Final Project Outline

Silvester, Almustapha Tran, Angelica Jane Iskander, Fatah

November 27, 2024

1 Part 1

$$\rho = \sqrt{(X - X_i)^2 + (Y - Y_i)^2} \tag{1}$$

$$\dot{\rho} = \frac{(X - X_i) * (\dot{X} - \dot{X}_i) + (Y - Y_i) * (\dot{Y} - \dot{Y}_i)}{\rho} \tag{2}$$

$$\phi = \arctan(\frac{Y - Y_i}{X - X_i}) \tag{3}$$

1.1 Section 1

The partial derivatives of equations (1), (2), and (3) with respect to X, $\dot{X}$, Y, and $\dot{Y}$ are:

For equation (1), $\rho = \sqrt{(X - X_i)^2 + (Y - Y_i)^2}$:

$$
\begin{aligned}
\frac{\partial \rho}{\partial X} &= \frac{\partial}{\partial X}\sqrt{(X-X_i)^2+(Y-Y_i)^2} \\
&= \frac{1}{2\sqrt{(X-X_i)^2+(Y-Y_i)^2}} \cdot \frac{\partial}{\partial X}[(X-X_i)^2+(Y-Y_i)^2] \\
&= \frac{1}{2\sqrt{(X-X_i)^2+(Y-Y_i)^2}} \cdot 2(X-X_i) \\
&= \frac{X-X_i}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}} \\
&= \frac{X-X_i}{\rho}
\end{aligned}
$$

$$
\frac{\partial \rho}{\partial \dot{X}} = \frac{\partial}{\partial \dot{X}}\sqrt{(X-X_i)^2+(Y-Y_i)^2} = 0
$$

$$
\begin{aligned}
\frac{\partial \rho}{\partial Y} &= \frac{\partial}{\partial Y}\sqrt{(X-X_i)^2+(Y-Y_i)^2} \\
&= \frac{1}{2\sqrt{(X-X_i)^2+(Y-Y_i)^2}} \cdot \frac{\partial}{\partial Y}[(X-X_i)^2+(Y-Y_i)^2] \\
&= \frac{1}{2\sqrt{(X-X_i)^2+(Y-Y_i)^2}} \cdot 2(Y-Y_i) \\
&= \frac{Y-Y_i}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}} \\
&= \frac{Y-Y_i}{\rho}
\end{aligned}
$$

$$
\frac{\partial \rho}{\partial \dot{Y}} = \frac{\partial}{\partial \dot{Y}}\sqrt{(X-X_i)^2+(Y-Y_i)^2} = 0
$$

For equation (2), $\dot{\rho} = \frac{(X-X_i)(\dot{X}-\dot{X}_i)+(Y-Y_i)(\dot{Y}-\dot{Y}_i)}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}}$:

$$\begin{aligned}
\frac{\partial \dot{\rho}}{\partial X} &= \frac{\partial}{\partial X}\left[\frac{(X-X_i)(\dot{X}-\dot{X}_i)+(Y-Y_i)(\dot{Y}-\dot{Y}_i)}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}}\right] \\
&= \frac{(\dot{X}-\dot{X}_i)\sqrt{(X-X_i)^2+(Y-Y_i)^2} - [(X-X_i)(\dot{X}-\dot{X}_i)+(Y-Y_i)(\dot{Y}-\dot{Y}_i)]\frac{X-X_i}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}}}{(X-X_i)^2+(Y-Y_i)^2} \\
&= \frac{(\dot{X}-\dot{X}_i)\rho - \dot{\rho}\rho\frac{X-X_i}{\rho}}{\rho^2} \\
&= \frac{\dot{X}-\dot{X}_i}{\rho} - \frac{\dot{\rho}(X-X_i)}{\rho^2}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \dot{\rho}}{\partial \dot{X}} &= \frac{\partial}{\partial \dot{X}}\left[\frac{(X-X_i)(\dot{X}-\dot{X}_i)+(Y-Y_i)(\dot{Y}-\dot{Y}_i)}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}}\right] \\
&= \frac{X-X_i}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}} \\
&= \frac{X-X_i}{\rho}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \dot{\rho}}{\partial Y} &= \frac{\partial}{\partial Y}\left[\frac{(X-X_i)(\dot{X}-\dot{X}_i)+(Y-Y_i)(\dot{Y}-\dot{Y}_i)}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}}\right] \\
&= \frac{(\dot{Y}-\dot{Y}_i)\sqrt{(X-X_i)^2+(Y-Y_i)^2} - [(X-X_i)(\dot{X}-\dot{X}_i)+(Y-Y_i)(\dot{Y}-\dot{Y}_i)]\frac{Y-Y_i}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}}}{(X-X_i)^2+(Y-Y_i)^2} \\
&= \frac{(\dot{Y}-\dot{Y}_i)\rho - \dot{\rho}\rho\frac{Y-Y_i}{\rho}}{\rho^2} \\
&= \frac{\dot{Y}-\dot{Y}_i}{\rho} - \frac{\dot{\rho}(Y-Y_i)}{\rho^2}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \dot{\rho}}{\partial \dot{Y}} &= \frac{\partial}{\partial \dot{Y}}\left[\frac{(X-X_i)(\dot{X}-\dot{X}_i)+(Y-Y_i)(\dot{Y}-\dot{Y}_i)}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}}\right] \\
&= \frac{Y-Y_i}{\sqrt{(X-X_i)^2+(Y-Y_i)^2}} \\
&= \frac{Y-Y_i}{\rho}
\end{aligned}$$

For equation (3), $\phi = \arctan(\frac{Y-Y_i}{X-X_i})$:

$$\begin{aligned}
\frac{\partial \phi}{\partial X} &= \frac{\partial}{\partial X} \arctan\left(\frac{Y-Y_i}{X-X_i}\right) \\
&= \frac{1}{1+\left(\frac{Y-Y_i}{X-X_i}\right)^2} \cdot \frac{\partial}{\partial X}\left(\frac{Y-Y_i}{X-X_i}\right) \\
&= \frac{1}{1+\left(\frac{Y-Y_i}{X-X_i}\right)^2} \cdot \frac{-(Y-Y_i)}{(X-X_i)^2} \\
&= -\frac{Y-Y_i}{(X-X_i)^2+(Y-Y_i)^2}
\end{aligned}$$

$$\frac{\partial \phi}{\partial \dot{X}} = \frac{\partial}{\partial \dot{X}} \arctan\left(\frac{Y-Y_i}{X-X_i}\right) = 0$$

$$\begin{aligned}
\frac{\partial \phi}{\partial Y} &= \frac{\partial}{\partial Y} \arctan\left(\frac{Y-Y_i}{X-X_i}\right) \\
&= \frac{1}{1+\left(\frac{Y-Y_i}{X-X_i}\right)^2} \cdot \frac{\partial}{\partial Y}\left(\frac{Y-Y_i}{X-X_i}\right) \\
&= \frac{1}{1+\left(\frac{Y-Y_i}{X-X_i}\right)^2} \cdot \frac{1}{X-X_i} \\
&= \frac{X-X_i}{(X-X_i)^2+(Y-Y_i)^2}
\end{aligned}$$

$$\frac{\partial \phi}{\partial \dot{Y}} = \frac{\partial}{\partial \dot{Y}} \arctan\left(\frac{Y-Y_i}{X-X_i}\right) = 0$$

1.2 Section 2

1.3 Section 3