HW 1: Introduction

Due Date: 2/4/2022

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Part A

Problem 1:

a: Show that $\cos(ct)$ and $\sin(ct)$ are solutions of the second order ODE

$$\ddot{u} + c^2 u = 0, \tag{1.1}$$

where c is a constant.

b: Linearize Eq.1.1, and write it in the form $\dot{x} = Ax$. (x is a two dimensional vector)

Problem 2: Provide a brief review of the vector norm:

$$\|x\|_p, \tag{1.2}$$

$p = 1, 2, \infty$, and the matrix norm

$$\|A\|_p. \tag{1.3}$$

$p = 1, 2, \infty$ and the Frobenius norm.

Also, discuss in more details the $L_{2,1}$ norm, and why it is more robust as error function.

Problem 3.a: A metric on a set X is a function (called distance function or simply distance)

$$d : X \times X \to [0, \infty),$$

where $[0, \infty)$ is the set of non-negative real numbers. What are the three axioms that should be satisfied for a function to be a distance function. Show a simple example.

b: Review and write the formula for the following distance functions, and discuss their common uses and disadvantages:

1. Euclidean Distance.
2. Manhattan Distance.
3. Chebyshev Distance.
4. Minkowski Distance.
5. Hamming Distance.

Part B

Problem 1: Consider the IVP:

$$\dot{x} = sin(t) - 2x, x(0) = 0. \tag{1.4}$$

which has the analytical solution:

$$x(t) = -\frac{1}{5}\cos(t) + \frac{2}{5}\sin(t) + \frac{1}{5}e^{-2t} \tag{1.5}$$

The purpose of this problem is to investigate the accuracy of some numerical methods in solving the IVP.

1. Choose a log-scale span of different time step τ. You can use the *logspace* function in MATLAB (or *numpy.logspace* in python) to create 20 points between -5 and 0 orders.
2. For each time step value, use the forward difference method given by:

$$x(t + \tau) = x(t) + \tau\dot{x}(t)$$

3. Consider the final time Tfinal = 10.
4. Find the relative error in the estimated trajectories.

$$E = \frac{\|X - S\|}{\|S\|}$$

   where X is the vector of trajectories found using your numerical estimation and S is the actual trajectories found using the analytical solution.
5. Plot the time step (in the x-axis) vs the error associated with it (in y-axis). Use the log-scale plot in x and y. (Matlab: loglog, Python: matplotlib.pyplot.loglog).
6. Write your comments and conclusions.
7. Repeat all the above using the backward difference method, central difference method, and rung-kutta method. Show the equations and details of your work.
8. Compare and comment on the error of each method.

Problem 2: In a programming language of your choice, list all the built-in functions for solving ODE numerically and discuss the method used in each function of them. Show some examples.