The multi-period competitive location problem of temporary retail facilities

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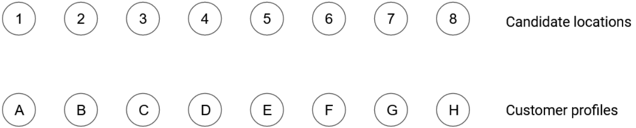
Temporary retail facilities

- Retail facilities meant to stay for a short amount of time [2].

Figure: Example of temporary retail facility from Nike. Extracted from https://weburbanist.com/2015/09/23/hot-pop-up-shops-14-imaginatively-risky-retail-designs/.

Competitive location problem

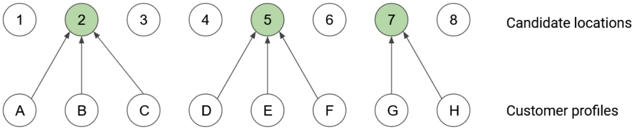
- Location decisions affect how customers patronize facilities, which impacts the estimated revenue of the company.



- Customers follow a rank-based discrete choice model [1].

Competitive location problem

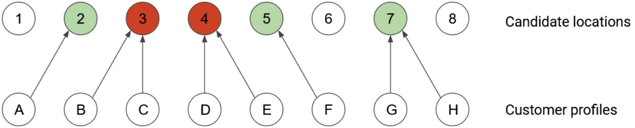
- Some configurations may be more profitable than others.



- For example, if $D : 4 \succ 5 \succ 1 \succ 2 \succ 3 \succ 6 \succ 7 \succ 8 \ldots$

Competitive location problem

- Customer profiles B, C, D, E have been stolen by competitors.



- The presence of competitors must be taken into account.

Multi-period aspect

- Consider the case with only one temporary facility.
- Plan how to move it from one location to the other over time.
- Decide the next location at the beginning of the time period.

Additional assumptions

- Competitors react to location decisions over time.
- No priority over competitors in securing a lease.
- In short, competitors plan to use a location in the next time period, this location cannot be used by the company.

Initial notation

- Set of candidate locations $\mathcal{L}$.
- Set of customer profiles $\mathcal{C}$.

Stages, states, actions, and uncertainty

- Stages $\{0, ..., N\}$ are time periods $\{0, ..., N\}$.
- States $x_k \in X_k = \mathcal{L} \cup \{0\}$ are current locations.
- Actions $u_k \in U_k(x_k) = \mathcal{L} \backslash \{x_k\}$ are next locations.
- Random variables $\omega_k \in \Omega_k(x_k) = \{\omega \,|\, \omega \subseteq \mathcal{L} \backslash \{x_k\}\}$ are next competitor locations, as location x_k is not seen as available.

Transition and cost functions

- The transition function, which gives the actual next location, is $x_{k+1} = f(x_k, u_k, \omega_k) = \begin{cases} u_k, & \text{if } u_k \notin \omega_k \\ 0, & \text{otherwise} \end{cases}$.
- The maintenance cost for a facility at location x is $m(x)$.
- The estimated revenue for facilities at locations $A \subseteq \mathcal{L}$ competing against locations $B \subseteq \mathcal{L}$ is $r(A, B)$.
- The cost function for $k < N$ is $g_k(x_k, u_k, \omega_k) = m(x_k) - r(\{f(x_k, u_k, \omega_k)\}, \omega_k)$.
- The cost function for $k = N$ is $g_N(x_k, u_k, \omega_k) = m(x_k)$.

Probability based on profitability

- The probability of the random variable ω_k being equal to realization ω is $P_k(\omega_k = \omega \,|\, x_k) = \frac{e^{\beta_k (r(\omega, \{x_k\}) - m(\omega))}}{\sum_{\bar{\omega} \in \Omega_k(x_k)} e^{\beta_k (r(\bar{\omega}, \{x_k\}) - m(\bar{\omega}))}}$.
- Assume a decaying effort in the competition over time with $\beta_k = \frac{1}{d^k}$, where d is the rationality decay parameter.

Bellman recurrence equations

- The Bellman recurrence equations fot this problem are:

$$J_N(x) = m(x) \qquad \forall x \in X_N$$
$$J_k(x) = \min_{u \in U_k(x_k)} Q_k(x,u) \qquad \forall x \in X_k, \forall k \in \{0, \dots N-1\}$$

$$Q_k(x,u) = m(x) + \mathbb{E}_{\omega_k}[-r(\{f(x,u,\omega_k)\}, \omega_k) + J_{k+1}(f(x,u,\omega_k))]$$

Implementation challenges

- The size of the support $\Omega_k(x_k) = \{\omega \,|\, \omega \subseteq \mathcal{L} \backslash \{x_k\}\}$ grows exponentially with the number of candidate locations $|\mathcal{L}|$.
- Work with a sampled support $\bar{\Omega}_k(x_k) \subset \Omega_k(x_k)$ of size s.

Implemented algorithms

- Backward solver: stochastic backward chaining algorithm.
  - There is no known analytical solution for this problem.
- Parametric solver: fitted value iteration algorithm.
  - Approximation of the $Q_k(x,u)$ values.
  - Linear architecture: $\tilde{Q}_k(x,u) = r_k^T \phi(x,u)$.
  - Feature vector $\phi(x,u)$ has three main set of features.
    - Distance between locations x and u in the rankings.
    - Maintenance costs of locations x and u.
    - Revenues of locations x and u versus location $\ell \in \mathcal{L}$.

Experimental instances

- Instance I^S has three candidate locations, two customer profiles, and a planning horizon of two time periods.
- Instance I^M has four candidate locations, eight customer profiles, and a planning horizon of six time periods.
- Instance I^G has ten candidate locations, thirteen customer profiles, and a planning horizon of six time periods.

Experimental questions

- #1 Are the exact and approximate approaches correctly implemented?
- #2 How does the rationality decay parameter d affect the expected profit?
- #3 How well can the parametric solver approximate the backward solver?

First question

(k, x)	$(0,0)$	$(0,1)$	$(0,2)$	$(0,3)$	$(1,0)$	$(1,1)$	$(1,2)$	$(1,3)$
$J_k(x)$ manual	0	-450	0	-450	0	-800	100	-800
$J_k(x)$ from π^*	0	-450	0	-450	0	-800	100	-800
$J_k(x)$ from $\tilde{\pi}$	50	-450	0	-450	100	-800	100	-800
$\tilde{J}_k(x)$ from $\tilde{\pi}$	-200	-400	-300	-400	-300	-600	100	-600

Table: Values of $J_k(x)$ and $\tilde{J}_k(x)$ obtained manually and by policies π^* and $\tilde{\pi}$.

- The algorithms seem to be correctly implemented.
- The approximated $\tilde{J}_k(x)$ values can be quite different.

Second question

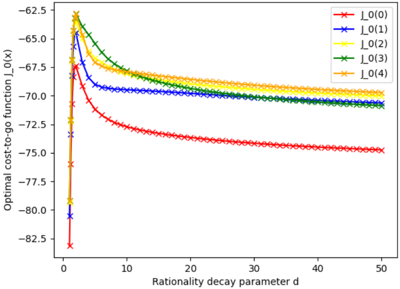


Figure: $J_0(x)$ values with varying values of the rationality decay parameter d for instance I^M.

Second question

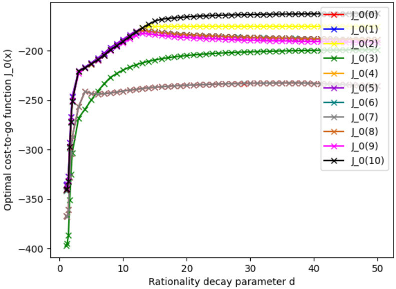


Figure: $J_0(x)$ values with varying values of the rationality decay parameter d for instance I^L.

Second question

- The rationality decay parameter d does not behave in an intuitive manner, and its value is hard to determine.

Third question

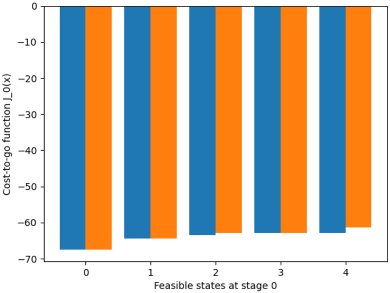


Figure: $J_0(x)$ values when applying optimal policy π^* in blue and when applying approximate policy $\tilde{\pi}$ in orange for instance I^M.

Third question

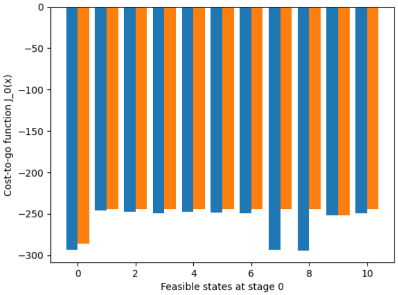


Figure: $J_0(x)$ values when applying optimal policy π^* in blue and when applying approximate policy $\tilde{\pi}$ in orange for instance I^L.

Third question

- The approximate policy $\tilde{\pi}$ has surprisingly a good performance, even though not for all states.
- The approximated $\tilde{J}_k(x)$ values can be quite different.

Conclusion

- The problem arises from recent developments in the retail industry and has many interesting challenges:
  - estimating the patronizing behavior of customers;
  - optimizing location decisions in a competitive environment;
  - modelling the rationality decay of the competitors;
  - enforcing constant change to keep the sense of urgency;
  - making strategic decisions under uncertainty.
- Result shows that some parameters may not be simple to set.
- Results also show that the parametric solver finds a reasonably good policy, even though not for all states.

Conclusion

- Unsuitable concepts seen on class:
  - approximation of the $J_k(x)$ values.
  - having an infinite horizon.
- Suitable concepts seen on class:
  - approximation with a rollout procedure.
  - approximation with a neural network.

References

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The end

Thank you!