

# Competitive Programming Notes

Raul Almeida

## Contents

<b>1</b>	<b>Tables</b>	<b>1</b>
<b>2</b>	<b>Algorithms</b>	<b>1</b>
2.1	Graph . . . . .	1
2.1.1	Articulations and Bridges . . . . .	1
2.1.2	Edmond Karp MaxFlow . . . . .	1
2.1.3	Euler Tour . . . . .	2
2.1.4	Floyd Warshall . . . . .	2
2.1.5	Kahn's Topological Sort . . . . .	2
2.1.6	Kruskal . . . . .	2
2.1.7	Lowest Common Ancestor . . . . .	2
2.1.8	Max Cardinality Bipartite Matching . . . . .	2
2.1.9	Prim's Algorithm . . . . .	2
2.1.10	Tarjan . . . . .	2
2.1.11	Kosaraju . . . . .	2
2.2	Math . . . . .	2
2.2.1	Floyd . . . . .	2
2.2.2	Combination . . . . .	2
2.2.3	Permutation . . . . .	2
2.3	Paradigm . . . . .	2
2.3.1	Coordinate Compression . . . . .	2
2.3.2	128 Bit Integers . . . . .	3
2.4	String . . . . .	3
2.4.1	Prefix Function (KMP) . . . . .	3

## 1 Tables

$n$	not-TLE	Example
$\leq [10..11]$	$\mathcal{O}(n!), \mathcal{O}(n^6)$	Enumerate permutations
$\leq [15..18]$	$\mathcal{O}(2^n n^2)$	TSP with DP
$\leq [18..22]$	$\mathcal{O}(2^n n)$	Bitmask DP
$\leq 100$	$\mathcal{O}(n^4)$	3D DP with $\mathcal{O}(n)$ loop
$\leq 400$	$\mathcal{O}(n^3)$	Floyd-Warshall
$\leq 2 \cdot 10^3$	$\mathcal{O}(n^2 \log n)$	2 nested loops + tree query
$\leq 5 \cdot 10^4$	$\mathcal{O}(n^2)$	Bubble/Selection/Insertion Sort
$\leq 10^5$	$\mathcal{O}(n \log^2 n)$	Build suffix array
$\leq 10^6$	$\mathcal{O}(n \log n)$	Merge Sort
$\leq 10^7$	$\mathcal{O}(n \log \log n)$	Totient function
$\leq 10^8$	$\mathcal{O}(n)$	Mathy solution often with IO bottleneck ( $n \leq 10^9$ )

10<sup>8</sup> operations per second

Sign	Type	Bits	Max	Digits
$\pm$	char	8	127	2
+	char	8	255	2
$\pm$	short	16	32 767	4
+	short	16	65 535	4
$\pm$	int/long	32	$2 \cdot 10^9$	9
+	int/long	32	$4 \cdot 10^9$	9
$\pm$	long long	64	$9 \cdot 10^{18}$	18
+	long long	64	$18 \cdot 10^{18}$	19
$\pm$	__int128	128	$17 \cdot 10^{37}$	38
+	__int128	128	$3 \cdot 10^{38}$	38

## 2 Algorithms

Algorithm	Time	Space
Articulations and Bridges	$\mathcal{O}(V + E)$	$\mathcal{O}(V + E)$
Bellman-Ford	$\mathcal{O}(VE)$	$\mathcal{O}(V + E)$
Dijkstra	$\mathcal{O}((V + E) \log V)$	$\mathcal{O}(V^2)$
Edmond Karp	$\mathcal{O}(VE^2)$	$\mathcal{O}(V + E)$
Euler Tour	$\mathcal{O}(E^2)$	
Floyd Warshall	$\mathcal{O}(V^3 + E)$	$\mathcal{O}(V^2 + E)$
Graph Check	$\mathcal{O}(V + E)$	$\mathcal{O}(V + E)$
Kahn	$\mathcal{O}(VE)$	$\mathcal{O}(V + E)$
Kruskal	$\mathcal{O}(E \log V)$	$\mathcal{O}(V + E)$
LCA	$\mathcal{O}(N \log N)$	$\mathcal{O}(N \log N)$
MCBM	$\mathcal{O}(VE)$	
Prim	$\mathcal{O}(E \log V)$	$\mathcal{O}(V + E)$
Tarjan	$\mathcal{O}(V + E)$	$\mathcal{O}(V + E)$
Extended Euclid	$\mathcal{O}(\log \min(a, b))$	$\mathcal{O}(1)$
Floyd (cycle)	$\mathcal{O}(V)$	$\mathcal{O}(1)$
Prime Fac. w/ Opt. Trial Div.	$\mathcal{O}(\pi(\sqrt{n}))$	$\mathcal{O}(n)$
Sieve of Eratosthenes	$\mathcal{O}(n \log \log n)$	$\mathcal{O}(n)$
Binary Search	$\mathcal{O}(\log N)$	
Coordinate Compression	$\mathcal{O}(N \log N)$	
KMP	$\mathcal{O}(N)$	
MUF	$\mathcal{O}(AM)$	$\mathcal{O}(N)$
Bottom-Up SegTree	$\mathcal{O}(\log N)$	$\mathcal{O}(N)$

A: Ackermann function

## 2.1 Graph

### 2.1.1 Articulations and Bridges

If vertex  $v$  is an **articulation point** and you remove it, the connected component to which it belongs becomes disconnected

If edge  $u, v$  is a **bridge** and you remove it, you can't reach  $v$  from  $u$

### 2.1.2 Edmond Karp MaxFlow

Ford-Fulkerson's method with BFS  $\rightarrow \mathcal{O}(VE)$  BFS calls,  $\mathcal{O}(E)$  per BFS

**Vertex weights:** if vertex  $V$  has a weight, create  $V_{in}$  (receives all in-edges of  $V$  and has an edge to  $V_{out}$ ) and  $V_{out}$  (receives an edge from  $V_{in}$  and has all out-edges of  $V$ ); edge  $\{V_{in}, V_{out}\}$  has the weight from  $V$

**MinCut:** run EdmondKarp;  $S - T$  sets are: all  $V$  that you can reach from the source with edges of positive residual capacity and all other  $V$

**MultiSource/MultiSink:** create a super source with infinite capacity pointing to all sources, analogous for sinks

**Max Cardinality Bipartite Matching:** use capacity 1 on all edges and apply the multi-source and multi-sink strategies

### 2.1.3 Euler Tour

Find the closest neighbor that has a path back to the current vertex to build an euler tour

**Euler path:** visits each edge once

**Tour/cycle/circuit** euler path that starts and ends at same node

**Undirected and has path:** every vertex has even degree or two have odd degree

**Undirected and has circuit:** every vertex has even degree

**Directed and has path:**  $\delta^+(v) - \delta^-(v) = 1$  for at most one  $v$ ,  $= -1$  for at most one  $v$ ,  $= 0$  for all other  $v$

**Directed and has circuit:**  $\delta^+(v) = \delta^-(v) \forall v \in V$

### 2.1.4 Floyd Warshall

Also works for SSSP ( $V \leq 400$ )

**Printing path:**  $p[i][j]$  set to  $i$  (last node that appears before  $j$  on the path), then  $p[i][j] = p[k][j]$  on update.

**Transitive Closure:** weight is boolean (init as 1 if there's an edge), update with bitwise OR

**Minimax/Maximin:**  $w[i][j]$  will be  $\min(w[i][j], \max(w[i][k], w[k][j]))$

**Finding negative/cheapest cycle:** init  $w[i][i] = \text{inf}$ ;  $\text{run}()$ ; any  $w[i][i] \neq \text{inf}$  is a cycle and the smallest is the cheapest; any  $w[i][i] < 0$  is negative cycle

This can also be used for finding SCCs (check with transitive closure)

### 2.1.5 Kahn's Topological Sort

Particular order (alphabetical)

### 2.1.6 Kruskal

Order edges by increasing weight, then use a MUF to know if each edge is useful (if it connects two previously disconnected vertices)

**Min Span Subgraph:** previously process fixed edges

**Min Span Forest:** count number of sets on the MUF

**2nd Best MST:** run kruskal; for each chosen edge, flag it as unavailable and run it without using that edge ( $O(VE)$ )

**Minimax:** max edge weight on the MST (maximin: min)

### 2.1.7 Lowest Common Ancestor

Binary lift to binary search the LCA or Euler Path

### 2.1.8 Max Cardinality Bipartite Matching

Jump from free to matched edges until you've used them all

### 2.1.9 Prim's Algorithm

Take smallest edge that leads to vertex  $v$

### 2.1.10 Tarjan

A node can reach any other node in its own SCC (DFS + stack)

### 2.1.11 Kosaraju

Get topological sort of a graph and then run DFS on the transposed graph following this topological sort.

Let  $C$  and  $C'$  be two strongly connected components in the graph  $G$ . If there is an edge  $\{C, C'\}$ , then after computing  $\text{tout}$  and  $\text{tin}$  in a DFS,  $\text{tout}[C] > \text{tout}[C']$ . Proof:

- If  $\text{tin}[C] < \text{tin}[C']$ ,  $C$  shows up first in the DFS, and since there is an edge to  $C'$ , it will be in  $C'$ 's subtree in the DFS tree; so  $\text{tout}[C] > \text{tout}[C']$
- If  $\text{tin}[C] > \text{tin}[C']$ ,  $C'$  shows up first, but since it has no path to  $C$ ,  $C$  will show up later and then have a greater  $\text{tout}$  value.

So when you do DFS on the transposed graph following topological sort, you will start on the root vertex (because it'll have the largest  $\text{tout}$  value), and only be able to visit the nodes in its SCC (because the edges that would lead to other SCCs doesn't exist in the transposed graph).

Remember that you have to "disable" the vertices in already processed SCCs.

## 2.2 Math

### 2.2.1 Floyd

Slow and fast (tortoise and hare)

### 2.2.2 Combination

A combination  ${}_nC_k = \binom{n}{k}$  ( $n$  chooses  $k$ ) refers to selecting  $k$  objects from a collection of  $n$  where the order of choice doesn't matter.

**Without repetition:** can't choose an element twice.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

**With repetition:** elements may be chosen more than once.  $\binom{n}{k} = \frac{(k+n-1)!}{k!(n-1)!}$

### 2.2.3 Permutation

A permutation  ${}_nP_k$  refers to selecting  $k$  objects from a collection of  $n$  where the order of choice matters.

**With repetition:** elements may be chosen more than once.  ${}_nP_k = n^k$

**Without repetition:** can't choose an element twice.  
 ${}_nP_k = \frac{n!}{(n-k)!}$

## 2.3 Paradigm

### 2.3.1 Coordinate Compression

Normalize vector access; can also be done with map/set but high constant

### 2.3.2 128 Bit Integers

GCC extension;  $2^{127} \cdot 10^{38}$

## 2.4 String

### 2.4.1 Prefix Function (KMP)

To find occurrences of  $s$  in  $t$ , use the string  $s\%+t$ , then look for  $pi[i] = s.length()$  on the " $t$  side"