

HW1 - Part B: Problem 1-Hint

Consider the IVP:

$$\dot{x} = \sin(t) - 2x, x(0) = 0.$$

which has the analytical solution:

$$x(t) = -\frac{1}{5}\cos(t) + \frac{2}{5}\sin(t) + \frac{1}{5}e^{-2t}$$

```
clearvars; close all; clc;
```

First, write your ODE function:

```
xdot = @(t,x) sin(t)-2*x;
```

and the true solution $x(t)$

```
xt = @(t) -0.2*cos(t) + 0.4*sin(t) + 0.2*exp(-2*t);
```

1. Choose a log-scale span of different time step τ . You can use the `logspace` function in MATLAB (or `numpy.logspace` in python) to create 20 points between -5 and 0 orders.

```
tau = logspace(-4,0,20)';
```

Initialize your variables:

```
E = zeros(size(tau)); %To store the error in the estimation
                        %since we will estimate the error for each tau
                        %then they are the same size

x0 = 0;                %given from the IVP
Tfinal = 10;
```

2. For each time step value, use the forward difference method given by:

$$x(t + \tau) = x(t) + \tau \dot{x}(t)$$

3. Consider the final time $T_{\text{final}} = 10$

```
for i=1:length(tau)
    tspan = (0:tau(i):Tfinal)'; %The time span

    X = x0; %initialize the estimated solution X with the initial condition
    for j=1:length(tspan)-1
```

```

        x = X(end) + tau(i)*xdot(tspan(j),X(end));
        X = [X; x];
    end
    % Now, the vector X has the trajectories obtained by forward difference
    % method

    %To find the true trajectories, you should use the true solution of the
    %ODE

    S = xt(tspan);

```

4. Find the relative error in the estimated trajectories.

$$E = \frac{\|X - S\|}{\|S\|}$$

where X is the vector of trajectories found using your numerical estimation and S is the actual trajectories found using the analytical solution.

```

    E(i) = norm(X-S)/norm(S);

    end %now the loop will go back and compute the error for the next value of tau

    loglog(tau,E,'-b')
    grid minor
    xlabel("$\tau$","Interpreter","latex")
    ylabel("$E=\frac{\|X-S\|}{\|S\|}$","Interpreter","latex")
    set(gca,'FontSize',15)

```

