Competitive Programming Notebook

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17

if (w > dist[v]) continue:

for (auto &[d, u]: adj[v])

6.4

6.5

C++ Template

1.3 Graph Check

```
1 // Usage: graphCheck(firstVertex, -1) (p stands
       for parent)
_{2} // _{0} (V+E) time & space
4 int UNVISITED = -1, EXPLORED = 0, VISITED = 1;
5 vector < vector < int >> adj(M);
6 vector<int> tin;
8 void graphCheck(int v, int p) { //vertex, parent
       tin[v] = EXPLORED;
       for (auto u: adj[v]) {
10
           if (tin[u] == UNVISITED) { //tree edge
               graphCheck(u, v);
12
           } else if (tin[u] == EXPLORED) {
               if (u == p)
                   ; //two way edge u <-> v
15
           ; //back edge v -> u
} else if (tin[u] == VISITED) {
17
               ; //forward/cross edge u-v
20
21
       tin[v] = VISITED;
23 }
```

1.4 Articulations and Bridges

```
1 // Usage: dfs(source, -1)
2 // Status: not tested
3 // O(V+E) time & space
5 int tk = 0;
6 vector < int > tin(M, -1);
7 vector < vector < int >> adj(M);
9 void dfs(int v, int p) {
      tin[v] = low[v] = tk++;
10
11
      int children = 0;
      for (auto u: adj[v]) {
12
           if (u == p) continue;
1.3
           else if (tin[u] == -1) {
               ++children;
15
               dfs(u, v);
17
               if (low[u] >= tin[v] && p != v)
                   ; //articulation point
18
               if (low[u] > tin[v])
                   ; //bridge u-v
               low[v] = min(low[v], low[u]);
21
          } else {
23
               low[v] = min(low[v], tin[u]);
24
```

1.5 Euler Tour

```
1 // Usage: tour(cyc.begin(), start\_vertex)
2 // Status: not tested
3 // Source: CP3 (pg. 205)
4 // O(E^2) time
5
6 list<int> cyc;
7 vector<vector<int>> adj(M);
8 vector<vector<bool>> traversed(M, vector<bool>(M, false));
```

```
10 //euler tour (list for fast insertion)
11 void tour(list<int>::iterator i, int v) {
      for (auto u: adj[v]) {
          if (!traversed[v][u]) {
               traversed[v][u] = true;
14
               for (auto t: adj[u])
15
                   if (t == v && !traversed[u][t]) {
                       traversed[u][t] = true;
17
               tour(cyc.insert(i, v), u);
          }
21
      }
22
23 }
```

1.6 Kahn's topological sort

```
1 // Status: tested (UVA11060)
_{2} // O(VE) time, O(V+E) space
4 vector < vector < int >> adj(M);
5 vector < int > sorted;
7 void kahn(int n) {
      vector<int> indeg(n, 0);
      vector < bool > valid(n, true);
      priority_queue < int > pq;
10
      for (int v = 0; v < n; ++v)
          for (auto u: adj[v])
13
               indeg[u]++;
      for (int v = 0; v < n; ++v)
15
           if (!indeg[v]) pq.push(v);
16
      while (!pq.empty()) {
18
19
           int v = pq.top(); pq.pop();
           sorted.push_back(v);
           valid[v] = false;
21
           for (auto u: adj[v])
               if (valid[u] && !(--indeg[u]))
24
                   pq.push(u);
25
26 }
```

1.7 Max Cardinality Bipartite Matching

```
1 // Status: not tested
2 // Source: CP3 (pg. 209)
3 // O(VE) time
5 vector < vector < int >> adj(M);
6 vector < int > match(M, -1);
7 vector < bool > visited(M);
9 bool augment(int left) { //match one on the left
      with one on the right
      if (visited[left]) return false;
      visited[left] = true;
      for (auto right: adj[left])
12
           if (match[right] == -1 ||
13
              augment(match[right])) {
14
               match[right] = left;
               return true;
16
17
       return false;
18 }
20 //usage
21 //(mcbm = V iff there's at least one way to
      completely match both sides)
22 int mcbm = 0; //number of matched vertices
23 match.assign(M, -1);
24 for (int v = 0; v < ls; ++v) {//ls = size of the
      left set
```

```
visited.assign(ls, false);
complete mcbm += augment(v);
}
```

1.8 Lowest Common Ancestor

```
1 // Status: not tested
_{2} // O(N log N) time, O(N log N) space
4 //--- binary lifting
5 int n, l = ceil(log2(n));
6 vector < vector < int >> adj;
7 int tk = 0;
8 vector < int > tin(n), tout(n);
9 vector < vector < int >> up(n, vector < int > (1+1)); //
11 void dfs(int v, int p) { // run dfs(root, root) to}
      initialize
       tin[v] = ++tk:
12
       up[v][0] = p;
for (int i = 1; i <= 1; ++i)
13
14
          up[v][i] = up[up[v][i-1]][i-1];
15
       for (int u : adj[v])
        if (u != p)
17
1.8
                dfs(u, v);
       tout[v] = ++tk;
19
20 }
22 bool ancestor(int v, int u) { // v is ancestor of u
      return tin[v] <= tin[u] && tout[v] >= tout[u];
23
25
26 int lca(int v, int u) {
       if (ancestor(v, u)) return v;
       if (ancestor(u, v)) return u;
for (int i = 1; i >= 0; --i)
28
29
           if (!ancestor(up[v][i], u))
               v = up[v][i];
31
32
       return up[v][0];
33 }
34
35 //--- euler path
36 using ii = pair<int, int>;
37 vector<ii> t;
38 vector < int > idx(n);
39 int tk = 1:
41 void dfs(int v, int d) { // call with dfs(root, 0);
      for (auto u : adj[v]) {
42
           st.update(tk, {d, v});
           tk++;
44
45
           dfs(u, d+1);
46
       idx[v] = tk:
47
48
       st.update(tk, {d, v});
       tk++;
49
50 }
52 int lca(int v, int u) {
       int 1 = idx[v], r = idx[u];
       return st.minquery(1, r).second; // .first is
           depth
```

1.9 Tarjan Strongly Connected Component

```
1 // Usage: Tarjan(N, adj)
2 // Status: tested (UVA247, UVA11838)
3 // O(V+E) time & space
4
5 vector<int> tin(M, -1), low(M, -1);
6 vector<vector<int> adj(M);
7 stack<int> S;
8 int tk = 0;
```

55 }

```
10 void dfs(int v) {
      low[v] = tin[v] = tk++;
      S.push(v);
12
      visited[v] = true;
      for (auto u: adj[v]) {
14
          if (tin[u] == -1)
15
              dfs(u);
          if (visited[u])
               low[v] = min(low[v], low[u]);
      if (low[v] == tin[v])
20
          while (true) {
              int u = S.top(); S.pop(); visited[u] =
                  false;
               if (u == v) break;
          }
25 }
```

1.10 Kosaraju SCC

```
1 // Usage: Tarjan(N, adj)
2 // not tested
3 // O(V+E) time & space (2 dfs calls)
5 int n; // number of vertices
6 vector < vector < int >> adj(n), adj_rev(n);
7 vector < bool > used(n);
8 vector < int > order, component;
10 void dfs1(int v) {
   used[v] = true;
    for (auto u: adj[v])
     if (!used[u])
13
        dfs1(u):
    order.push_back(v);
15
16 }
18 void dfs2(int v) {
19
   used[v] = true;
   component.push_back(v);
    for (auto u: adj_rev[v])
      if (!used[u])
        dfs2(u):
24 }
26 void kosaraju() {
    for (int i = 0; i < n; ++i)
      if (!used[i]) dfs1(i);
    used.assign(n, false);
    reverse(order.begin(), order.end());
32
    for (auto v: order)
     if (!used[v]) {
34
35
        dfs2(v);
        // ...process vertices in component
37
        component.clear();
38
```

1.11 Bellman-Ford SSSP

1.12 Kruskal MST

```
1 // Usage: Kruskal(V, E, edges) (weighted edges)
2 // Status: tested (UVA1174)
3 // O(E log V) time, O(V+E) space
5 using iii = pair<int, pair<int, int>>; //weight,
       two vertices
6 vector<iii> edges;
7 UnionFind muf;
9 int kruskal() {
      int cost = 0;
10
11
       sort(edges.begin(), edges.end());
       for (auto a: edges) {
13
           int w = a.first;
           pair<int, int> e = a.second;
14
15
           if (!muf.isSameSet(e.first, e.second)) {
16
               cost += w:
               muf.unionSet(e.first, e.second);
18
19
      }
20
       return cost;
21 }
```

1.13 Edmond Karp MaxFlow

```
1 // Status: tested (CSES1694, CSES1695)
_{2} // _{0} (VE_{2}) time, _{0} (V+E) space
4 vector < vector < int >> capacity(M, vector < int > (M,
      0)), adj(M);
5 vector < pair < int , int >> mc; //mincut edges
7 int bfs(int s, int t, vi &par) {
       fill(all(par), -1);
9
       par[s] = -2;
       queue < pair < int , int >> q; q.push({s, inf});
10
       while (!q.empty()) {
11
           int v = q.front().first,
12
                    flow = q.front().second;
13
           q.pop();
15
           for (auto u: adj[v])
               if (par[u] == -1 && capacity[v][u]) {
16
                    par[u] = v;
                    int new_flow = min(flow,
18
                        capacity[v][u]);
                    if (u == t) return new_flow;
                    q.push({u, new_flow});
20
               }
21
       return 0:
23
24 }
25
26 int maxflow(int s, int t) {
       int flow = 0;
27
       vi par(M);
28
       int new_flow;
29
       while ((new_flow = bfs(s, t, par))) {
30
           flow += new_flow;
31
32
           int v = t;
           while (v != s) {
33
               int p = par[v];
34
               capacity[p][v] -= new_flow;
               capacity[v][p] += new_flow;
36
37
               v = p;
           }
```

```
return flow;
41 }
42
43 void mincut(int s, int t) {
      maxflow(s, t);
44
      stack < int > st;
45
      vector < bool > visited(n, false);
      vector<pair<int, int>> ans;
47
       st.push(s); // changed from 0 to s
      while (!st.empty()) {
49
           int v = st.top(); st.pop();
51
           if (visited[v]) continue;
           visited[v] = true;
52
           for (auto u: adj[v])
54
               if (capacity[v][u] > 0)
55
                   st.push(u);
                   ans.push_back({v, u});
57
58
      mc.clear();
      for (auto &[v, u] : ans)
60
61
           if (!visited[u])
               mc.push_back({v, u});
63 }
```

1.14 Floyd Warshall APSP

```
1 // Usage: FloydWarshall(n, edges)
2 // Status: tested (UVA821, UVA1056)
3 // O(V^3 + E) time, O(V^2 + E) space
5 struct edge { int v, u, w; };
6 const int inf = 0x3f3f3f3f;
7 vector < vector < int >> weight(M, vector < int > (M, inf));
8 vector < edge > edges;
10 void floydWarshall(int n) {
11
      for (auto e: edges)
           weight[e.v][e.u] = e.w;
12
      for (int k = 0; k < n; ++k)
13
           for (int i = 0; i < n; ++i)</pre>
               for (int j = 0; j < n; ++j)
                   if (max(weight[i][k],
16
                       weight[k][j]) < inf)
                        weight[i][j] =
17
                            min(weight[i][j],
                            weight[i][k]+weight[k][j]);
18 }
```

2 Math

2.1 Sieve of Eratosthenes

```
1 // Status: not tested
 2 // O(n log log n) time, <math>O(n) space
 4 bitset <11234567> pr;
 5 vector<int> factors(M, 0);
 6 vector < int > primes;
 8 void sieve(int n) {
        pr.set();
        for (int i = 2; i*i <= n; ++i)
            if (pr[i]) { //factors[i] == 0
11
                 primes.push_back(i);
                 for (int p = i*i; p <= n; p += i) {
    pr[p] = false;</pre>
 13
14
15
                     factors[p]++;
                 }
16
            }
17
18 }
20 // O(1) for small n, O(sieve_size) else
21 bool isPrime(int n) {
```

2.2 Prime Factors w/ Optimized Trial Divisions

```
1 // Status: not tested
2 // Source: CP3 (pg. 238)
3 // O(pi(sqrt(n))) time, O(n) space
5 vector < int > primes;
6 vector<pair<int, int>> factors;
8 void pf(int n) {
       for (auto p: primes) {
          if (p*p > n) break;
10
           int i = 0;
11
           while (!(n%p)) {
12
              n /= p;
15
          factors.push_back({p, i});
17
       if (n != 1) factors.push_back({n, 1});
18
```

2.3 Extended Euclid for solving Linear Diophantine Equations

```
1 // Status: not tested
2 // Source: CP3 (pg. 242)
3 // O(\log min(a, b)) time
5 int x, y, d;
6 void extendedEuclid(int a, int b) {
      if (b == 0) { x = 1; y = 0; d = a; return; }
      extendedEuclid(b, a%b);
      int x1 = y;
      int y1 = x - (a/b)*y;
10
      x = x1;
      y = x1;
12
13 }
15 void solve(int a, int b, int c, int i) { //i
      solutions
      extendedEuclid(a, b);
      if (d%c) return;
17
      x *= c/d;
      y *= c/d;
19
20
      do {
          cout << x << ", " << y << '\n';
21
          x += b/d;
22
           y = a/d;
23
      } while (--i);
24
25 }
```

2.4 Floyd's algorithm cycle-finding

```
1 // Status: not tested
2 // Source: CPHB (p. 156)
3 // O(V) time
4
5 int findCycle(int x) {
6    int a, b;
7    a = succ(x);
8    b = succ(succ(x));
9    while (a != b) {
10        a = succ(a);
11        b = succ(succ(b));
```

```
a = x;
      while (a != b) {
14
15
          a = succ(a);
          b = succ(b);
17
      int first = a; // first element in cycle
      b = succ(a);
      int length = 1;
      while (a != b) {
          b = succ(b);
23
          length++;
24
25 }
```

3 Paradigm

3.1 Coordinate Compression

3.2 128 Bit Integers

```
1 // Status: not tested
2 // Source: GEMA (YouTube)
4 // cout, cerr, etc; pode dar over/underflow
5 ostream& operator << (ostream& out, __int128 x) {</pre>
      if (x == 0) return out << 0;
      string s; bool sig = x < 0; x = x < 0 ? -x : x;
      while(x > 0) s += x % 10 + '0', x /= 10;
      if (sig) s += '-';
      reverse(s.begin(), s.end());
10
      return out << s;
12 }
14 // cin, etc; pode dar over/underflow
istream& operator>>(istream& in, __int128& x) {
      char c, neg = 0; while(isspace(c = in.get()));
      if(!isdigit(c)) neg = (c == '-'), x = 0;
      else x = c - '0';
      while(isdigit(c = in.get())) x = (x \ll 3) + (x
19
         << 1) - '0' + c;
20
      x = neg ? -x : x; return in;
21 }
```

3.3 Binary Search (but beautiful)

```
13 }
14
15 // nice - binary steps
16 int k = 0;
17 for (int b = n/2; b > 0; b /= 2)
18     while (k+b < n && array[k+b] <= x)
19         k += b;
20 if (array[k] == x)
21     // found</pre>
```

4 String

4.1 Prefix Function (KMP)

```
1 // Status: not tested
2 // Source: CP-Algorithms
3 // O(N) time
5 vector<int> prefix(string s) {
      int n = s.length();
       vector<int> pi(n, 0); // can be optimized if
           you know max prefix length
       for (int i = 1; i < n; ++i) {
           int j = pi[i-1];
           while (j > 0 && s[i] != s[j])
10
11
                j = pi[j-1];
           if (s[i] == s[j])
13
               j++;
14
           pi[i] = j;
15
       return pi;
16
17 }
```

5 Structure

5.1 Merge/Disjoint Union-Find

```
1 // Usage: UnionFind(N);
_{2} // Status: tested (UVA11503)
3 // O(Ackermann * N) time, O(N) space
5 struct UnionFind {
      int N;
       vi par, rk, count;
       {\tt UnionFind(int\ N)\ :\ N(N),\ par(N),\ rk(N,\ 0),}
           count(N, 1) {
           rep(i, 0, N) par[i] = i;
10
11
12
      int findSet(int i) {
13
           return par[i] == i ? i : (par[i] =
               findSet(par[i]));
15
       int unionSet(int a, int b) {
17
           int x = findSet(a), y = findSet(b);
1.8
           if (x != y)
19
               count[x] = count[y] =
20
                   (count[x]+count[y]);
           if (rk[x] < rk[y])</pre>
21
               par[x] = y;
22
           else {
               par[y] = x;
24
               if (rk[x] == rk[y])
26
                   rk[x]++;
           }
27
28
           return count[x];
29
30
       bool isSameSet(int i, int j) {
31
           return findSet(i) == findSet(j);
32
33
34 };
```

5.2 Bottom-Up Segment Tree

```
1 // Usage: SegTree(N);
2 // Source: CP Handbook
3 // Status: not tested
4 // Complexity:
5 // build: O(n)
6 // query: O(log n)
7 // modify: O(log n)
8 // + uses less space than top-down 4n segtree (2n
      here)
10 struct SegTree {
    unsigned int n;
    vector<int> tree;
    SegTree(vector<int> v) : n(v.size()), tree(2*n) {
     for (int i = 0; i < n; ++i)
15
        modify(i, v[i]);
18
    int query(int a, int b) {
    a += n, b += n;
20
      int ans = 0;
      while (a <= b) {
        if (a%2 == 1) ans += tree[a++];
23
        if (b\%2 == 0) ans += tree[b--];
        a >>= 1; b >>= 1;
26
27
      return ans;
29
    void modify(int k, int x) {
     k += n;
31
      tree[k] += x;
      for (k /= 2; k >= 1; k /= 2)
         tree[k] = tree[k<<1] + tree[(k<<1) + 1];</pre>
34
36 };
```

5.3 Segment Tree

```
1 // Usage: SegTree(N)
2 // Complexity:
3 // build: O(n)
4 // query: O(n)
5 // modify: O(n)
7 struct SegTree {
      int N;
      vi st, A;
10
      SegTree(int N): N(N), st(4*n), A(n) {
          init();
13
      void init() { build(1, 0, n-1); }
15
16
      int left(int i) { return i*2; }
      int right(int i) { return i*2+1; }
18
19
      void build(int v, int tl, int tr) {
          if (tl == tr) st[v] = a[tl];
21
          else {
22
              int tm = (tl+tr)/2;
              build(left(v), tl, tm);
               build(right(v), tm+1, tr);
               st[v] = max(st[left(v)], st[right(v)]);
          }
27
29
      int maxquery(int v, int t1, int tr, int 1, int
          if (1 > r) return -1;
          if (1 == tl && r == tr) return st[v];
33
          int tm = (t1+tr)/2;
          int q1 = maxquery(left(v), tl, tm, l,
34
              min(r, tm));
```

```
int q2 = maxquery(right(v), tm+1, tr,
              max(1, tm+1), r);
           return max(q1, q2);
36
37
38
       int maxquery(int 1, int r) {
39
40
           return maxquery(1, 0, n-1, 1-1, r-1);
41
42
       void update(int v, int tl, int tr, int p, int
           new val) {
           if (tl == tr) st[v] = new_val;
44
45
           else {
               int tm = (tl+tr)/2:
46
               if (p <= tm)
                   update(left(v), tl, tm, p,
48
                       new_val);
                   update(right(v), tm+1, tr, p,
50
                       new_val);
               st[v] = max(st[left(v)], st[right(v)]);
           }
52
53
      }
54
       void update(int p, int new_val) {
5.5
           update(1, 0, n-1, p-1, new_val);
56
57
58 };
```

6 Extra

6.1 Bashrc

```
1 xmodmap -e 'clear lock' -e 'keycode 66=Escape' #
      caps -> esc
2 alias e=vim
4 BASE_CP="/home/raul/cp2022"
6 alias c='g++ -Wall -Wconversion -Wfatal-errors -g
      -02 -std=gnu++17 -fsanitize=undefined, address;
7 alias c14='g++ -Wall -Wconversion -Wfatal-errors
      -g - 02 - std = gnu + + 14
      -fsanitize=undefined, address,
8 alias p3='pypy3 -m py_compile'
10 tp () {
       [ -f "$1.cpp" ] && echo "$1.cpp already
          exists";
       [ ! -f "$1.cpp" ] && tail -n +2
          $BASE_CP/code/extra/template.cpp > $1.cpp
           && vim $1.cpp;
13 }
14
15 clip () {
      .
if [ -f "$1" ];
16
          cat $1 | clip.exe;
18
19
       else
           echo "$1 not found"
20
21
      fi
22 }
```

6.2 Vim

```
1 set et ts=2 sw=2 ai si cindent sta
2 set is tm=50 nu noeb sm "cul
3 sy on
```

6.3 Generator

```
1 #include <bits/stdc++.h>
2 using namepsace std;
3
```

```
4 int main(int argc, char *argv[]) {
5    cin.tie(0); ios_base::sync_with_stdio(0);
6    if (argc < 2) {
7        cout << "usage: " << argv[0] << " <seed>\n";
8        exit(1);
9    }
10    srand(atoi(argv[1]));
11    // use rand() for random value
12 }
```

6.4 C++ Template

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 using l1 = long long;
4
5 int main() {
6    ios_base::sync_with_stdio(0);
7    cin.tie(0);
8 }
```

6.5 Stress