AE461A

Aircraft Design - I

Project End Term Report

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Abstract

Within the framework of AE461A, our focus centers on the development of electric aircraft tailored specifically for pilot training and short-range flights. Our attention is drawn to the realm of similar innovations, notably exemplified by the ongoing manufacturing progress of the PIPISTREL VELIS ELECTRO. This endeavor encapsulates our commitment to exploring and advancing the domain of electric aircraft, aligning with the burgeoning landscape of sustainable aviation solutions.

Motivation

The motivation behind transitioning towards small electric aircraft for pilot training lies in the urgent need for sustainable aviation solutions. The aviation industry faces increasing pressure to reduce its environmental impact, and electric aircraft offer a promising avenue to achieve this goal. By opting for electric propulsion in pilot training aircraft, we aim to significantly mitigate both noise pollution and carbon emissions commonly associated with traditional aviation fuel. Moreover, these electric aircraft present a prime opportunity to revolutionize the training landscape, offering quieter operations that are more conducive to both urban and suburban environments. Embracing electric aviation not only aligns with global sustainability targets but also signifies a progressive step towards a cleaner, quieter, and more accessible future for aviation.

Aircraft Design Report

CONTENTS

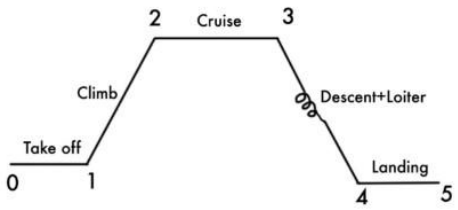
- **Mission Requirement-**

Total Passengers	2
Passenger Weight	2*85=170 kg
Baggage Weight	30 kg
Total Max. Payload	200 kg
Cruise Speed	55-65 m/s
Stall speed	30-35 m/s
Range	280 km
Endurance	5600 s(~90 mins)
L/D_{max}	12-17
battery efficiency (η_b)	0.95
propeller efficiency (η_p)	0.9
Battery SED (LIB)	0.4 kW-hr/kg
Service Ceiling	3.5 km
Cruise Ceiling	3 km
Max Rate of Climb	3.3m/s
Runway length	500 m

- **Weight Estimation**

1. Total Take-off Weight Estimation:

We identify the mission segments of the flight mission profile. For every mission profile chosen, there are some basic segments, which are universal for each of them. These segments are shown in the following figure below:



Maneuver	Label
Take off	0 to 1
Climb	1 to 2
Cruise	2 to 3
Loiter	3 to 4
Landing	4 to 5

$W_O = W_{payload} + W_{empty} + W_{fuel}$
For our aircraft we will be using electric batteries instead of fuel.
And the weight of electric battery will be calculated through iteration at later stage, for that we need to find total takeoff weight(payload+empty) which will be starting weight for battery weight iteration program

$W_O = W_{payload} + W_{empty}$

$1 = \frac{Wpayload}{WO} + \frac{Wempty}{WO}$

$W_{payload} = W_O[1 - \frac{Wempty}{WO}]$

$W_O = \frac{Wpayload}{1 - \frac{Wempty}{WO}}$

We know that,

$W_{payload} = W_{crew} + W_{luggage}$

$W_{payload} = 170 + 30$

$W_{payload} = 200$ kg

We also know that,

$\frac{Wempty}{WO}$=aW_O+b

Where a and b are considered after analysis of historical data of similar kind of aircrafts

Using the data of prop trainer aircraft, we have taken a =$1.39*10^{-6}$ and b=0.64

$\frac{Wempty}{WO}$=(0.00000139)W_O+0.64

Putting this expression of $\frac{Wempty}{WO}$ in W_O formula

W_O=$\frac{Wpayload}{1-\frac{Wempty}{WO}}$

W_O=$\frac{Wpayload}{1-[(0.00000139)WO+0.64]}$=$\frac{200}{1-[(0.00000139)WO+0.64]}$

Solving this Equation we get,

W_O= 556.752 kg

2. Calculation of Battery Weight (by iteration)

Calculating the value of C_L: C_L= $\frac{2*Wt0*g}{\rho Sv2}$

Calculating the value of C_D:

$C_D = (D/L) * C_L$

$C_D = (D/L) * \frac{2*Wo*g}{\rho*S*V^2}$

Calculating the value of (E_R The energy required):

$E_R = Drag\ Force * Cruise\ speed * Endurance$

$\Rightarrow E_R = D * v * \Delta t$

$\Rightarrow E_R = (1/2) * \rho * v^3 * S * C_D * \Delta t$

Keeping C_D from above,

$\Rightarrow E_R = \frac{Wo*v\ *g*\Delta t}{\frac{L}{D}}$

Once E_R is known, the weight of the battery (W_B) can be calculated as follows:

$W_B = E_R / \{(\eta_b * \eta_p) * SED\}$

Sample calculation of Battery weight is shown below:

Taking W_O=556.752 kg

V=65 m/s

Δt=5600 s

L/D=15

Taking rho at cruise altitude(3000m), ρ=1.1209

$E_R = \frac{Wo*v\ *g*\Delta t}{\frac{L}{D}}$

$E_R = \frac{556.752*65\ *g*5600}{15}$=132538154.1 W-s

Converting W-s into kW-hr

E_R =132538154.1 W-s=$\frac{132538154.1}{3600*1000}$=36.8161 kW-hr

$W_B = E_R / \{(\eta_b * \eta_p) * SED\}$

$W_B = 36.8161/\{(0.95 * 0.9) * 0.4\}$ =107.649 kg

Now We can calculate True Total Take off weight using battery weight as follows

W_O = St_ratio * W_{t0} + $W_{payload}$ + $W_{battery}$

Here St_ratio= Structural ratio=0.65

W_{t0} is takeoff weight in $(n\text{-}1)^{th}$ iteration, whereas W_O is takeoff weight in n^{th} iteration

W_O=(0.65*556.752)+200+107.649

W_O=669.5378 kg

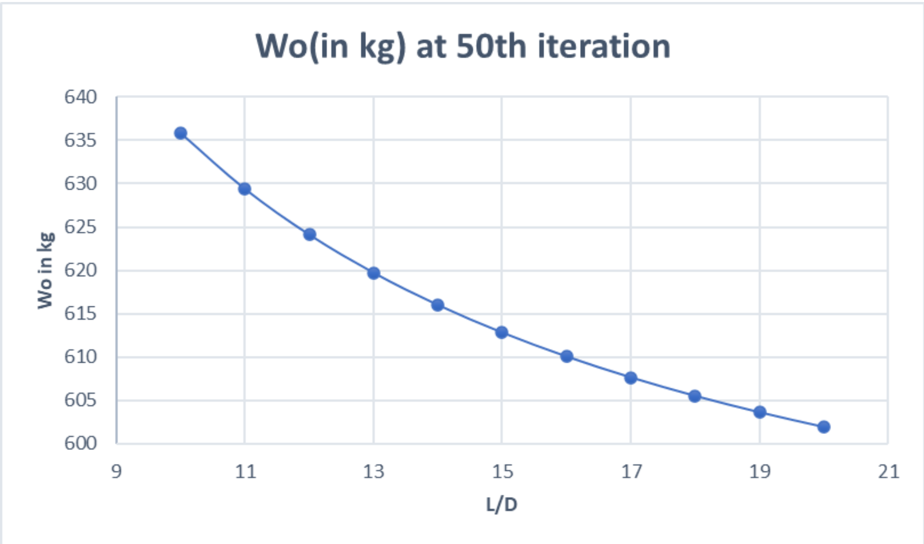
We did such 50 iterations using code until the value of W_O becomes almost constant

At the end of 50th iteration we found W_O values at different L/D ratio as follows

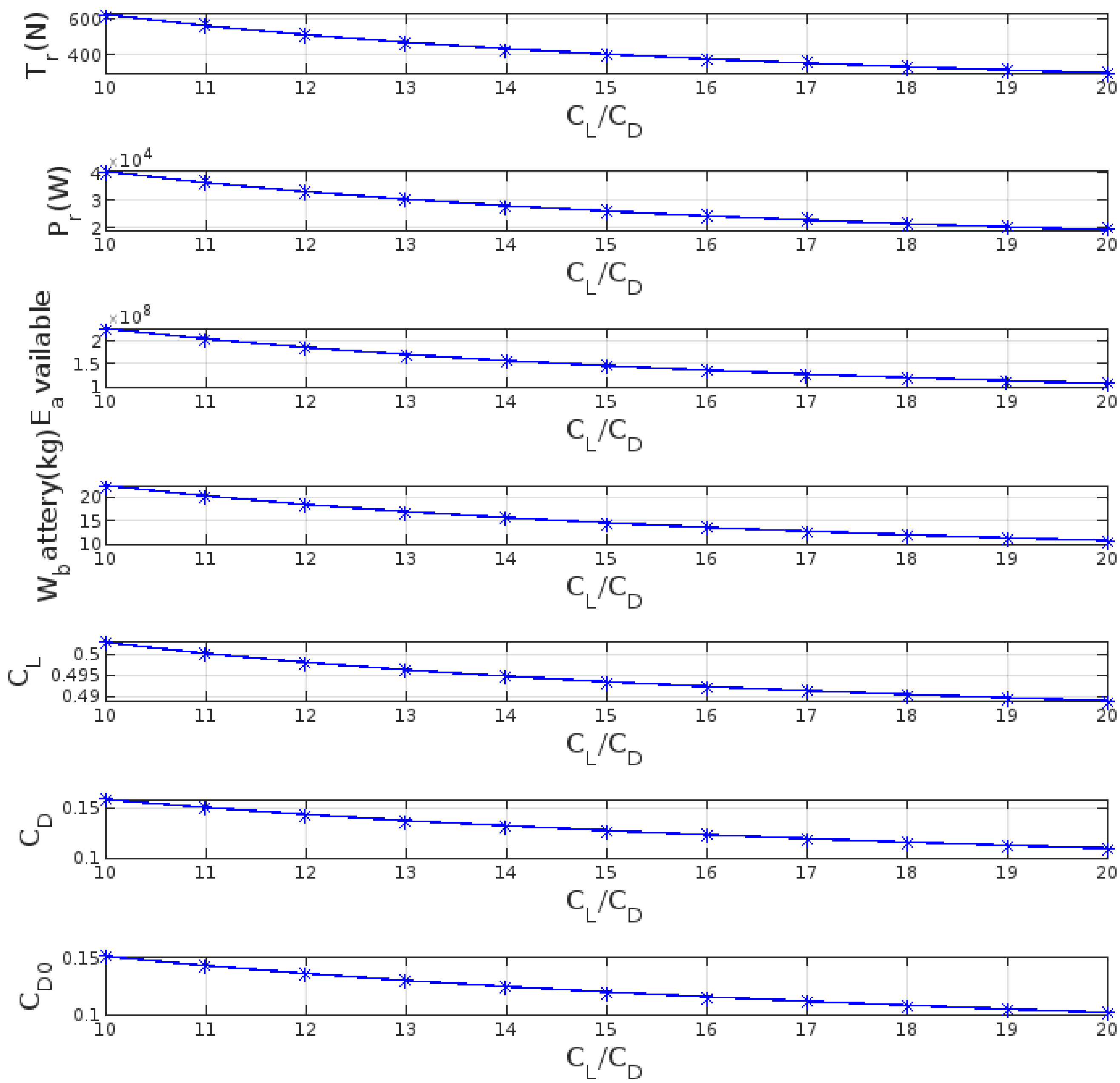
Table:

L/D	Wto(in N)(Total weight at 50th iteration)	Wto(in kg)
10	6238.029124	635.8847222
11	6174.711215	629.4302972
12	6122.919979	624.1508643
13	6079.770441	619.7523385
14	6043.266268	616.03122
15	6011.982065	612.8422084
16	5984.872886	610.0787855
17	5961.155247	607.6610853
18	5940.23016	605.5280489
19	5921.631846	603.6321963
20	5904.992656	601.9360506

Plot:



The Plots for T_r(Thrust required), P_r(Power required),E_a(Energy Available),W_b(Battery weight),C_L,C_D,C_{Do} Vs L/D are given below
These values are at the **50th** iteration. Calculations for them are done in the code.



Iterations for finding the value of W_{t0} are shown in the Matlab code below:

```
g0 = 9.81;
iteration = 50;
Wt0 = zeros(11, iteration);
Wt0(1) = 556.752 * g0;
Wpayload = 200 * g0;
St_ratio = 0.65; %structural ratio(strucutre weight ratio)
wing_span = 10.7;
AR = 12;
k = 1/(pi*0.85*AR);
rho = 1.225;
Area = wing_span^2/AR;
SED = 0.4; %Specific Energy Density
LD = 10:20; %L/D range
eeta_battery = 0.95; %battery efficiency
safety_factor = 1.2;
eeta_propeller = 0.9; %Propeller efficiency
V = 65; %cruise speed in m/s
Endurance = 5600; %Endurance in seconds
Thrust_required = zeros(1, length(LD));
Power_required = zeros(1, length(LD));
Energy_required = zeros(1, length(LD));
W_battery = zeros(1,length(LD));
Cl = zeros(1,length(LD));
Cd = zeros(1,length(LD));
Cd0 = zeros(1,length(LD));
for a = 1:length(LD)
    for i = 1:(iteration)
        Ecruise = (Wt0(a, i) * V * Endurance) / LD(a);
        Erequired = (safety_factor * Ecruise) / (eeta_propeller * eeta_battery);
        Wbattery = Erequired / (SED * 3600000);
        Wt0(a, i + 1) = St_ratio * Wt0(a, i) + Wpayload + Wbattery;
    end
    Thrust_required(a) = Wt0(a, iteration) / LD(a);
    Power_required(a) = Thrust_required(a) * V;
    Energy_required(a) = Power_required(a) * Endurance;
    W_battery(1,a) = Wbattery;
    Cl(1,a) = sqrt(2*Wt0(a,iteration)/(rho*V^2*Area));
    Cd(1,a) = sqrt(2*Thrust_required(1,a)/(rho*V^2*Area));
    Cd0(1,a) = Cd(1,a) - k*Cl(1,a)^2;

end
```

- **Wing Loading & Power Loading Calculation**

1. Plots for Critical Flight Parameters

i. Maximum take-off weight estimation:
Since our planned aircraft is electrical, we have already done the task of calculating the Total Take-off weight (with accurate battery weight estimation using the iteration method) The total take-off weight obtained at different values of L/D were shown before.
Let us take (L/D) = 15 for cruise flight, which also satisfies the drag coefficient value estimated in the next section. Based on this,
W_{TO} = 6011.982065 N

ii. C_{D0} estimation:
For an aircraft in the preliminary design phase, the coefficient CD0 can also be determined as an average value from aircraft with similar performance and configuration:

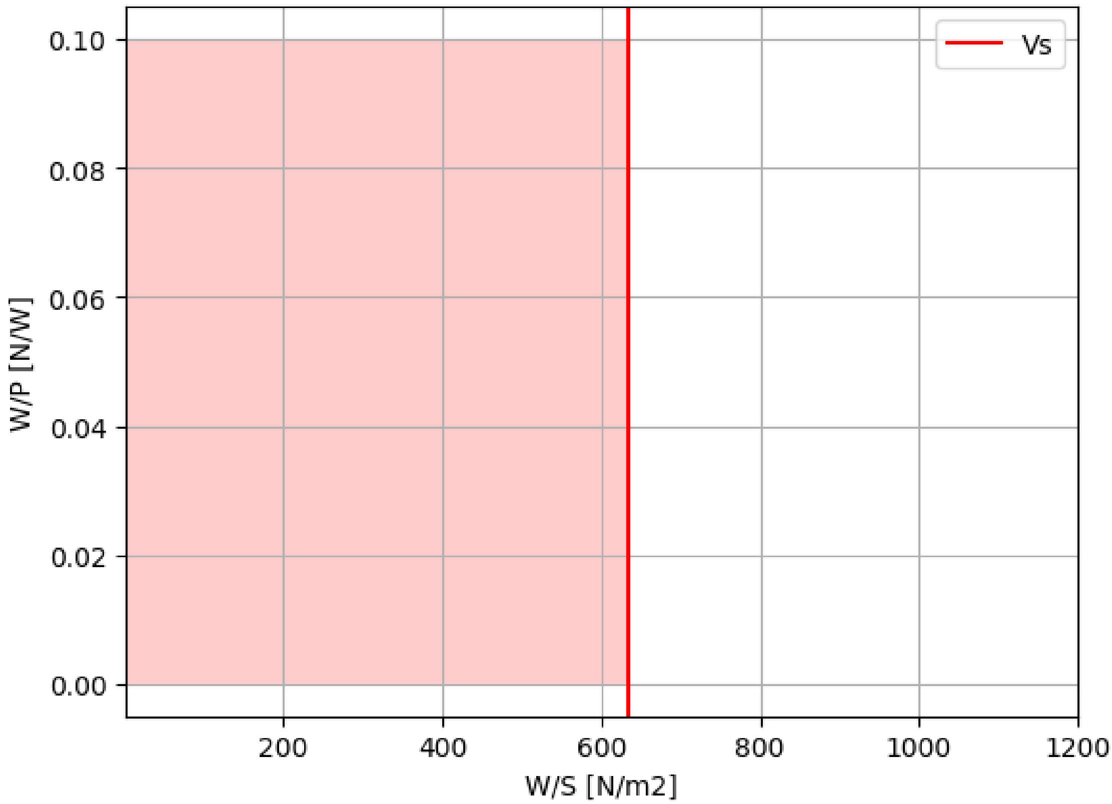
$$C_{Do} = \frac{C_{Do1} + C_{Do2} + C_{Do3} + C_{Do4} + C_{Do5}}{5}$$

Typical values of C_{D0} for microlight aircraft lie in the range of 0.02-0.035.
C_{DO1} = 0.019057
C_{DO2} = 0.01892
C_{DO3} = 0.022405
C_{DO4} = 0.019888
C_{DO5} = 0.021463
(values taken from historical data of microlight aircraft)
At this stage, the coefficient CD0 is calculated as an average value:
After calculation, C_{Do} = 0.203466

iii. Equation for stall speed:
For most aircraft, the stall speed shall not exceed some minimum defined value.

$$\left(\frac{W}{S}\right)_{V_s} = \frac{1}{2}\rho V_s^2 C_{lmax}$$

ρ is air density at sea level, 1.225 kg/m3
From the statistics, the following parameters were determined from the data available for microlight, jet aircrafts and reference aircraft are velis electro.
For propeller- driven aircraft,
C_{lmax} = 1.23, V_s = 29m/s
W/S obtained is 633.588375 N/m^2



In general, lower stall speed is required as it results in a safer flight. Lower stall speed results in a safer take-off and landing. The take-off and landing speed are usually slightly higher than the stall speed (1.1Vs to 1.3Vs).
Note that W/S doesn't depend on the variation of W/P.
We have used matlab and python code for the calculation and plotting.

iv. <u>Equation for Maximum speed:</u>

If in design requirements cruise speed requirements are defined instead of maximum speed requirements, there shall be taken a 20 % to 30 % greater maximum speed. This is because the cruise speed for propeller driven aircraft is calculated for 75 % to 80 % of engine power. So, the air vehicle's maximum speed is as follows:

$V_{max} = 1.23\ V_c = 80$ m/s

(We have taken cruise velocity 65m/s in our initial requirements)

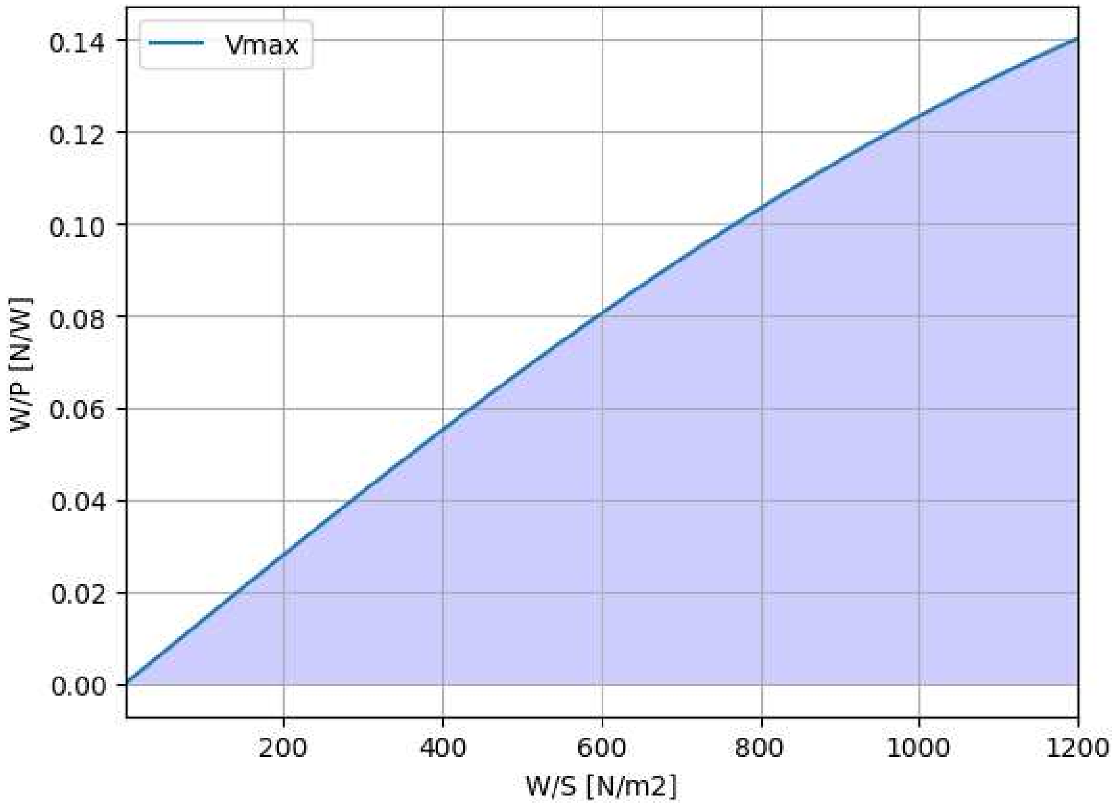
For propeller-driven aircraft:

The following equation is used for the matching plot construction:

$$\left(\frac{W}{P_{SL}}\right)_{V_{max}} = \frac{\eta_P}{\frac{1}{2}\rho_o V_{max}^3 C_{D_o} \frac{1}{\left(\frac{W}{S}\right)} + \frac{2K}{\rho\sigma V_{max}}\left(\frac{W}{S}\right)}$$

where η_p is propeller efficiency coefficient equal to 0.9.

we have previously calculated k in lab exercise 2 for the AR = 12 and it is 0.037571246.



We have used matlab and python code for the calculation and plotting.
As Vmax is in denominator, in case when it is increasing, the value of power loading (W/P) is decreasing. Consequently, any value of Vmax greater than the one specified in the requirements satisfies the maximum speed requirements, and the region below the graph is acceptable.

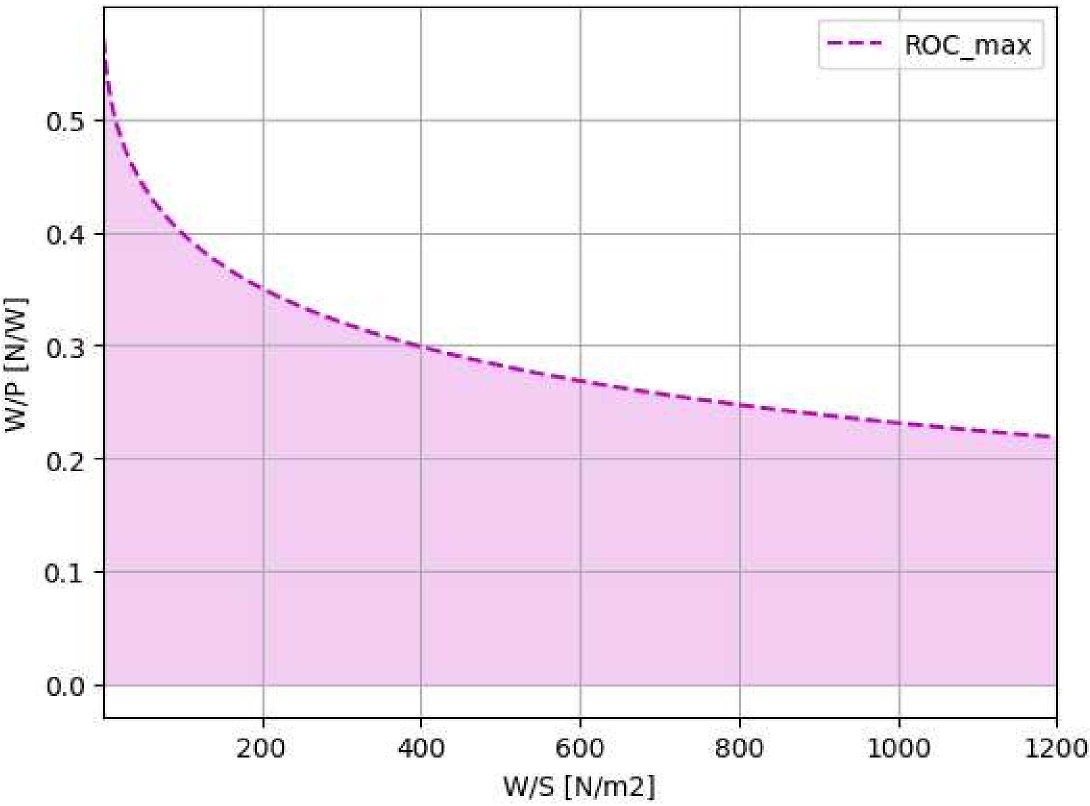
v. <u>Equation for rate of climb:</u>

For prop-driven aircraft:
Following equation is derived in this case:

$$\left(\frac{W}{P}\right)_{\mathrm{ROC}} = \frac{1}{\dfrac{\mathrm{ROC}}{\eta_{\mathrm{P}}} + \sqrt{\dfrac{2}{\rho\sqrt{\dfrac{3C_{D_o}}{K}}}\left(\dfrac{W}{S}\right)}\left(\dfrac{1.155}{(L/D)_{\max}\,\eta_{\mathrm{P}}}\right)}$$

where η_p is propeller efficiency coefficient in climbing flight, and we have taken it 0.9. L/D_{max} is taken as 18 and all other values are same which we have determined earlier.

We have used the ROC = 1.5, it is at cruise altitude of 3km.



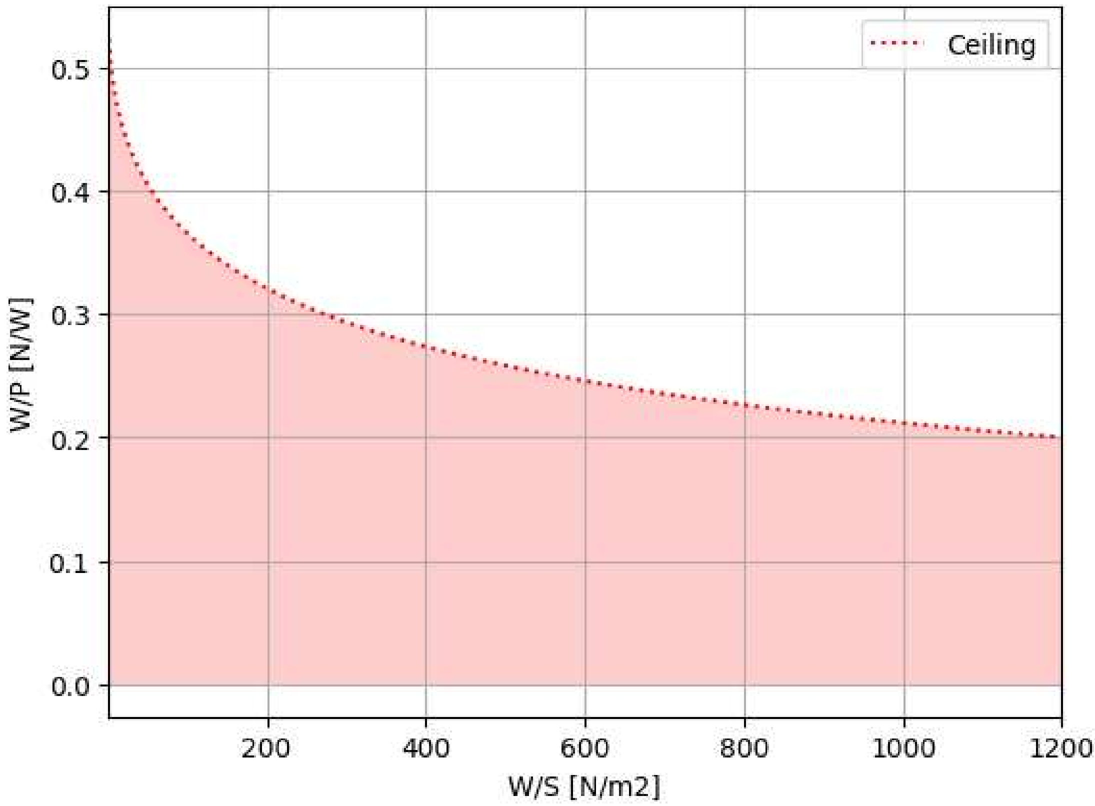
The ROC value is a denominator in the equations, so when the rate of climb is increasing, the value of power loading (W/P) is decreasing. Consequently, any value of ROC greater than the one specified complies with the rate of clime requirements, and the region below the graph is acceptable.

vi. <u>Equation for Ceiling altitude:</u>
It is generally defined for several types of ceiling:
absolute ceiling – The absolute ceiling is an altitude where the aircraft flight ROC is zero;
service ceiling – The service ceiling is an altitude where the aircraft flight ROC is 0.5 m/s;
cruise ceiling – The cruise ceiling is an altitude where the aircraft flight ROC is 1.5 m/s;
combat ceiling – The combat ceiling is an altitude where the fighter can take altitude with a speed of 5 m/s. This altitude is defined only for combat aircraft.

For prop-driven aircraft:
The expression for the power loading (W /P), as a function of wing loading (W /S) and ROC is

$$\left(\frac{W}{P_{SL}}\right)_C = \frac{\sigma_C}{\dfrac{ROC_C}{\eta_P} + \sqrt{\dfrac{2}{\rho_C\sqrt{\dfrac{3C_{D_o}}{K}}}\left(\dfrac{W}{S}\right)}\left(\dfrac{1.155}{(L/D)_{max}\,\eta_P}\right)}$$

According to the design requirements, the cruise ceiling for the air vehicle is 3000m, hence ROC at this point is 1.5m/s.
ρ must be calculated at service ceiling.



The ROC and ρ_c values represent a denominator in the equations, so when the altitude is increasing, ρ_c is decreasing, and the relative air density is decreasing as well. Whereas ρ_c value is in denominator of denominator, in case of altitude increasing, power loading (W/P) is decreasing. Consequently, any altitude greater then defined h_c altitude satisfies the ceiling requirements, and the region below the graph is acceptable.

vii. <u>Equation for take-off run distance</u>:

In EASA CS VLA 51, it is defined that the range should not exceed 500 m to clear an up to 15 m tall obstacle. Thus, S_{TO} = 500m is taken in the calculations.

For prop-driven aircraft:

The take-off speed is a little bit greater than the stall speed, we have taken,

$V_{TO} = 1.2V_s = 34.8$ m/s

The matching plot in this case is calculated with the equation:

$$\left(\frac{W}{P}\right)_{S_{TO}} = \frac{1-\exp\left(0.6\rho g C_{DG} S_{TO}\frac{1}{W/S}\right)}{\mu-\left(\mu+\frac{C_{DG}}{C_{LR}}\right)\left[\exp\left(0.6\rho g C_{DG} S_{TO}\frac{1}{W/S}\right)\right]}\cdot\frac{\eta_P}{V_{TO}}.$$

The parameter in this equation is calculated as follows:

$C_{DOTO} = C_{DOLG} + C_{DOLG} + C_{DOHLD\text{-}TO} = 0.0203466 + 0.009 + 0.0055 = 0.034866$

where, C_{DOTO} = zero lift-drag coefficient during take-off.

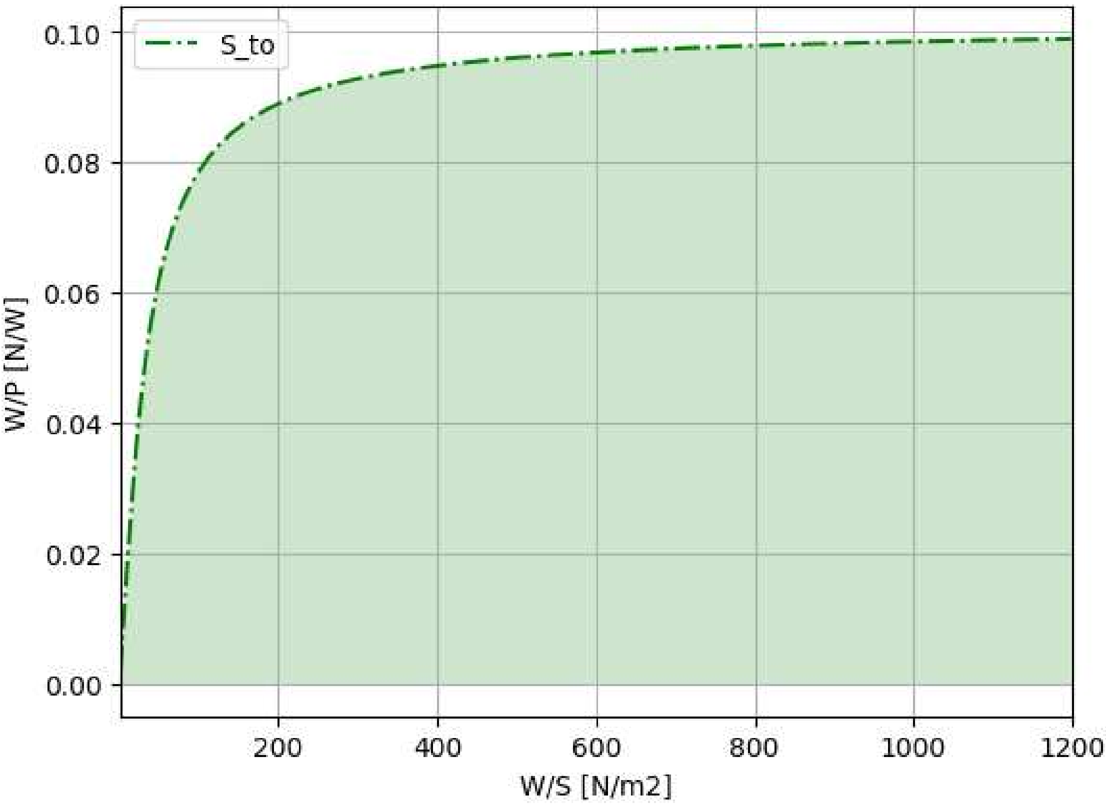
C_{DOLG} = landing gear drag coefficient; (its typical value is between 0.006- 0.012, in this case we have taken the average value)

$C_{DOHLD\text{-}TO}$ = high lift device drag coefficient (its typical value is between 0.003-0.008, in this case we have taken the average value)

The aircraft's take-off lift coefficient is as follows:

$C_{LTO} = C_{LC} + \Delta C_{LflapTO} = 0.3 + 0.55 = 0.85$
where C_{LC} is the aircraft's cruise lift coefficient, which is usually about 0.3 for subsonic aircraft, and $\Delta C_{LflapTO}$ is high lift devices' lift coefficient in take-off configuration (its typical value is between 0.3- 0.8, in this case we have taken the average value)

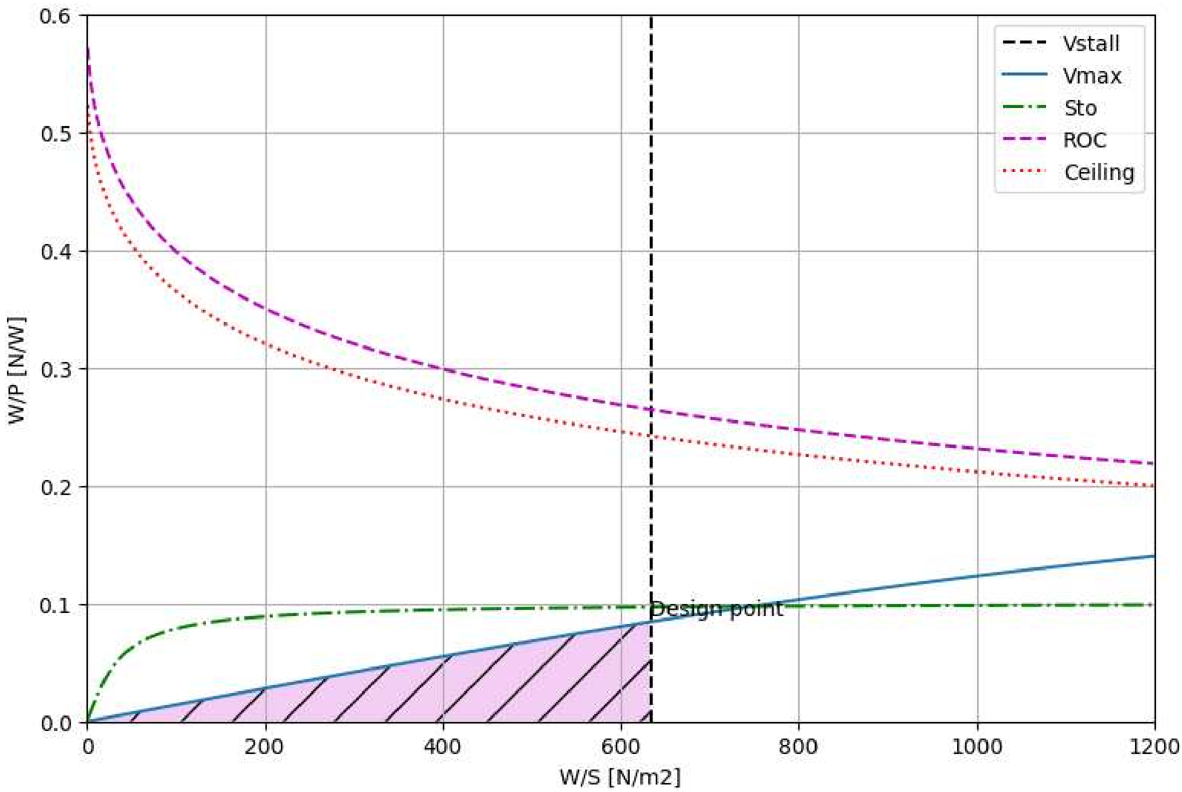
The air vehicle's drag coefficient in take-off configuration is:
$C_{DTO} = C_{DOTO} + kC_L^2 = 0.061991$
And,
$C_{DG} = C_{DTO} - \mu C_{LTO} = 0.019491$
Where μ is runway friction coefficient and we have take it 0.05 (for concrete/asphalt runway) the take-off rotation lift coefficient is taken equal to C_{LTO}, that is, 0.85.



The equation numerator and denominator contain an exponential value of STO parameter. If the take-off runway is increasing, the exponential parameter is increasing as well, so the power loading (W/P) value is also increasing. Consequently, any value of STO greater than the one specified does not satisfy the take-off run requirements, and the region above the graph is not acceptable.

2. **Matching plot**

In this step we have plotted all the performance parameters for prop-driven and jet-driven each and we concluded a desirable region in which a aircraft is operatable. By plotting this, we calculated the design point for the aircraft, there will be only one design point which shows the lowest engine power in the case of prop-driven aircraft.
The design point makes it possible to obtain two parameters: corresponding wing loading $(W/S)_d$ and power loading $(W/P)_d$ or $(T/W)_d$.
The area corresponds to the shaded region is acceptable for the designing purposes.



3. Estimation of Wing Area & Powerplant selection:

The following equations is used:

$$S = \frac{W_{TO}}{W/S_d}$$

$$P = \frac{W_{TO}}{W/P_d}$$

$$T = W_{TO} * T/W_d$$

$$S = \frac{W_{TO}}{W/S_d}$$

$$S = \frac{6011.982065}{633.588375}$$

$S = 9.4887$ m^2

And,

$$P = \frac{W_{TO}}{W/P_d}$$

$$P = \frac{6011.982065}{0.084499}$$

P = 71.148 kW

Selection of powerplant:

Off-the-shelf power plant selection (need to choose a power plant available in the market which is close to the requirements) based on estimated Engine Power (P) or Engine Thrust (T)

$S = 9.4887$ m^2

P = 71.148 kW

We have few options here:

Battery Name	**SED(kW-hr/kg)**
LiPo Battery	0.14-0.2
Lithium Cobalt Oxide (LCO)	0.15-0.2
Lithium ion	0.26-0.27

While studying the developments in lithium ion battery's performance we found that Researchers have succeeded in making rechargeable pouch-type lithium batteries with a record-breaking energy density of over 700 W-h/kg.

So, For our calculation it is safe to assume we are using Lithium ion battery of 0.4 kW-hr/kg

There are few concerns regarding safety when using these LIBs-

Lithium-ion batteries (LIBs) can be dangerous to handle and transport.

They can cause fires and electrical shocks if not packaged and handled safely.

LIBs are prone to “thermal runaway,” which is a ballistic reaction that can cause an immediate flame.
These flames can spread to other cells and packaging.
LIB fires can burn at very high temperatures and spread hazardous gases such as carbon monoxide and hydrogen cyanide. They can also spread very fast.

Hence it is very important to handle these batteries with care and keep them away from flames during flight.

Considering SED=0.4 kW-hr/kg

E= Energy= Power * time
E= Power required * Endurance
E= 71.148 k-W * 5600 s
E= 110.6746 kW-hr

Weight of battery required can be calculated to provide this energy,
For LIB battery= 142 kg
As a rough estimate, individual lithium-ion batteries for use in small aircraft might typically weigh anywhere from 5 kg to 20 kg or more, depending on their capacity, energy density, and safety features (commonly used in electric aviation applications)

<u>About battery connections:</u>

Series Connection: When batteries are connected in series, the voltage adds up while the capacity (energy) remains the same. Connecting batteries in series is suitable when we need to achieve a higher voltage for your aircraft's power system. However, doing so won't increase the total energy capacity; it only affects the voltage.

Parallel Connection: When batteries are connected in parallel, the voltage remains the same, but the capacity (energy) adds up. Connecting batteries in parallel is suitable when we need to increase the total energy capacity (current or runtime) without changing the voltage.

- **Wing planform geometrical Calculations:**

We are designing an electric powered pilot training aircraft, for which the calculation of W/S and W_o has already been done. These are the values -

Calculation of Wing area and Power:

$$S = \frac{W_{TO}}{W/S_d}$$

W/S=633.588375 N/m^2 =64.5859 kg/m^2
W=6011.982065 N= 612.842 kg
L/D = 15(The operational L/D ratio)

$$S = \frac{6011.982065}{633.588375}$$

Wing Area= S = 9.48878 m^2

We know that,

$$L = \frac{\rho v^2 SCL}{2} = W$$

$$C_{Ld} = \frac{2(W/S)}{\rho v^2}$$

V_d=V=cruise velocity for the mission= 65m/s
Endurance= t= 5600 s= 1.55 hrs

Aspect ratio= AR= b^2/S
b=wingspan
S= wing area
Tapper ratio=λ=TR=C_t/C_R=Tip chord/Root chord
S=b/2(C_R)(1+λ)

C_R=2S/b(1+λ)
C_t=C_R* λ
Cruise altitude= 3 km
Hence rho =1.1209 kg/m^3

We will consider Tapper ratio from historical data= 0.6
We already have the wing loading parameter= 69.518 kg/m^2
Let's vary the W/S and AR to see its effect on geometrical parameters which define wing planform

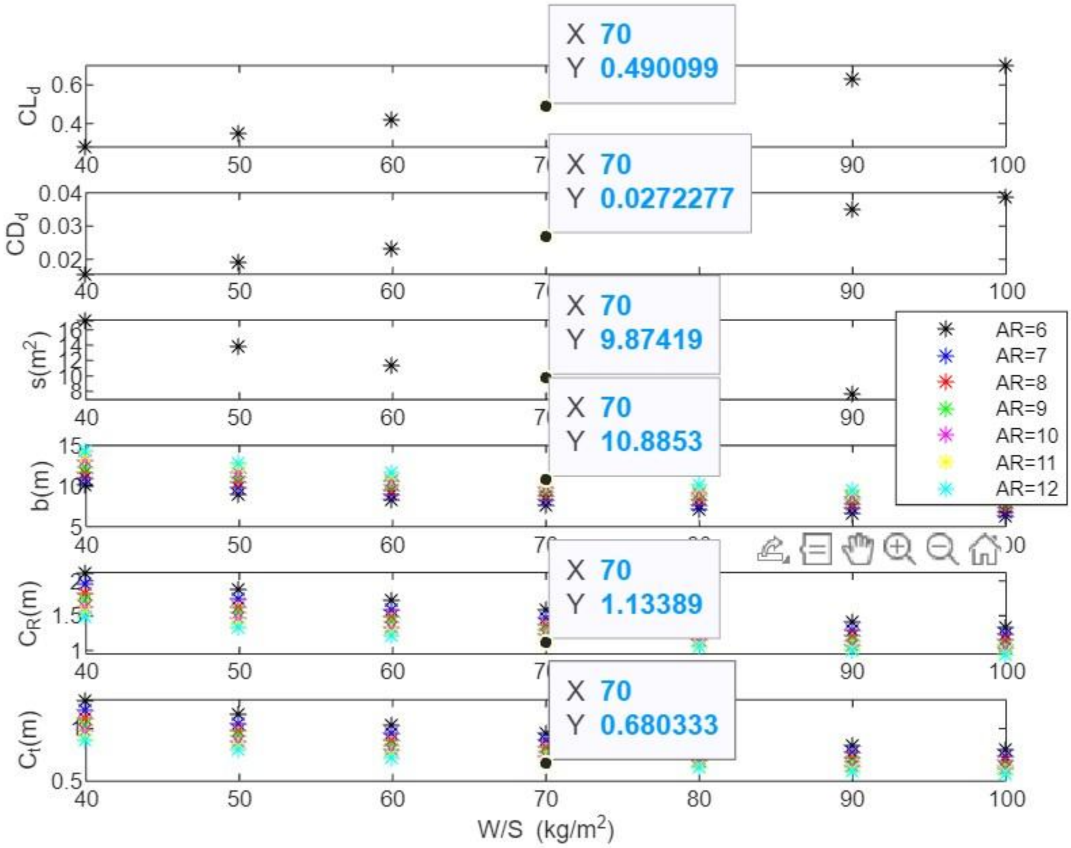
Tapper axis- It is an axis along which similar points(lets say quarter chord length) of all cross sectional chords lie.
We will taper the wing along the quarter chord length and will give back sweep of angle λ

To determine the complete geometry of wing we need to decide following factors

1. Aspect ratio- taken as 12
2. Tapper ratio- taken as 0.6
3. Tapper axis- tapered about quarter chord line axis

We wrote the matlab subroutine for the calculation of C_{Ld}, C_{Dd}, S, b, C_R, C_t with varying W/S(wing loading) and AR(Aspect ratio from 6 to 12)

The code is attached at the end of this report



We have already obtained W/S= 64.5859 kg/m^2, taking AR as 12, the values of C_{Ld}, C_{Dd}, S, b, C_R, C_t are as follows-

C_{Ld}=0.49

C_{Dd}=0.0272

S=9.874 m^2

b=10.88 m

C_R=1.133 m
C_t =0.68 m

- **Wing Airfoil:**

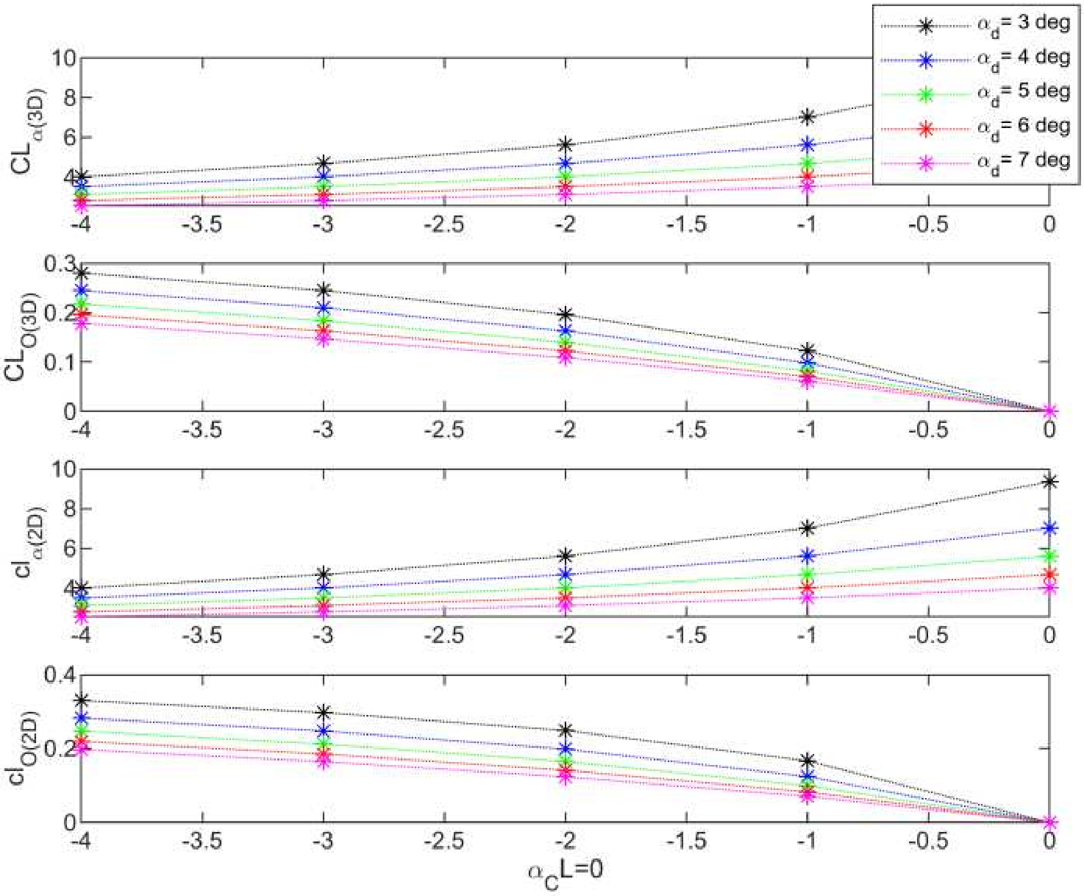
$$L=\frac{\rho v^2 SCL}{2}$$

$$C_{L\alpha}=C_{Ld}/(\alpha_d-\alpha_{CL=0})$$
$$C_{Ld}=C_{L\alpha}*(\alpha_d-\alpha_{CL=0})$$

$$C_{L\alpha,3D}=C_{Ld}/(\alpha_d-\alpha_{CL=0})$$
$$C_{L0,3D}= -\alpha_{CL=0}*C_{L\alpha,3D}$$

$$C_{L\alpha,2D}=C_{L\alpha,3D}/(1-(C_{L\alpha,3D}/pi*e*AR))$$
$$C_{L0,2D}= -\alpha_{CL=0}*C_{L\alpha,2D}$$

Using the parameters obtained till now we made calculations for $C_{L\alpha,3D}$,$C_{L0,3D}$,$C_{L\alpha,2D}$,$C_{L0,2D}$ by varying the $\alpha_{CL=0}$ from -4 to 0 and α_d from 3° to 7°
The code is attached at the end of this report

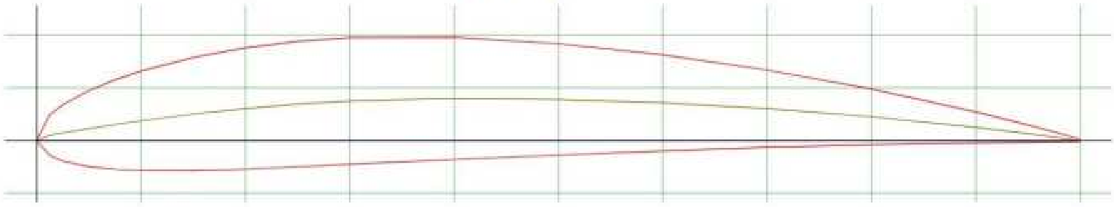


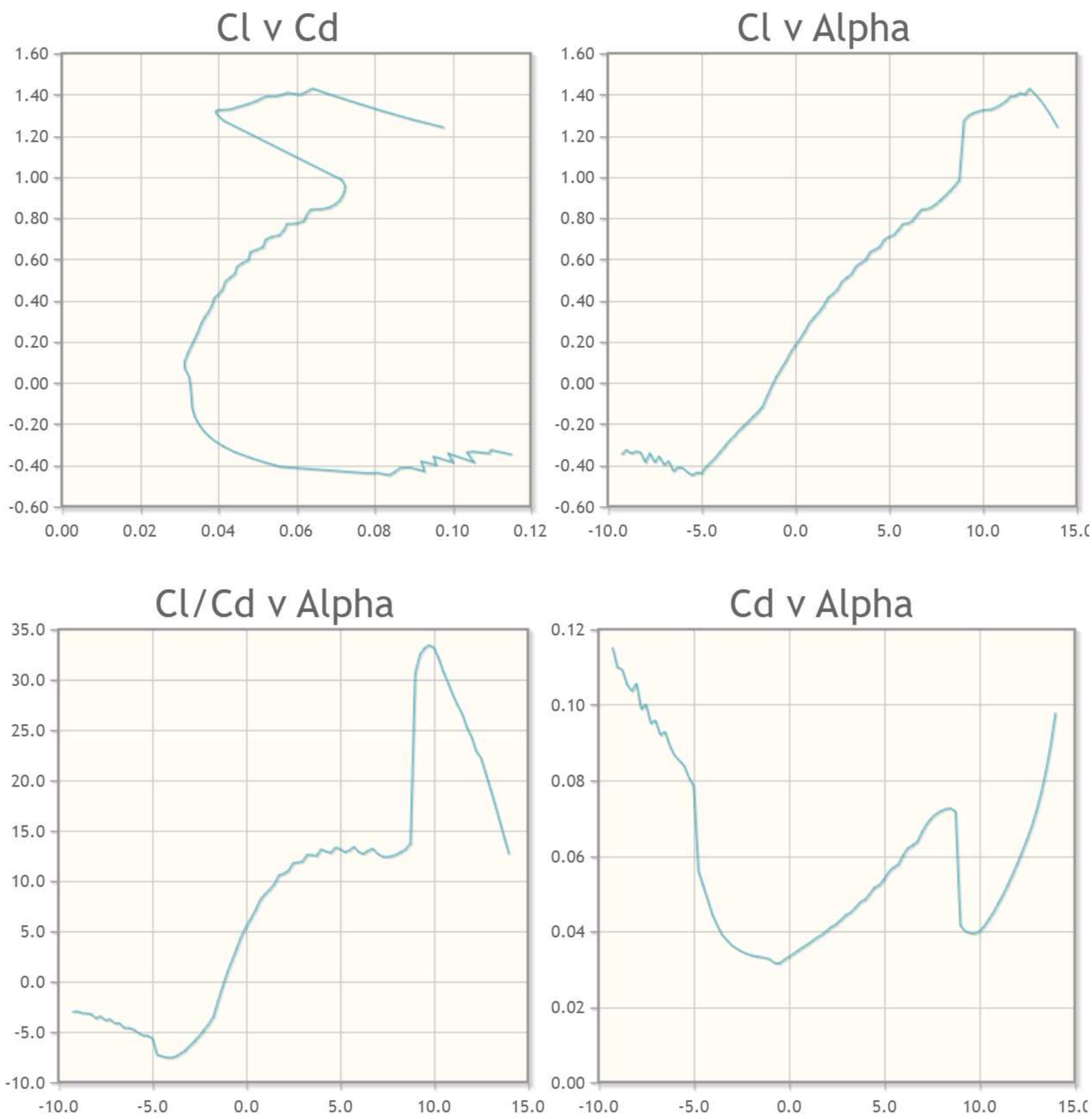
Now, we need to choose appropriate $C_{L\alpha,2D}$,$C_{L0,2D}$ (by searching in airfoil database)

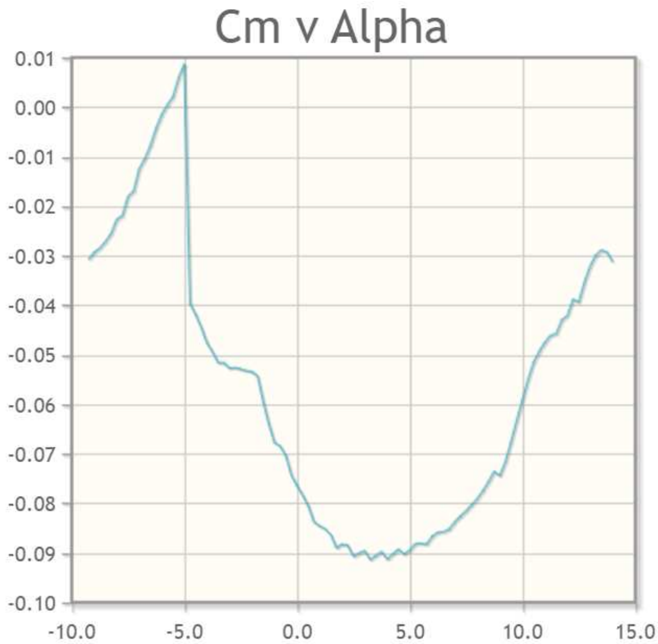
We found that NACA4412 at Reynolds no 50k has
$C_{L\alpha,2D}$=4.679 (per rad)=0.0816 (per deg)
$C_{L0,2D}$=0.198*(pi/180)=0.0034
$\alpha_{CL=0}$= -2°
For these values the α_d= 4°
Based on this the values of other parameters
$C_{L\alpha,3D}$=4.67916
$C_{L0,3D}$=0.1633

The plots for NACA4412 Airfoil are given below:

Airfoil NACA4412

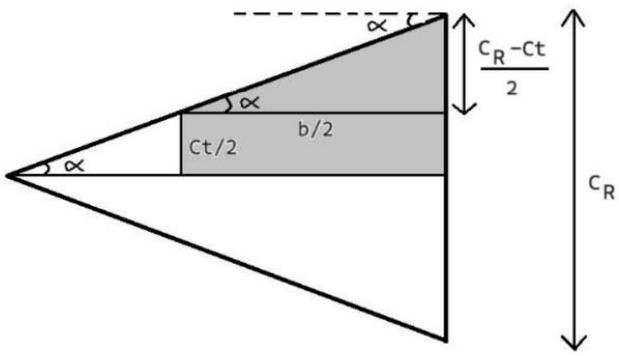






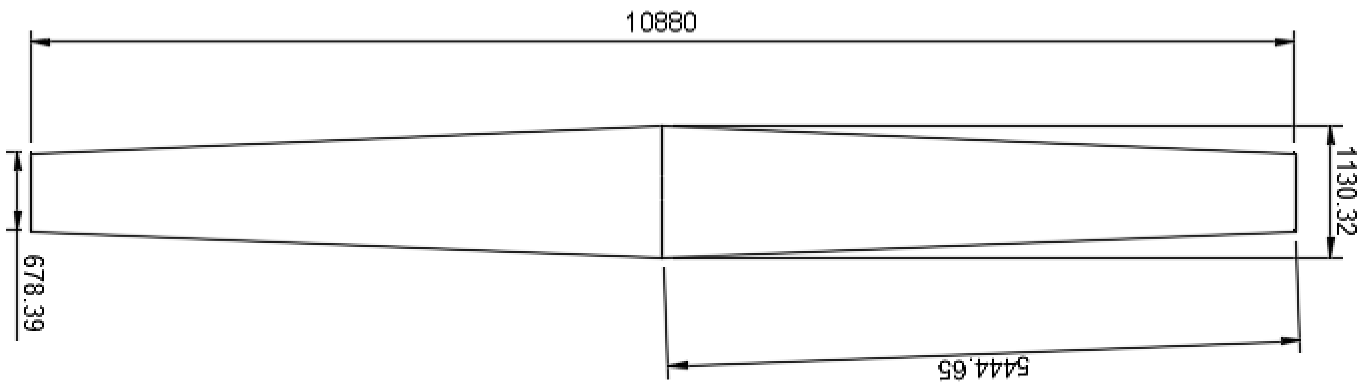
Designing Wing Planform:

The list of parameters that will be helpful to design wing planform
Aspect ratio- taken as 12
Tapper ratio- taken as 0.6
Tapper axis- tapered about quarter chord line axis
S=9.874 m2
b=10.88 m
C_R=1.133 m
C_t =0.68 m



From figure, tanα=(C_R-C_t)/b
α=tan^{-1}((CR-Ct)/b)
Hence, α=2.260°

- **Wing 3D CAD Design:**



- **Neutral Point**

1. Important Aircraft Point Calculation:

i. Neutral point, $CL_{\alpha t}$ and S_t/S calculation:

The equation for the calculation of neutral point is,

$$\bar{X}_{NP} = \frac{C_{L_{\alpha w}}\bar{X}_{ac_w} + \eta\left(\frac{S_t}{S}\right)\left(1-\frac{\partial\epsilon}{\partial\alpha}\right)C_{L_{\alpha_t}}\bar{X}_{ac_t}}{C_{L_{\alpha w}} + \eta\left(\frac{S_t}{S}\right)\left(1-\frac{\partial\epsilon}{\partial\alpha}\right)C_{L_{\alpha_t}}}$$

Since we have already done the task of wing airfoil selection, We have all the data which is fully dependent on wing:

$CL_{\alpha w} = 4.67916\ rad^{-1}$ (calculated)

$\frac{d\epsilon}{d\alpha} = 2 * \frac{CL\alpha w}{\pi e AR}$ (It is assumed that $CL\alpha \approx CL\alpha w$)

Here, e = 0.706 and AR = 12

$$\frac{d\epsilon}{d\alpha} = 0.3516$$

Mean aerodynamic centre of the wing is calculated by,

$$X_{ac,w} = \frac{2}{s}\int_0^{b/2} c(y)^2\ dy$$

By taking parameters of wing, s = 9.874 m^2, b = 10.88 mm

from the parameters of wing and cad model, the equation for chord length as a function of y can be written as:

c(y) = -0.083y+1.133

By putting all parameters and equation, after solving the integral, the mean aerodynamic centre calculated is:

$$\overline{X_{ac,w}} = 0.9256$$

After analysing and taking the fuselage length around 6.2m, we decided to take the mean aerodynamic centre of tail from the leading edge of wing is,

$$\overline{X_{ac,t}} = 4.82625$$

The efficiency factor for the calculation is taken as, η = 0.8

After fixing all other values, for the estimation of Neutral point, we have plotted the curve between neutral point and $CL_{\alpha t}$ by varying the S_t/S value from 0.1 to 0.6. We obtained the following plot:

By analysing some more aircrafts, we have decided to keep the value of S_t/S is 0.2, as typically it's in the range of 0.2-0.3.

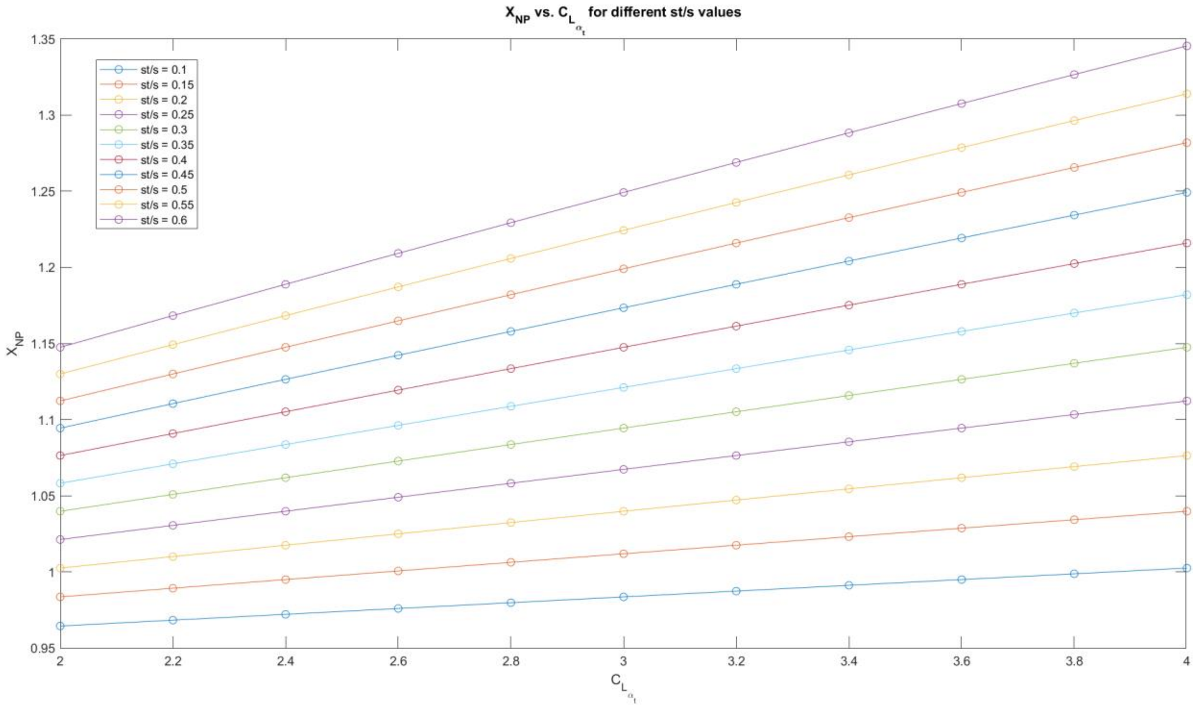
Also, as reference for a similar aircraft (similar weight and planform area), the $CL_{\alpha t}$ value we decided to take is around 2.4.

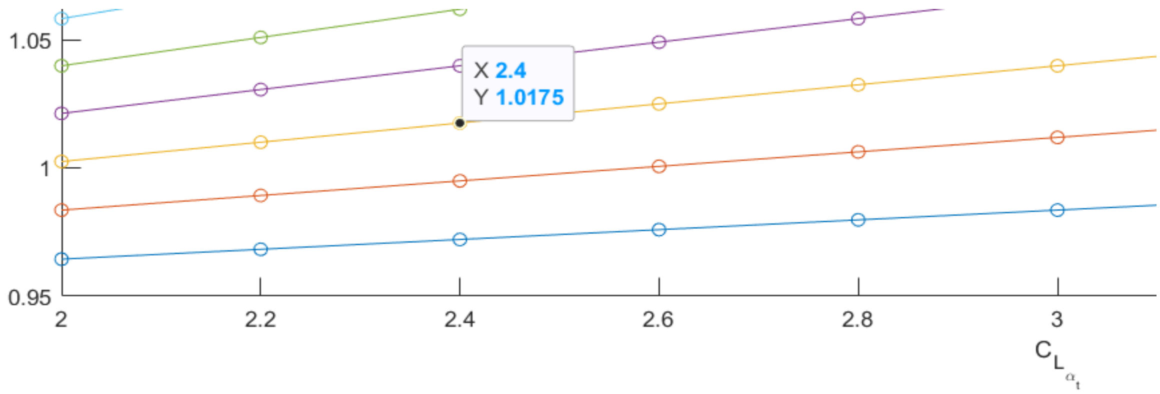
By fixing these two values, the neutral point obtained is,

$$\overline{X_{NP}} = 1.0175$$

ii. <u>$C_{L\alpha}$ calculation:</u>

The equation for CL_{α} is:





$$C_{L_\alpha} = C_{L_{\alpha w}} + \eta\left(\frac{S_t}{S}\right)\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)C_{L_{\alpha t}}$$

We have calculated all the values in previous step, by putting these values CL_{α} obtained is:

$CL_{\alpha} = 4.92814\ \text{rad}^{-1}$

iii. <u>Calculation of Centre of gravity:</u>

By taking Static margin of 10%, the location of centre of gravity from the leading edge of wing is:

S.M. = $\overline{X_{NP}} - \overline{X_{cg}}$

$$\overline{X_{cg}} = \overline{X_{NP}} - 0.1$$
$$\overline{X_{cg}} = 1.0175 - 0.1$$
$$\overline{X_{cg}} = 0.9175$$

Since, $\overline{X_{cg}} < \overline{X_{ac,w}}$ All above considered values are valid for stable flight operations.

$X_{cg} = \overline{X_{cg}} * \bar{c}$

$X_{cg} = 0.831713$ m

iv. Calculation of C_{mo}:

The equation for C_{mo} is:

$$C_{m_0} = -(-SM \times C_{L_\alpha})\alpha_{design}$$

We already have static margin, 0.1, CL_{α}. we have calculated previously and α_{design} we have calculated in previous exercise, and it was 4°.

By putting all the values,

C_{mo} = 0.0344049

2. **Tail Settling Angle Calculation:**

With the help of expression of C_{mo}, the angle i_t can be calculated.

$$C_{m_0} = C_{m_{acw}} + C_{L_{0w}}(\bar{X}_{cg} - \bar{X}_{ac_w}) - \eta\left(\frac{S_t}{S}\right) C_{L_{\alpha_t}}(\bar{X}_{ac_t} - \bar{X}_{cg}) i_t$$

Here $C_{m,ac,w} = C_{mo} + C_L(\frac{dC_m}{dC_L})$

We got C_{mo} from the NACA4412 airfoil graph of C_m vs α.

C_{mo} obtained is -0.075

$C_L = \frac{W}{\frac{1}{2}*\rho*v^2*s}$

C_L = 0.476 (at cruise)

$C_{m,ac,w}$ = -(0.075) –(0.1*0.476)

$C_{m,ac,w}$ = -0.1226

and C_{Low} can be written directly from the graph of NACA airfoil and it is 0.2.

i_t obtained from the equation is,

i_t = -1.396°

Matlab Code for all above calculations and plots is given at the end of this report

- **Horizontal Tail:**

i. <u>Calculation of tail volume ratio:</u>

$\overline{V_H} = S_t * \frac{l_t}{s*\bar{c}}$ (all parameters are previously calculated)

$\overline{V_H} = 0.78175$

ii. <u>Calculation of St:</u>

$S_t = S_{t/}\ S*S$

$S_t = 0.2*9.874$

$S_t = 1.9748\ m^2$

iii. <u>Calculation of root chord, tip chord, and span:</u>

Using historical data and by analysis of typical values of taper ratios, we have considered the AR of the tail as 8 and the taper ratio as 0.75.

Lets calculate b using AR = b^2/S_t

$b=\sqrt{AR}/S_t$

b = 3.9747 m

Now, $S_t = \frac{b}{2} * C_{R,t}(1+\lambda)$

$C_{R,t} = 0.568$ m

Now, with the help of taper ratio,

$C_{t,t} = \lambda * C_{R,t}$

$C_{t,t} = 0.426$ m

Let's take tapered axis as quarter chord line, then

$\Lambda = \tan^{-1}((Cr - Ct)/b)$

$\Lambda = 2.04459°$

iv. <u>Calculation of Ostwald efficiency factor:</u>

$$e = 1.78(1 - 0.045AR^{0.68}) - 0.64$$

As we have AR = 8

e = 0.810592

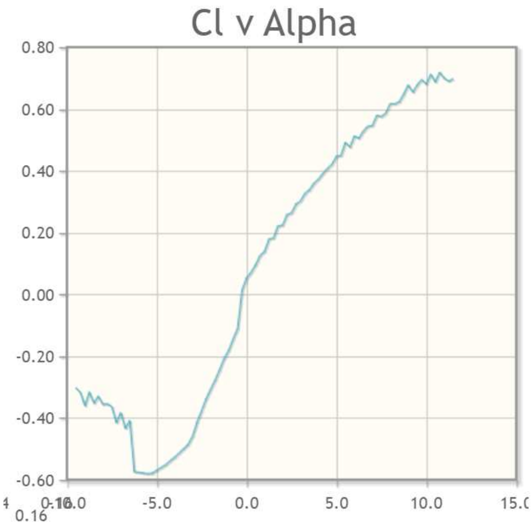
v. <u>$CL_{\alpha t(2D)}$ Calculation</u>:

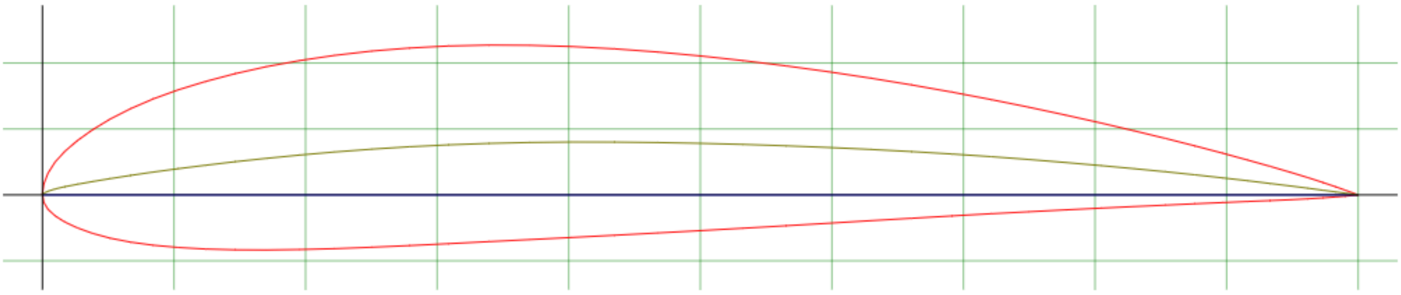
$$C_{L_{\alpha_t}(2D)} = \frac{C_{L_{\alpha_t}(3D)}}{1 - \frac{C_{L_{\alpha_t}(3D)}}{\pi e_t AR_t}}$$

$CL_{\alpha t(2D)} = 2.7205$

1. **Airfoil Selection:**

We found that NACA4415 is an appropriate choice for all of the requirements of the horizontal tail and satisfy the values of $C_{L\alpha 2D}$.





2. **Elevator Calculation:**

we have S_t=1.9748 m^2 Hence,

S_e =(S_e / S_t)* S_t

For different **τ** we get different S_e

The relation between **τ** and S_e/S_t was taken from plot provided in one of the exercises.

Control surface Effectiveness (τ)	Se/St	Elevator surface area m^2 (Se)
0.22	0.1	0.19748
0.36	0.2	0.39496
0.5	0.3	0.59244
0.58	0.4	0.78992
0.66	0.5	0.9874
0.74	0.6	1.18488
0.8	0.7	1.38236

From historical data of Se/St ratio of various aircrafts, we chose to select Se/St=0.3
τ (control surface effectiveness) associated with it would be 0.5

Hence the S_e= 0.59244

Determine chord ($\bar{c}$) and span(b) of the control surface using area information

For the horizontal tail we know following parameters:
S_t = 1.9748 m^2
b_t = 3.9747 m
$\bar{c_t} = 0.4968\, m$
For the calculation of Elevator parameters:
We already have, S_e = 0.59244 m^2

Choosing the wingspan for elevator:
Control Effectiveness: A smaller elevator might offer more concentrated control effectiveness within a smaller area. It can potentially provide sharper and more precise control over the pitch compared to a larger elevator.

Aerodynamic Interference: A smaller elevator might reduce potential aerodynamic interference with the airflow around the tail section, which could improve overall aerodynamic efficiency. However, it might also reduce the effectiveness of the control surface in certain flight conditions.

Hence, we selected elevator with wingspan b=3.5 meters
Now, with the help of wingspan, $\bar{c_e} = \frac{S_e}{b}$
$\bar{c_e} = 0.169268\, m$

Calculation of control surface derivatives ($C_{L\delta e}$, $C_{m\delta e}$)using finalized, $\boldsymbol{C_{L\alpha t}}$ and $\boldsymbol{V}_H$

$$C_{L_{\delta_e}} = \eta \left(\frac{S_t}{S}\right) C_{L_{\alpha_t}} \tau$$
$$C_{m_{\delta_e}} = -\eta V_H C_{L_{\alpha_t}} \tau$$

we have already calculated $CL_{\alpha t}$ and V_H, the obtained values were:

$CL_{\alpha t}$ = 2.4 rad^{-1} and V_H = 0.78175

S_t/S = 0.2, η = 0.8 and τ = 0.5

By putting all these values in the equation, we will obtain

$CL_{\delta e}$ = 0.8*0.2*2.4*0.5

$CL_{\delta e}$ = 0.192 rad^{-1}

and Similarly, $Cm_{\delta e}$ = -0.8*0.78175*2.4*0.5

$Cm_{\delta e}$ = -0.75048 rad^{-1}

3. **Performance of the design for various trim conditions.**

With the help of following equations, the various parameters at trim condition have been calculated by iterating the δ_e angle.
The equations are:

$$\delta_{trim}(i,1) = \delta_e\left(\frac{\pi}{180}\right)$$
$$\alpha_{trim}(i,1) = -(C_{m_0} + C_{m_{\delta_e}}\delta_{e_{trim}}(i,1)$$
$$C_L(i,1) = C_{L_0} + C_{L_\alpha}\alpha_{trim}(i,1) + C_{L_{\delta_e}}\delta_{e_{trim}}(i,1)$$
$$T_R(i,1) = \left(\frac{1}{2}\rho s V_{trim}^2(i,1)\right)(C_{D_0} + kC_L(i,1)^2)$$
$$P_R(i,1) = T_R(i,1) \times V_{trim}(i,1)$$

V_{trim}^2 (i,1)= 2*(W/S)/(ρ*CL_{trim}(i,1))

Here, all the parameters we have calculated earlier in the lab exercise 6 and 7.
C_{mo} = 0.0344049
$Cm_{\delta e}$ = -0.75048 $radian^{-1}$
CL_α = 4.92814 rad^{-1}
$CL_{\delta e}$ = 0.192 $radian^{-1}$
ρ = 1.1209 (at cruise altitude = 3km)
W/S = 633.588
S = 9.874
C_{D0} = 0.203466
k = 0.037571246

CL_0 can be calculated by using the formula:

$$C_{L0} = C_{L0w} + \eta * \left(\frac{S_t}{S}\right) * C_{L\alpha t} * i_t$$

by using, $C_{L0w} = 0.2, \frac{S_t}{S} = 0.2, \eta = 0.8, C_{L\alpha t} = 2.4\ and\ i_t = -1.396°$

$$C_{L0} = -0.336064$$

Cm_α can be calculated by using the formula:

$$C_{m\alpha} = C_{L\alpha w} * (\overline{X_{cg}} - \overline{X_{ac,w}}) - (\eta * \left(\frac{S_t}{S}\right) * C_{L\alpha t} * \left(1 - \frac{d\epsilon}{d\alpha}\right) * \overline{X_{ac,t}} - \overline{X_{cg}}$$

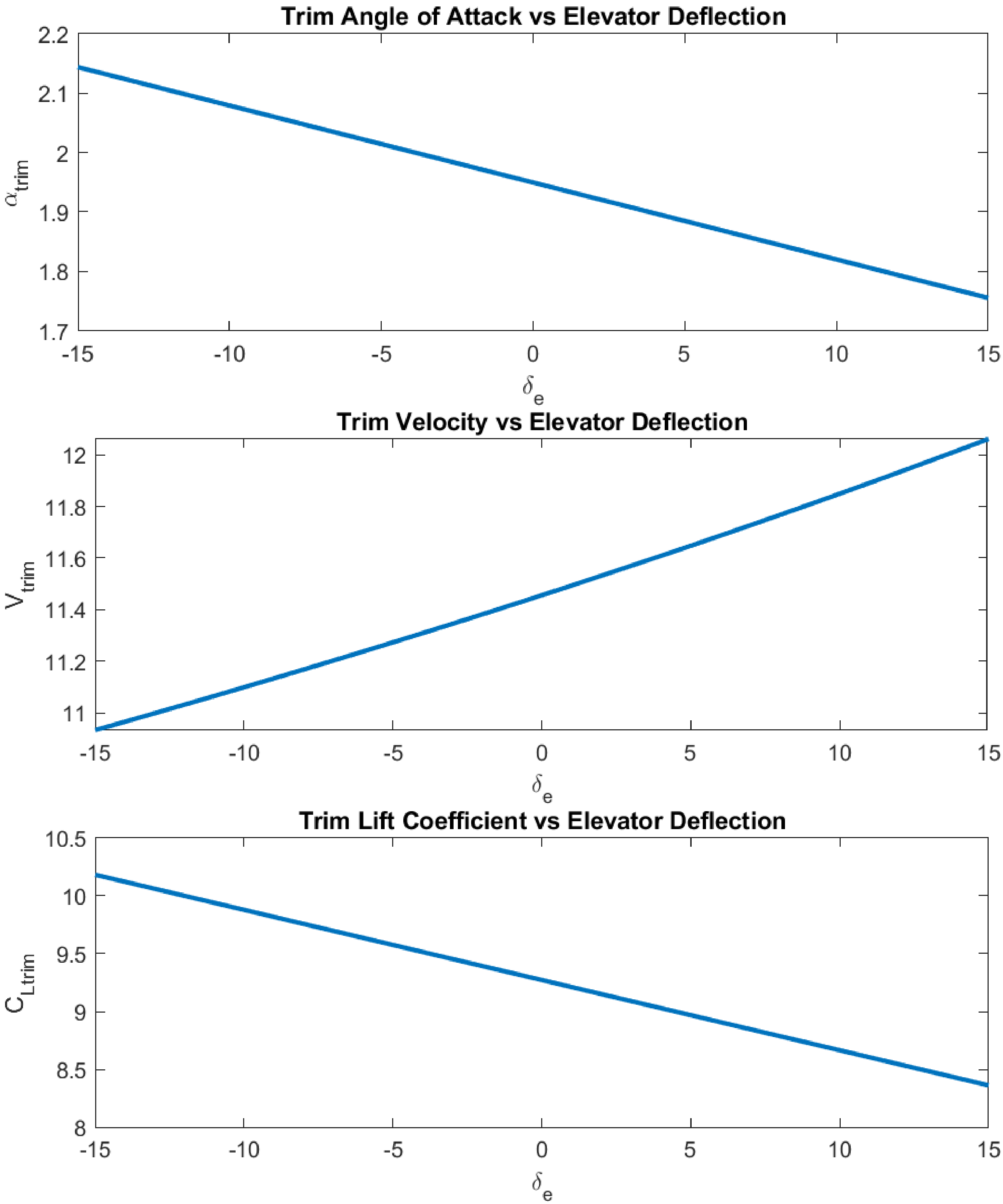
by using, $C_{L\alpha w} = 4.67916, \overline{X_{cg}} = 0.9175, \overline{X_{ac,w}} = 0.9256, \frac{d\epsilon}{d\alpha} = 0.3516\ and\ \overline{X_{ac,t}} = 4.82625$

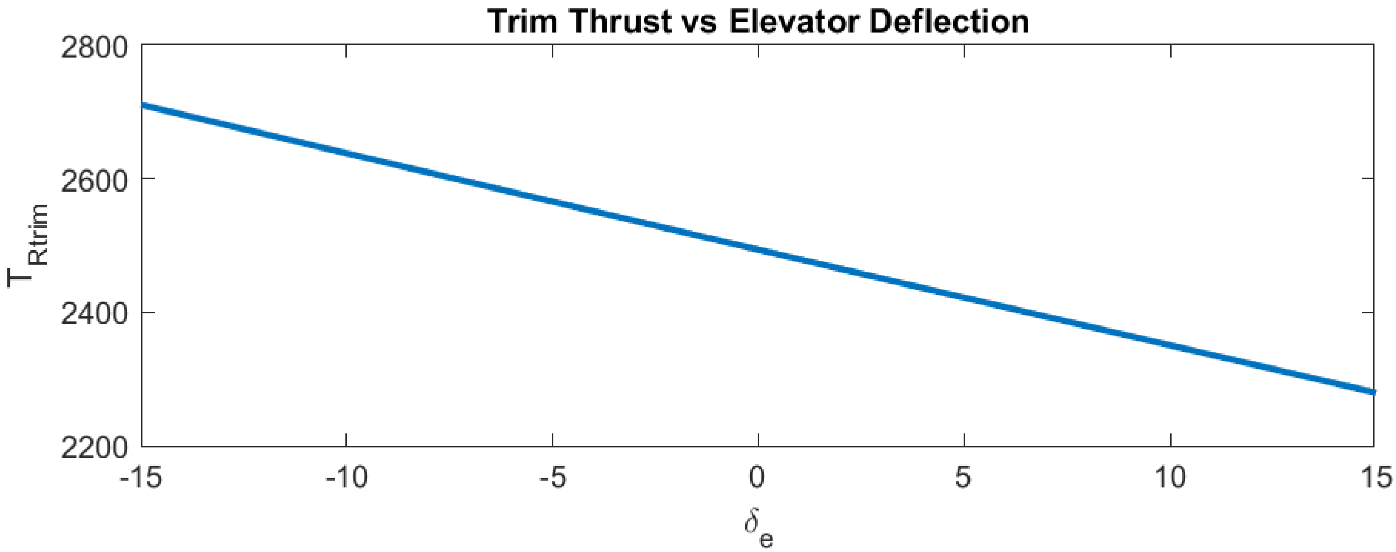
$$C_{m\alpha} = -1.0111\ rad^{-1}$$

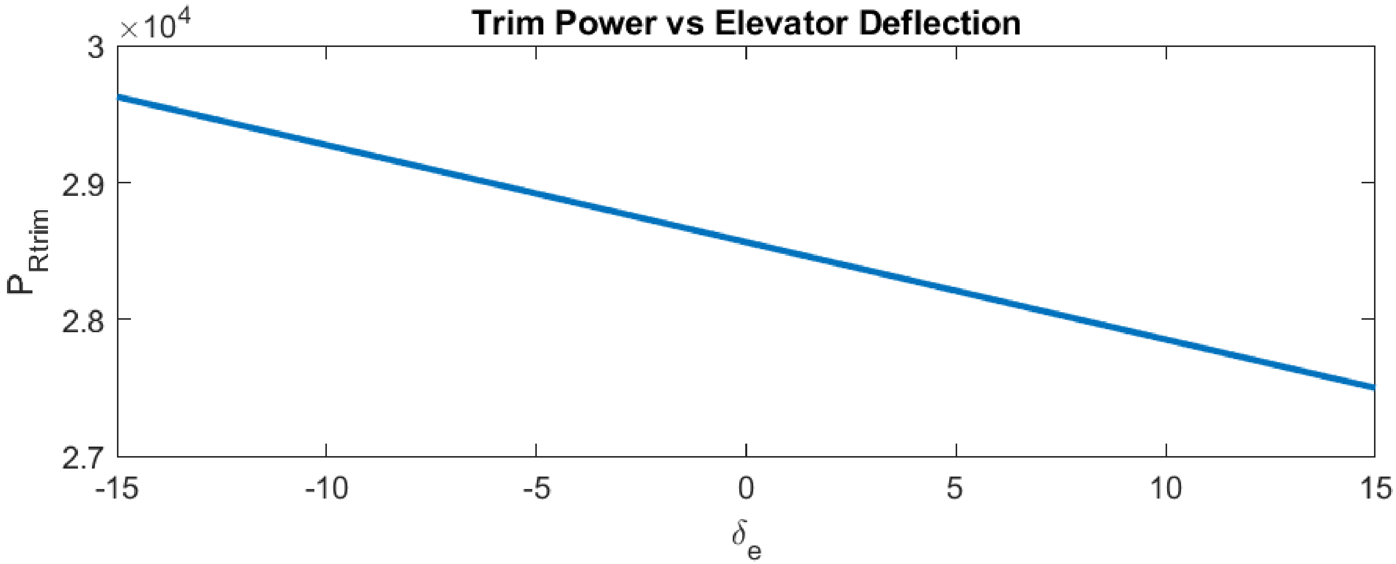
At the trip conditions the iterated values at different δ_e are:

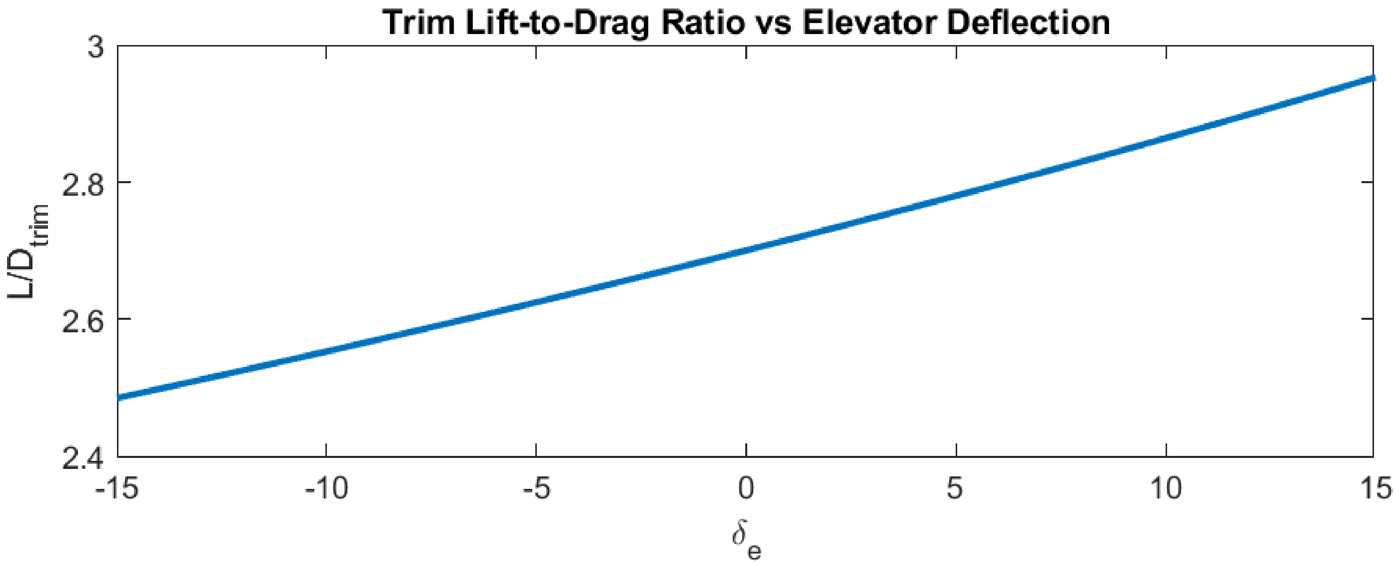
```
alpha_trim:

2.1439   2.1310   2.1180   2.1051   2.0921   2.0792   2.0662   2.0533   2.0403   2.0273   2.0144
2.0014   1.9885   1.9755   1.9626   1.9496   1.9367   1.9237   1.9108   1.8978   1.8848   1.8719
1.8589   1.8460   1.8330   1.8201   1.8071   1.7942   1.7812   1.7683   1.7553

CL_trim:

10.1793   10.1188   10.0583   9.9978   9.9373   9.8768   9.8163   9.7558   9.6953   9.6349   9.5744
9.5139   9.4534   9.3929   9.3324   9.2719   9.2114   9.1509   9.0904   9.0299   8.9695   8.9090
8.8485   8.7880   8.7275   8.6670   8.6065   8.5460   8.4855   8.4250   8.3646

V_trim:

10.9334   10.9661   10.9990   11.0322   11.0657   11.0996   11.1337   11.1682   11.2030   11.2381
11.2735   11.3093   11.3454   11.3819   11.4187   11.4559   11.4935   11.5314   11.5697   11.6084
11.6475   11.6869   11.7268   11.7671   11.8078   11.8490   11.8905   11.9325   11.9750   12.0179
12.0613

T_R:

1.0e+03 *
2.7099   2.6954   2.6809   2.6665   2.6520   2.6375   2.6231   2.6086   2.5942   2.5798   2.5654
2.5510   2.5366   2.5222   2.5079   2.4935   2.4792   2.4649   2.4506   2.4363   2.4220   2.4077
2.3935   2.3792   2.3650   2.3508   2.3366   2.3224   2.3083   2.2941   2.2800


P_R:

1.0e+04 *
2.9629   2.9558   2.9488   2.9417   2.9346   2.9275   2.9205   2.9134   2.9063   2.8992   2.8921
2.8850   2.8779   2.8708   2.8637   2.8566   2.8495   2.8423   2.8352   2.8281   2.8210   2.8139
2.8068   2.7997   2.7926   2.7855   2.7784   2.7713   2.7642   2.7571   2.7500

L/D:

2.4849   2.4982   2.5117   2.5254   2.5392   2.5531   2.5671   2.5813   2.5957   2.6102   2.6249
2.6397   2.6546   2.6698   2.6851   2.7005   2.7161   2.7319   2.7478   2.7640   2.7803   2.7967
2.8134   2.8302   2.8472   2.8645   2.8819   2.8994   2.9172   2.9352   2.9534
```

MATLAB Code is added at the end of report

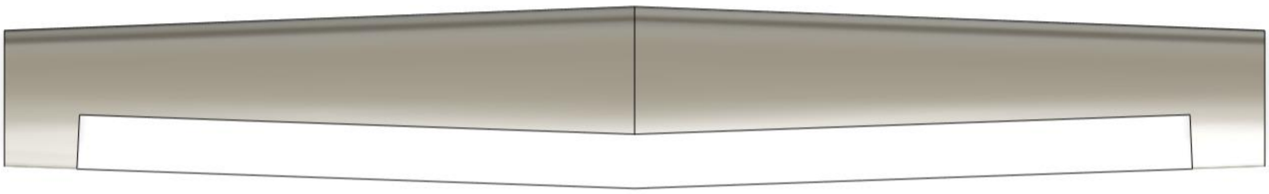

Trim Angle of Attack vs Elevator Deflection
2.2
2.1
2
1.9
1.8
1.7
α_{trim}
-15
-10
-5
0
5
10
15
δ_e
Trim Velocity vs Elevator Deflection
12
11.8
11.6
11.4
11.2
11
V_{trim}
-15
-10
-5
0
5
10
15
δ_e
Trim Lift Coefficient vs Elevator Deflection
10.5
10
9.5
9
8.5
8
C_{Ltrim}
-15
-10
-5
0
5
10
15
δ_e

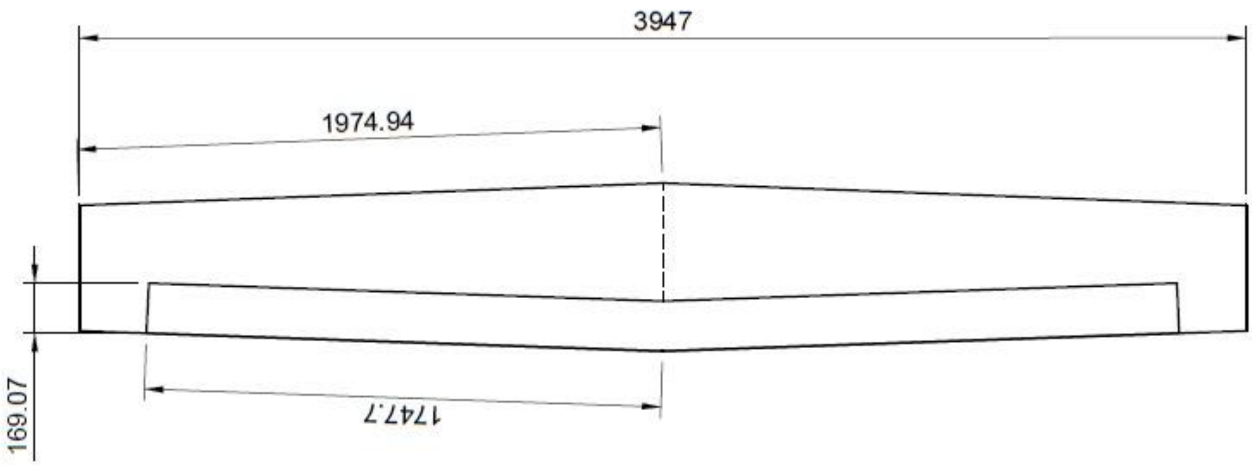

Trim Thrust vs Elevator Deflection
2800
2600
2400
2200
T_{Rtrim}
-15
-10
-5
0
5
10
15
δ_e


Trim Power vs Elevator Deflection
$\times 10^4$
3
2.9
2.8
2.7
P_{Rtrim}
-15
-10
-5
0
5
10
15
δ_e


Trim Lift-to-Drag Ratio vs Elevator Deflection
3
2.8
2.6
2.4
L/D_{trim}
-15
-10
-5
0
5
10
15
δ_e

4. Tail and Elevator 3D CAD Design



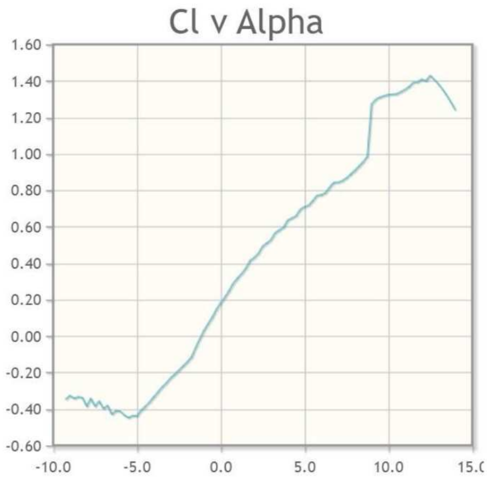


- **V-n Diagram:**

Plotting the V-n diagram of the design under various structural design considerations.
Let's take,
Maximum positive limit load factor as +3, and
Maximum negative limit load factor be be -1.8
(values were suggested by prof during class)

Now,
L=nW
For n=3, L=3W
Wing Airfoil- NACA4412

The plot of $C_{L\alpha,2D}$ Vs α for NACA4412



<u>For 2-D case:</u>

$C_{L,max}=C_{Lo}+C_{L,\alpha}*\alpha_{stall}$

For NACA4412 airfoil which is the airfoil for our wing,

$C_{L,o}=0.2$

α_{stall}(for linear part)=8°

$C_{L,\alpha}=0.1$ per degree

Hence,

$C_{L,max}=0.2+(0.1*8)$

$C_{L,max}=1$

<u>For 3-D case:</u>

$C_{L\alpha,3D}=\frac{CL\alpha,2D}{1+\frac{CL\alpha,2D}{\pi*e*AR}}=\frac{0.1}{1+(0.03757*0.1)}=0.0996$

$C_{L0,3D}= -\alpha_{CL=0}*C_{L\alpha,3D}=2*0.0996=0.1992$

$C_{L,max,3D}=C_{Lo,3D}+C_{L,\alpha,3D}*\alpha_{stall}$
$C_{L,max,3D}=0.1992+0.0996*8$

$C_{L,max,3D}=0.996$
$V_{\infty}=\sqrt{\frac{2*n*W/S}{\rho*Cl}}$

Let's define some points
Point S, Stall speed(V_S) at level flight and Load factor (n)=1
Point A, positive load factor limit(n)=3, maneuvering speed
Point C, Design Cruise speed (V_C), n=1
Point D, Design Diving speed(V_D)=1.4*V_C

From point A to C, C_L is reduced to increase velocity.

Calculation of V_{stall}
$V_{stall}=\sqrt{\frac{2*n*W/S}{\rho*Cl(max)}}=\sqrt{\frac{2*1*(681.9716)}{1.225*(0.996)}}=33.4349$ m/s

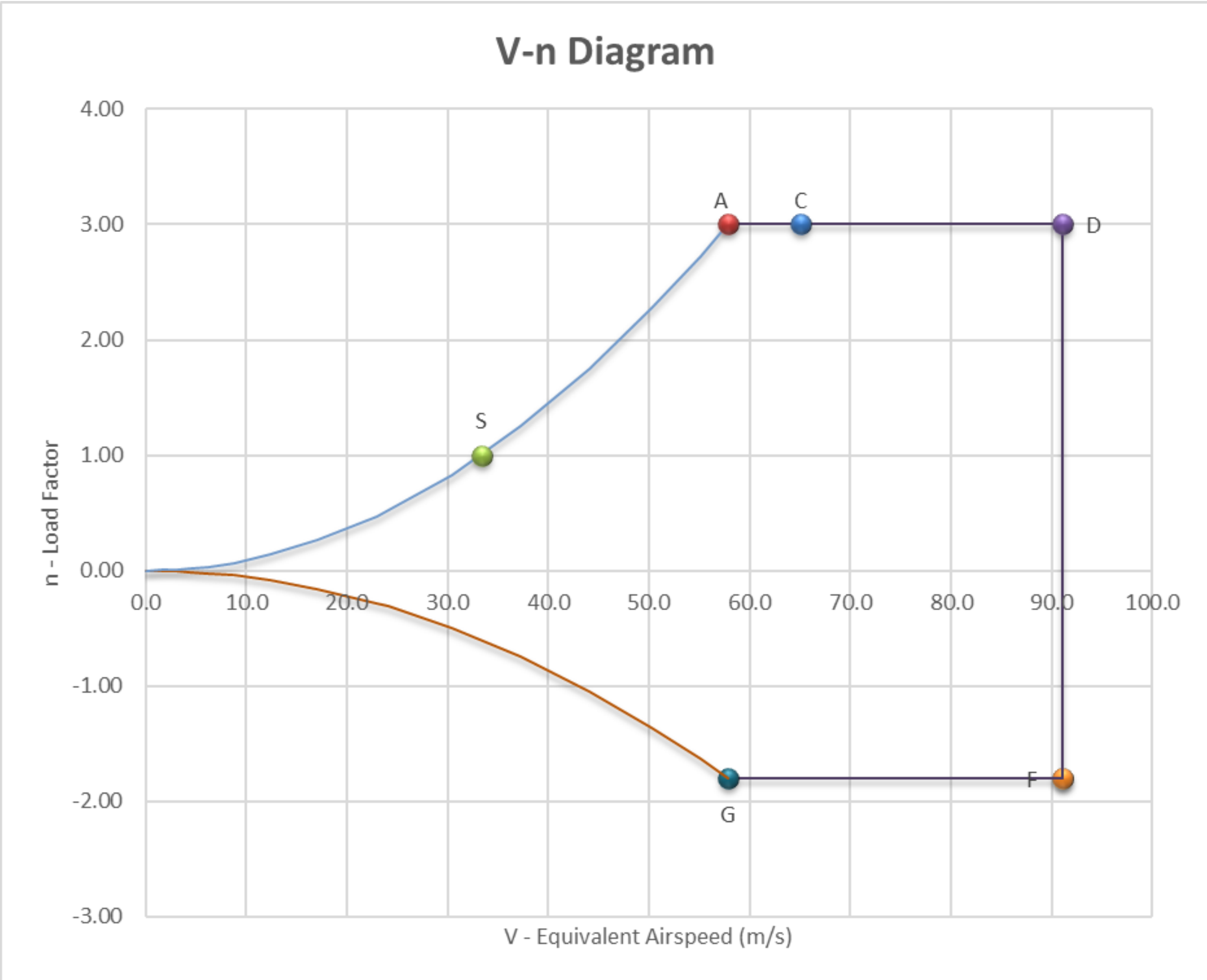
Calculation of V_A=
At cruise altitude (3 km), Rho=1.1209

But for V-n diagram we plot Equivalent air- speed(at sea-level) on x axis hence rho=1.225

$V_A=\sqrt{\frac{2*n*W/S}{\rho*Cl(max)}}=\sqrt{\frac{2*3*(681.9716)}{1.225*(0.996)}}=57.905$ m/s

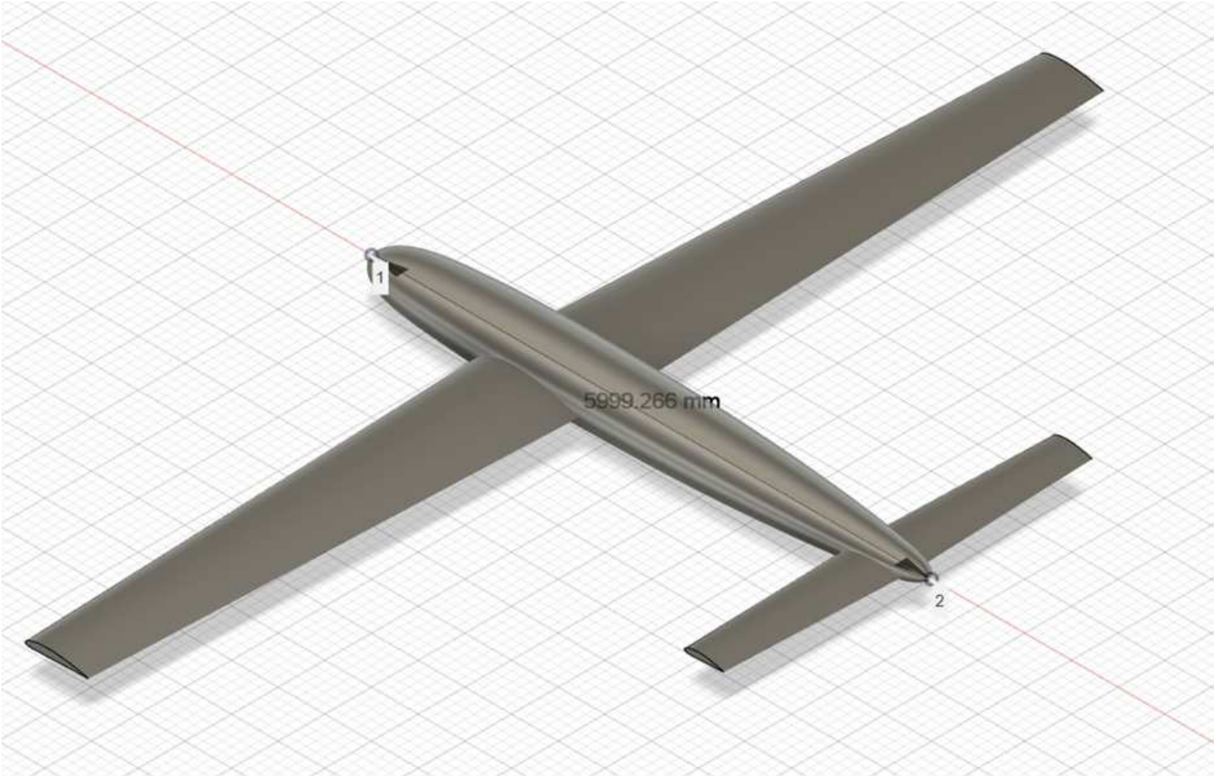
From mission profile, the design cruise speed was 65 m/s

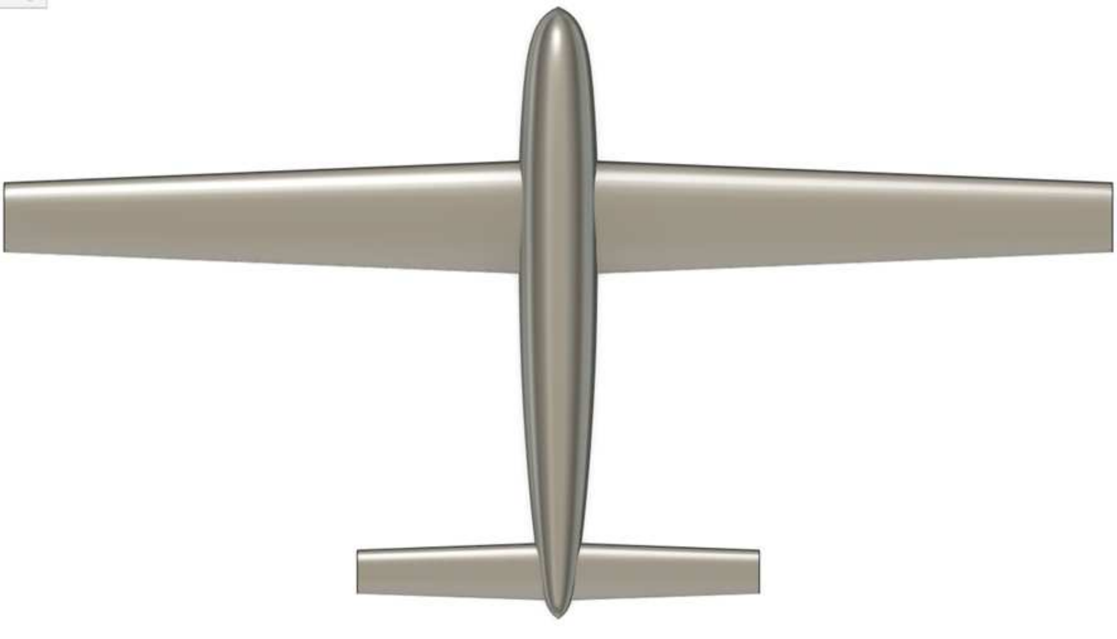
Hence the V_C= 65 m/s
Hence the V_D= 1.4*65=91 m/s

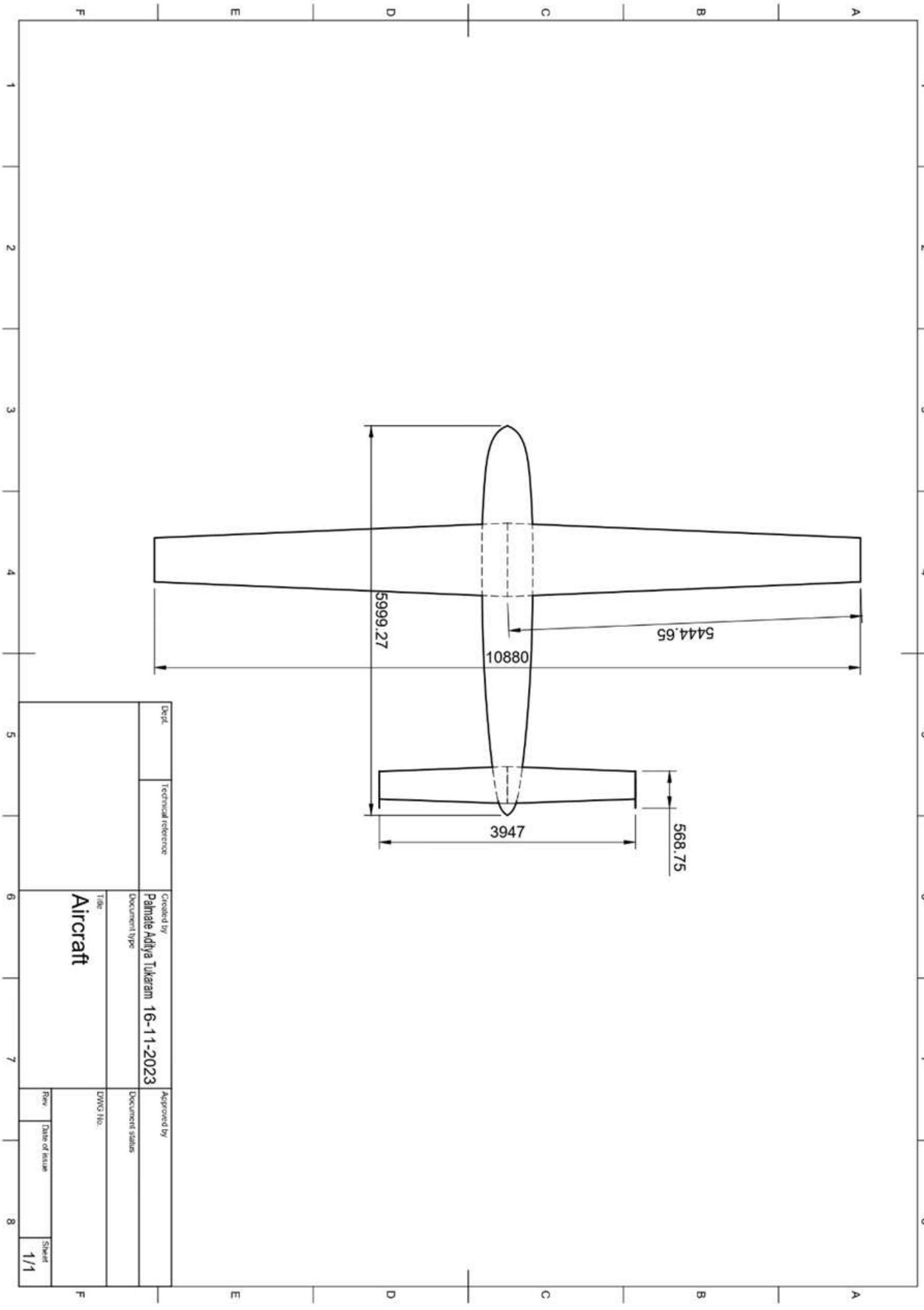


Point	V(m/s)	n(Load factor)
S	33.435	+1
A	57.905	+3
C	65	+3
D	91	+3
F	91	-1.8
G	57.79	-1.8

- **Engineering Drawing of Aircraft:**







5999.27
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Dept.
Technical reference
Created by
Palmate Aditya Tukaram
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Title
Aircraft
DWG No.
Rev
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1/1

- **MATLAB Codes**

Google Drive Link for all codes and cad designs-[Links_Group_3]

1. **Battery- weight iteration**

```
g0 = 9.81;
iteration = 50;
Wt0 = zeros(11, iteration);
Wt0(1) = 556.752 * g0;
Wpayload = 200 * g0;
St_ratio = 0.65; %structural ratio(strucutre weight ratio)
wing_span = 10.7;
AR = 12;
k = 1/(pi*0.85*AR);
rho = 1.225;
Area = wing_span^2/AR;
SED = 0.4; %Specific Energy Density
LD = 10:20; %L/D range
eeta_battery = 0.95; %battery efficiency
safety_factor = 1.2;
eeta_propeller = 0.9; %Propeller efficiency
V = 65; %cruise speed in m/s
Endurance = 5600; %Endurance in seconds
Thrust_required = zeros(1, length(LD));
Power_required = zeros(1, length(LD));
Energy_required = zeros(1, length(LD));
W_battery = zeros(1,length(LD));
Cl = zeros(1,length(LD));
Cd = zeros(1,length(LD));
Cd0 = zeros(1,length(LD));
for a = 1:length(LD)
    for i = 1:(iteration)
        Ecruise = (Wt0(a, i) * V * Endurance) / LD(a);
        Erequired = (safety_factor * Ecruise) / (eeta_propeller * eeta_battery);
        Wbattery = Erequired / (SED * 3600000);
        Wt0(a, i + 1) = St_ratio * Wt0(a, i) + Wpayload + Wbattery;
    end
    Thrust_required(a) = Wt0(a, iteration) / LD(a);
    Power_required(a) = Thrust_required(a) * V;
    Energy_required(a) = Power_required(a) * Endurance;
    W_battery(1,a) = Wbattery;
    Cl(1,a) = sqrt(2*Wt0(a,iteration)/(rho*V^2*Area));
    Cd(1,a) = sqrt(2*Thrust_required(1,a)/(rho*V^2*Area));
    Cd0(1,a) = Cd(1,a) - k*Cl(1,a)^2;

end
```

2. Matching Plot Code

Vstall

```
[ ] WbyS_Vs = 0.5*rho0*pow(Vs,2)*ClMax
```

```
[ ] print("W/S for Vstall condition = ",WbyS_Vs)

    W/S for Vstall condition =  633.588375
```

```
[ ] plt.axvline(WbyS_Vs,color='r')
    plt.grid();plt.xlabel('W/S [N/m2]');
    plt.ylabel("W/P [N/W]");
    plt.legend(['Vs']);
    plt.xlim((1,1200));
    plt.fill_between(WbyS[:int(WbyS_Vs)],0.1,facecolor='r',alpha=0.2);
```

Vmax

```
[ ] WbyP_Vmax = sigma*etaP/((0.5*rho*pow(Vmax,3)*Cd0/(WbyS)) + ((2*K*WbyS/(rho*Vmax))))
```

```
[ ] plt.plot(WbyS,WbyP_Vmax,"-");
    plt.grid();plt.xlabel('W/S [N/m2]');
    plt.ylabel("W/P [N/W]");
    plt.legend(['Vmax']);
    plt.xlim((1,1200));
    plt.fill_between(WbyS,WbyP_Vmax,facecolor='b',alpha=0.2);
```

ROC

```
[ ] # Cdi
    Cdi = Cd0
    CL_Roc = np.sqrt(Cd0/K)
    Cd_Roc = Cd0 + Cdi
    # cl/cd max
    ClbyCd_max = CL_Roc/Cd_Roc
    # rate of climb (m/s)
    ROC_cruise = 1.5
    # W/P
    WbyP_ROC = 1/((ROC_cruise/etaP) + (np.sqrt(2*WbyS/\
                (rho*np.sqrt(3*Cd0/K))))*(1.155/(ClbyCd_max*etaP)))
```

```
[ ] plt.plot(WbyS,WbyP_ROC,"--m");
    plt.grid();plt.xlabel('W/S [N/m2]');
    plt.ylabel("W/P [N/W]");
    plt.legend(['ROC_max']);
    plt.xlim((1,1200));
    plt.fill_between(WbyS,WbyP_ROC,facecolor='m',alpha=0.2);
```

Ceiling

```
[ ] #  ceiling altitude (m)
    h_celing = 3000 # cruise ceiling
    rho_celing = getRHO(h_celing)
    sigma_ceiling = rho_celing/rho0
    ROC_ceiling = 1.5
    # W/P for ceiling
    WbyP_ceiling = sigma_ceiling/((ROC_ceiling/etaP) + (np.sqrt(2*WbyS/\
                    (rho_celing*np.sqrt(3*Cd0/K))))*(1.155/(ClbyCd_max*etaP)))
```

```
[ ] plt.plot(WbyS,WbyP_ceiling,":r");
    plt.grid();plt.xlabel('W/S [N/m2]');
    plt.ylabel("W/P [N/W]");
    plt.legend(['Ceiling']);
    plt.xlim((1,1200));
    plt.fill_between(WbyS,WbyP_ceiling,facecolor='r',alpha=0.2);
```

Take off run

```
[ ] mu = 0.05
    S_to = 500 # take off run distance (m)
    rho = getRHO(3000)
    Vr = 1.2*Vs
    CL_R = 2*WbyS/(rho*Vr*Vr)
    CL_to = 0.85
    CD0_HLDto = 0.0055
    CD0_LG = 0.009
    CD0_to = Cd0 + CD0_LG + CD0_HLDto
    CD_to = CD0_to + K*pow(CL_to,2)
    CD_G = CD_to - mu*CL_to
```

```
[ ] TbyW_Sto = (mu-(mu + (CD_G/CL_R))*(np.exp(0.6*rho*g0*CD_G*S_to/WbyS)))/\
                    (1-np.exp(0.6*rho*g0*CD_G*S_to/WbyS))
    WbyP_Sto = (etaP/Vto)/TbyW_Sto
```

```
[ ] plt.plot(WbyS,WbyP_Sto,"-.g");
    plt.grid();plt.xlabel('W/S [N/m2]');
    plt.ylabel("W/P [N/W]");
    plt.legend(['S_to']);
    plt.xlim((1,1200));
    plt.fill_between(WbyS,WbyP_Sto,facecolor='g',alpha=0.2);
```

Matching Plot

```
[ ]  plt.figure(figsize=(9,6));
     plt.axvline(x=WbyS_Vs, color='k', linestyle='--')
     plt.plot(WbyS,WbyP_Vmax,"-");
     plt.plot(WbyS,WbyP_Sto,"-.g");
     plt.plot(WbyS,WbyP_ROC,"--m");
     plt.plot(WbyS,WbyP_ceiling,":r");

     min_WbyP = np.minimum(WbyP_Vmax, WbyP_Sto)
     plt.fill_between(WbyS[:633],min_WbyP[:633],facecolor='m',hatch = "/",alpha=0.2);

     plt.annotate("Design point", (633,0.09))

     plt.grid();
     plt.xlabel('W/S [N/m2]');
     plt.ylabel("W/P [N/W]");
     plt.legend(['Vstall','Vmax','Sto','ROC','Ceiling']);
     plt.xlim((0,1200));
     plt.ylim((0,0.6));
```

Design point

```
[ ]  WbyS_design = 633.588375

     WbyP_design = sigma*etaP/((0.5*rho*pow(Vmax,3)*Cd0/(WbyS_design)) + ((2*K*WbyS_design/(rho*Vmax))))
```

```
[ ]  print("W/S for design point = ",WbyS_design)

     W/S for design point =  633.588375
```

```
[ ]  print("W/P for design point = ",WbyP_design)

     W/P for design point =  0.08449955354714783
```

3. Calculation of $C_{L,d}$, $C_{D,d}$, S, b, C_R, C_t with varying W/S and AR

```
z=3000 ;
v= 50;
w= 691.193;
TR=0.6;
W=w*9.81
```

```
W = 6.7806e+03
```

```
rho=getRHO(z)
```

```
rho = 1.1209
```

```
CL_CD= 18;

j=0;
for AR= 6:1:12
j=j+1;
A_R(j,1)=AR;
e(j,1)= 1.78*(1-0.045*AR^0.68)-0.64;
k(j,1)=1/(pi*e(j,1)*AR);
i=0;
for w_s= 40:10:100
    i=i+1;
    W_S(i,j)=w_s;
    CL_d(i,j)=(2*w_s*9.81)/(rho*v^2);
    S(i,j)=w/w_s;
    b(i,j) = sqrt(AR*S(i,j));
    Cr(i,j)= (2*S (i,1))/(b (i,j)*(1+TR));
    Ct(i,j)=Cr(i,j) *TR;
    CD_d(i,j)= CL_d(i,j)/CL_CD;
    CDO (i,j)= CD_d(i,j)-k(j,1)*CL_d(i,j)^2;
end
end
```

4. Calculations for $C_{L\alpha,3D}$,$C_{L0,3D}$,$C_{L\alpha,2D}$,$C_{L0,2D}$ by varying the $\alpha_{CL=0}$ from -4 to 0 and α_d from 3° to 7°

```
CL_d =0.49;
AR=12;
W= 691.193*9.81;
s=9.874;
e=0.706;
k=1/(pi*e*AR);

j = 0;
for aoa_d =3* (pi/180):1* (pi/180):7* (pi/180)
    j = j+1;
AOA_d(j,1)=aoa_d* (180/pi);
    i = 0;
for aoa_OCL = -4* (pi/180):1* (pi/180):0* (pi/180)
    i = i+1;
AOA_OCL (i,j)= aoa_OCL* (180/pi);
CL_alpha (i,j)= CL_d/(aoa_d-aoa_OCL);
cl_alpha (i,j) = CL_alpha (i,j)/(1-(CL_alpha (i,j)/(pi*e*AR)));
CLO(i,j)= -CL_alpha (i,j) *aoa_OCL;
clO(i,j)=-cl_alpha (i,j) *aoa_OCL;
end
end
```

5. Code for variation in Neutral point with varying $C_{L\alpha t}$ and S_t/S

```
% Given values
CL_alpha_w = 4.67916;

e = 0.706;
AR = 12;
x_ac_t = 4.82625;
x_ac_w = 0.9256;
n = 0.8;
% Range of CL_alpha_t and st/s values
CL_alpha_t_range = 2:0.2:4;
st_over_s_range = 0.1:0.05:0.6;
% Initialize arrays to store results
X_NP_values = zeros(length(CL_alpha_t_range), length(st_over_s_range));
% Calculate X_NP for different CL_alpha_t and st/s values
for i = 1:length(CL_alpha_t_range)
   for j = 1:length(st_over_s_range)
      CL_alpha_t = CL_alpha_t_range(i);
      st_over_s = st_over_s_range(j);

      % Formula for X_NP
      X_NP = ((CL_alpha_w * x_ac_w) + (n * (st_over_s) * (1 - e) * CL_alpha_t * x_ac_t)) / ...
           (CL_alpha_w + (n * (st_over_s) * (1 - e) * CL_alpha_t));

      % Store results
      X_NP_values(i, j) = X_NP;
   end
end
% Plot X_NP vs. CL_alpha_t for different st/s values
figure;
for j = 1:length(st_over_s_range)
   plot(CL_alpha_t_range, X_NP_values(:, j), '-o', 'DisplayName', ['st/s = ' num2str(st_over_s_range(j))]);
   hold on;
end
hold off;
xlabel('C_{L_{\alpha_t}}');
ylabel('X_{NP}');
title('X_{NP} vs. C_{L_{\alpha_t}} for different st/s values');
legend('show');
```

6. Trim condition iteration

```
clc;
clear;
% Define constants
Cm0 = 0.0344049;
Cm_delta_e = -0.75048; % in per radian
CL0 = -0.336064;
CL_alpha = 4.92814;
Cm_alpha = -1.0111;
CL_delta_e = 0.192;
W_S = 633.588;
CD0 = 0.203466;
rho = 1.1209; % air density at cruise altitude =3km
k = 0.037571246; % a constant value
S = 9.874; % reference area
b = 10.88; % wing span
% Initialize variables
delta_e_range = -15:1:15; % range of delta_e values
alpha_trim = zeros(size(delta_e_range));
CL_trim = zeros(size(delta_e_range));
V_trim_2 = zeros(size(delta_e_range));
TR = zeros(size(delta_e_range));
PR = zeros(size(delta_e_range));
L_D = zeros(size(delta_e_range));
% Iterate over delta_e values
for i = 1:length(delta_e_range)
    delta_e = delta_e_range(i) * (pi / 180);
    % Calculate alpha_trim, CL_trim, TR_trim, PR_trim
    alpha_trim(i) = -(Cm0 + Cm_delta_e * delta_e) / Cm_alpha;
    CL_trim(i) = CL0 + CL_alpha * alpha_trim(i) + CL_delta_e * delta_e;
    V_trim_2(i) = 2 * (W_S) / (rho * CL_trim(i));
    TR(i) = 0.5 * rho * V_trim_2(i) * (CD0 + k * (CL_trim(i))^2) * S;
    PR(i) = TR(i) * sqrt(V_trim_2(i));
    L_D(i) = CL_trim(i) / (CD0 + (k * (CL_trim(i))^2));
end
disp("alpha_trim:");
disp(alpha_trim);
disp("CL_trim:");
disp(CL_trim);
disp("V_trim:");
disp(sqrt(V_trim_2));
disp("T_R:");
disp(TR);
disp("P_R:");
disp(PR);
disp("L/D:");
disp(L_D);
```