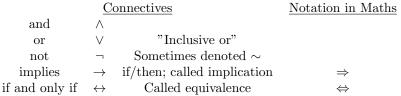
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Task: Recall enough propositional logic to see how it matches up with set theory.

Definition: A <u>proposition</u> is any declarative sentence that is either true or false.

0.1 Connectives



0.1.1 Truth Table of the Connectives

Let P, Q be propositions:

P	Q	$P \wedge Q$	
F	F	F	
F	Т	F	
Т	F	F	
Т	Т	Т	

F F F F T T T T T T T T T	Р	Q	$P \vee Q$
T F T	F	F	F
	F	Τ	Т
TTTT	Τ	F	Т
1 1 1	Т	Т	Т

Р	¬P	
F	T	
Т	F	

Р	Q	$P \rightarrow Q$
F	F	Τ
F	Т	Т
Т	F	F
Т	Т	Т

P	Q	$P \leftrightarrow Q$
F	F	Т
F	Т	F
T	F	F
Τ	Т	Т

Priority of the Connectives

Highest to Lowest: $\neg, \land, \lor, \rightarrow, \leftrightarrow$

0.2 Important Tautologies

$$\begin{array}{cccc} (P \to Q) & \leftrightarrow & (\neg P \lor Q) \\ (P \leftrightarrow Q) & \leftrightarrow & [(P \to Q) \land (Q \to P)] \\ \neg (P \land Q) & \leftrightarrow & (\neg P \lor \neg Q) \\ \neg (P \lor Q) & \leftrightarrow & (\neg P \land \neg Q) \end{array} \right\} \text{ De Morgan Laws}$$

As a result, \neg and \lor together can be used to represent all of \neg , \land , \lor , \rightarrow , \leftrightarrow .

Less obvious: One connective called the sheffer stroke P|Q (which stands for "not both P and Q" or "P nand Q") can be used to represent all of \neg , \wedge , \vee , \rightarrow , \leftrightarrow since $\neg P \leftrightarrow P|P$ and $P \vee Q \leftrightarrow (P|P) \mid (Q|Q)$.

Recall if $P \rightarrow Q$ is a given implication, $Q \rightarrow P$ is called the <u>converse</u> or $P \rightarrow Q$. $\neg Q \rightarrow \neg P$.

0.3 Indirect Arguments/Proofs by Contradiction/Reductio as absurdum

Based on the tautology (P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)

Example: Famous argument that $\sqrt{2}$ is irrational.

Proof:

Suppose $\sqrt{2}$ is rational, then it can be expressed is fraction form $\frac{a}{b}$. Let us assume that our fraction is in the lowest term, i.e. their only common divisor is 1.

Then,

$$\sqrt{2} = \frac{a}{b}$$

Squaring both sides, we have

$$2 = \frac{a^2}{b^2}$$

Multiplying both sides by B^2 yields

$$2b^2 = a^2$$

Since a^2b^2 , we can conclude that a^2 is even because whatever the value of b^2 has to be multiplied by 2. If a^2 is even, then a is also even. Since a is even, no matter what the value of a is, we can always find an integer that if we divide a by 2, it is equal to that integer. If we let that integer be k, then $\frac{a}{b} = k$ which means that a = 2k.

Substituting the value of 2k to a, we have $2b^2 = (2k)^2$ which means that $2b^2 = 4k^2$. dividing both sides by 2 we have $b^2 = 2k^2$. That means that the value b^2 is even, since whatever the value of k you have to multiply it by 2. Again, is b^2 is even, then b is even.

This implies that both a and b are even, which means that both the numerator and the denominator of our fraction are divisible by 2. This contradicts our **assumption** that $\frac{a}{b}$ has no common divisor except 1. Since we found a contradiction, our assumption is, therefore, false. Hence the theorem is true.

qed