n);
vector<bool> used(n);
vector<int> order, component;

void dfs1(int v) {
  used[v] = true;
  for (auto u: adj[v])
    if (!used[u])
      dfs1(u);
  order.push_back(v);
}

void dfs2(int v) {
  used[v] = true;
  component.push_back(v);
  for (auto u: adj_rev[v])
    if (!used[u])
      dfs2(u);
}

void kosaraju() {
  for (int i = 0; i < n; ++i)
    if (!used[i]) dfs1(i);

  used.assign(n, false);
  reverse(order.begin(), order.end());

  for (auto v: order)
    if (!used[v]) {
      dfs2(v);
      // ...process vertices in component
      component.clear();
    }
}
```

## 1.11 Bellman-Ford SSSP

```
// Status: tested (UVA1112, UVA10449)
// O(VE) time, O(V+E) space
const int inf = 0x3f3f3f3f;
vector<vector<pair<int, int>>> adj(M);
vector<int> dist(M, inf);

void bellmanFord(int n) {
    for (int i = 0; i < n-1; ++i)
        for (int v = 0; v < n; ++v)
```

```
            for (auto &[u, w]: adj[v])
                if (dist[v] != inf)
                    dist[u] = min(dist[u],
                        dist[v]+w);
}

//check if there are negative cycles
bool cycle(int n) {
    bool ans = false;
    for (int v = 0; v < n; ++v)
        for (auto &[u, w]: v)
            ans |= dist[v] != inf && dist[u] >
                dist[v]+w;
}
```

## 1.12 Kruskal MST

```
// Usage: Kruskal(V, E, edges) (weighted edges)
// Status: tested (UVA1174)
// O(E log V) time, O(V+E) space

using iii = pair<int, pair<int, int>>; //weight,
    two vertices
vector<iii> edges;
UnionFind muf;

int kruskal() {
    int cost = 0;
    sort(edges.begin(), edges.end());
    for (auto a: edges) {
        int w = a.first;
        pair<int, int> e = a.second;
        if (!muf.isSameSet(e.first, e.second)) {
            cost += w;
            muf.unionSet(e.first, e.second);
        }
    }
    return cost;
}
```

## 1.13 Edmond Karp MaxFlow

```
// Status: tested (CSES1694, CSES1695)
// O(VE^2) time, O(V+E) space

vector<vector<int>> capacity(M, vector<int>(M,
    0)), adj(M);
vector<pair<int, int>> mc; //mincut edges

int bfs(int s, int t, vi &par) {
    fill(all(par), -1);
    par[s] = -2;
    queue<pair<int, int>> q; q.push({s, inf});
    while (!q.empty()) {
        int v = q.front().first,
            flow = q.front().second;
        q.pop();
        for (auto u: adj[v])
            if (par[u] == -1 && capacity[v][u]) {
                par[u] = v;
                int new_flow = min(flow,
                    capacity[v][u]);
                if (u == t) return new_flow;
                q.push({u, new_flow});
            }
    }
    return 0;
}

int maxflow(int s, int t) {
    int flow = 0;
    vi par(M);
    int new_flow;
    while ((new_flow = bfs(s, t, par))) {
        flow += new_flow;
        int v = t;
        while (v != s) {
            int p = par[v];
            capacity[p][v] -= new_flow;
            capacity[v][p] += new_flow;
            v = p;
        }
    }
    return flow;
}

void mincut(int s, int t) {
    maxflow(s, t);
    stack<int> st;
    vector<bool> visited(n, false);
    vector<pair<int, int>> ans;
    st.push(s); // changed from 0 to s
    while (!st.empty()) {
        int v = st.top(); st.pop();
        if (visited[v]) continue;
        visited[v] = true;
        for (auto u: adj[v])
            if (capacity[v][u] > 0)
                st.push(u);
            else
                ans.push_back({v, u});
    }
    mc.clear();
    for (auto &[v, u] : ans)
        if (!visited[u])
            mc.push_back({v, u});
}
```

## 1.14 Floyd Warshall APSP

```
// Usage: FloydWarshall(n, edges)
// Status: tested (UVA821, UVA1056)
// O(V^3 + E) time, O(V^2 + E) space

struct edge { int v, u, w; };
const int inf = 0x3f3f3f3f;
vector<vector<int>> weight(M, vector<int>(M, inf));
vector<edge> edges;

void floydWarshall(int n) {
    for (auto e: edges)
        weight[e.v][e.u] = e.w;
    for (int k = 0; k < n; ++k)
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                if (max(weight[i][k],
                    weight[k][j]) < inf)
                    weight[i][j] =
                        min(weight[i][j],
                        weight[i][k]+weight[k][j]);
}
```

# 2 Math

## 2.1 Sieve of Eratosthenes

```
// Status: not tested
// O(n log log n) time, O(n) space

bitset<11234567> pr;
vector<int> factors(M, 0);
vector<int> primes;

void sieve(int n) {
    pr.set();
    for (int i = 2; i*i <= n; ++i)
        if (pr[i]) { //factors[i] == 0
            primes.push_back(i);
            for (int p = i*i; p <= n; p += i) {
                pr[p] = false;
                factors[p]++;
            }
```

```
        }
}

// O(1) for small n, O(sieve_size) else
bool isPrime(int n) {
    int sieve_size = 11234567;
    if (n <= sieve_size) return pr[n];
    for (auto p: primes) // only works if n <=
        primes.back()^2
        if (!(n%p)) return false;
    return true;
}
```

## 2.2 Prime Factors w/ Optimized Trial Divisions

```
// Status: not tested
// Source: CP3 (pg. 238)
// O(pi(sqrt(n))) time, O(n) space

vector<int> primes;
vector<pair<int, int>> factors;

void pf(int n) {
    for (auto p: primes) {
        if (p*p > n) break;
        int i = 0;
        while (!(n%p)) {
            n /= p;
            i++;
        }
        factors.push_back({p, i});
    }
    if (n != 1) factors.push_back({n, 1});
}
```

## 2.3 Extended Euclid for solving Linear Diophantine Equations

```
// Status: not tested
// Source: CP3 (pg. 242)
// O(log min(a, b)) time

int x, y, d;
void extendedEuclid(int a, int b) {
    if (b == 0) { x = 1; y = 0; d = a; return; }
    extendedEuclid(b, a%b);
    int x1 = y;
    int y1 = x - (a/b)*y;
    x = x1;
    y = x1;
}

void solve(int a, int b, int c, int i) { //i
    solutions
    extendedEuclid(a, b);
    if (d%c) return;
    x *= c/d;
    y *= c/d;
    do {
        cout << x << ", " << y << '\n';
        x += b/d;
        y -= a/d;
    } while (--i);
}
```

## 2.4 Floyd's algorithm cycle-finding

```
// Status: not tested
// Source: CPHB (p. 156)
// O(V) time

int findCycle(int x) {
    int a, b;
    a = succ(x);
    b = succ(succ(x));
    while (a != b) {
        a = succ(a);
        b = succ(succ(b));
    }
    a = x;
    while (a != b) {
        a = succ(a);
        b = succ(b);
    }
    int first = a; // first element in cycle
    b = succ(a);
    int length = 1;
    while (a != b) {
        b = succ(b);
        length++;
    }
}
```

# 3 Paradigm

## 3.1 Coordinate Compression

```
// Status: not tested
// Source: GEMA ICMC (YouTube)
// O(N log N) time

vector<int> v, vals, cv; // all of the same size,
    cv = compressed v
vals = v;
sort(vals.begin(), vals.end());
vals.erase(unique(vals.begin(), vals.end()),
    vals.end());
for (int i = 0; i < n; ++i) {
    int idx = lower_bound(vals.begin(),
        vals.end(), v[i]) - vals.begin();
    cv[i] = idx;
}
```

## 3.2 128 Bit Integers

```
// Status: not tested
// Source: GEMA (YouTube)

// cout, cerr, etc; pode dar over/underflow
ostream& operator<<(ostream& out, __int128 x) {
    if (x == 0) return out << 0;
    string s; bool sig = x < 0; x = x < 0 ? -x : x;
    while(x > 0) s += x % 10 + '0', x /= 10;
    if (sig) s += '-';
    reverse(s.begin(), s.end());
    return out << s;
}

// cin, etc; pode dar over/underflow
istream& operator>>(istream& in, __int128& x) {
    char c, neg = 0; while(isspace(c = in.get()));
    if(!isdigit(c)) neg = (c == '-'), x = 0;
    else x = c - '0';
    while(isdigit(c = in.get())) x = (x << 3) + (x
        << 1) - '0' + c;
    x = neg ? -x : x; return in;
}
```

## 3.3 Binary Search (but beautiful)

```
// Status: not tested
// Source: CPHB
// O(log N) time

// std
int l = 0, r = n-1;
while (l <= r) {
```

```
    int m = l+(r-l)/2;
    if (array[m] == x)
        // found
    if (array[m] > x) r = m-1;
    else l = m+1;
}

// nice - binary steps
int k = 0;
for (int b = n/2; b > 0; b /= 2)
    while (k+b < n && array[k+b] <= x)
        k += b;
if (array[k] == x)
    // found
```

# 4 String

## 4.1 Prefix Function (KMP)

```
// Status: not tested
// Source: CP-Algorithms
// O(N) time

vector<int> prefix(string s) {
    int n = s.length();
    vector<int> pi(n, 0); // can be optimized if
        you know max prefix length
    for (int i = 1; i < n; ++i) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}
```

# 5 Structure

## 5.1 Merge/Disjoint Union-Find

```
// Usage: UnionFind(N);
// Status: tested (UVA11503)
// O(Ackermann * N) time, O(N) space

struct UnionFind {
    int N;
    vi par, rk, count;

    UnionFind(int N) : N(N), par(N), rk(N, 0),
        count(N, 1) {
        rep(i, 0, N) par[i] = i;
    }

    int findSet(int i) {
        return par[i] == i ? i : (par[i] =
            findSet(par[i]));
    }

    int unionSet(int a, int b) {
        int x = findSet(a), y = findSet(b);
        if (x != y)
            count[x] = count[y] =
                (count[x]+count[y]);
        if (rk[x] < rk[y])
            par[x] = y;
        else {
            par[y] = x;
            if (rk[x] == rk[y])
                rk[x]++;
        }
        return count[x];
    }

    bool isSameSet(int i, int j) {
        return findSet(i) == findSet(j);
    }
};
```

## 5.2 Bottom-Up Segment Tree

```
// Usage: SegTree(N);
// Source: CP Handbook
// Status: not tested
// Complexity:
//  build: O(n)
//  query: O(log n)
//  modify: O(log n)
//  + uses less space than top-down 4n segtree (2n
    here)

struct SegTree {
  unsigned int n;
  vector<int> tree;

  SegTree(vector<int> v) : n(v.size()), tree(2*n) {
    for (int i = 0; i < n; ++i)
      modify(i, v[i]);
  }

  int query(int a, int b) {
    a += n, b += n;
    int ans = 0;
    while (a <= b) {
      if (a%2 == 1) ans += tree[a++];
      if (b%2 == 0) ans += tree[b--];
      a >>= 1; b >>= 1;
    }
    return ans;
  }

  void modify(int k, int x) {
    k += n;
    tree[k] += x;
    for (k /= 2; k >= 1; k /= 2)
      tree[k] = tree[k<<1] + tree[(k<<1) + 1];
  }
};
```

## 5.3 Segment Tree

```
// Usage: SegTree(N)
// Complexity:
//  build: O(n)
//  query: O(n)
//  modify: O(n)

struct SegTree {
    int N;
    vi st, A;

    SegTree(int N): N(N), st(4*n), A(n) {
        init();
    }

    void init() { build(1, 0, n-1); }

    int left(int i) { return i*2; }
    int right(int i) { return i*2+1; }

    void build(int v, int tl, int tr) {
        if (tl == tr) st[v] = a[tl];
        else {
            int tm = (tl+tr)/2;
            build(left(v), tl, tm);
            build(right(v), tm+1, tr);
            st[v] = max(st[left(v)], st[right(v)]);
        }
    }
```

```
29
30      int maxquery(int v, int tl, int tr, int l, int
            r) {
31          if (l > r) return -1;
32          if (l == tl && r == tr) return st[v];
33          int tm = (tl+tr)/2;
34          int q1 = maxquery(left(v), tl, tm, l,
                min(r, tm));
35          int q2 = maxquery(right(v), tm+1, tr,
                max(l, tm+1), r);
36          return max(q1, q2);
37      }
38
39      int maxquery(int l, int r) {
40          return maxquery(1, 0, n-1, l-1, r-1);
41      }
42
43      void update(int v, int tl, int tr, int p, int
            new_val) {
44          if (tl == tr) st[v] = new_val;
45          else {
46              int tm = (tl+tr)/2;
47              if (p <= tm)
48                  update(left(v), tl, tm, p,
                        new_val);
49              else
50                  update(right(v), tm+1, tr, p,
                        new_val);
51              st[v] = max(st[left(v)], st[right(v)]);
52          }
53      }
54
55      void update(int p, int new_val) {
56          update(1, 0, n-1, p-1, new_val);
57      }
58  };
```

# 6 Extra

## 6.1 C++ structs

```
1  struct example {
2    vector<int> a;
3    vector<bool> b = vector<bool>(5); // default
         value
4    int i;
5    example(int _i) : a(_i), i(_i) {};
6    bool operator< (example& e) { return i < e.i; }
7  }
8
9  example e = example(3);
10 example f(3);
```

## 6.2 Bashrc

```
1  xmodmap -e 'clear lock' -e 'keycode 66=Escape' #
       caps -> esc
2  alias e=vim
3
4  BASE_CP="/home/raul/cp2022"
5
6  alias c='g++ -Wall -Wconversion -Wfatal-errors -g
       -O2 -std=gnu++17 -fsanitize=undefined,address'
7  alias c14='g++ -Wall -Wconversion -Wfatal-errors
       -g -O2 -std=gnu++14
       -fsanitize=undefined,address'
8  alias p3='pypy3 -m py_compile'
9
10 tp () {
11      [ -f "$1.cpp" ] && echo "$1.cpp already
            exists";
12      [ ! -f "$1.cpp" ] && tail -n +2
            $BASE_CP/code/extra/template.cpp > $1.cpp
            && vim $1.cpp;
13 }
```

```
14
15 clip () {
16      if [ -f "$1" ];
17      then
18          cat $1 | clip.exe;
19      else
20          echo "$1 not found"
21      fi
22 }
```

## 6.3 Vim

```
1  set et ts=2 sw=2 ai si cindent sta
2  set is tm=50 nu noeb sm "cul
3  sy on
```

## 6.4 Generator

```
1  #include <bits/stdc++.h>
2  using namepsace std;
3
4  int main(int argc, char *argv[]) {
5    cin.tie(0); ios_base::sync_with_stdio(0);
6    if (argc < 2) {
7      cout << "usage: " << argv[0] << " <seed>\n";
8      exit(1);
9    }
10   srand(atoi(argv[1]));
11   // use rand() for random value
12 }
```

## 6.5 C++ Template

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  using ll = long long;
4
5  int main() {
6      ios_base::sync_with_stdio(0);
7      cin.tie(0);
8  }
```

## 6.6 Stress

```
1  for (( I=0; I < 5; I++ )); do
2    ./gen $I > a.in
3    ./brute < a.in > expected.txt
4    ./a.out < a.in > output.txt
5    if diff -u expected.txt output.txt; then : ; else
6      echo "--> input:"; cat a.in
7      echo "--> expected output:"; cat expected.txt
8      echo "--> received output:"; cat output.txt
9      break
10   fi
11   echo -n .
12 done
```