1 Spectral K-means

2 Fuzzy C-Means

We selected the **generalized suppressed Fuzzy C-Means** (gs-FCM) algorithm, an improvement over traditional FCM, which often shows multimodal behavior near cluster boundaries (Fig. 1a). This issue, where fuzzy memberships remain high for unrelated clusters, was addressed by Höppner and Klawonn [2].

The suppressed Fuzzy C-Means (s-FCM) algorithm [1] enhances convergence speed and classification accuracy without minimizing the traditional objective function J_{FCM}. It introduces a suppression step during each iteration to reduce non-winner fuzzy memberships, which is mathematically equivalent to virtually reducing the distance to the winning cluster's prototype (Fig. 1b) [4].

Szilágyi et al. [4] defined the quasi-learning rate η of s-FCM, analogous to learning rates in competitive algorithms:

$$\eta(m, \alpha, u_{wk}) = 1 - \frac{\delta_{wk}}{d_{wk}} = 1 - \left(1 + \frac{1-\alpha}{\alpha u_{wk}}\right)^{(1-m)/2}.$$

Building on this, gs-FCM modifies the learning rate to decay linearly with increasing winner membership u_{wk}, as proposed in sg_ρ-FCM [3]:

$$\eta(u_{wk}) = 1 - \rho u_{wk}, \quad \text{where } 0 \leq \rho \leq 1.$$

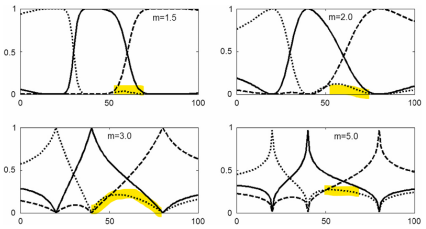
This approach ensures a logical adaptation of membership weighting, expressed as:

$$\alpha_k = \left[1 - u_w + u_w \left(1 - f(u_w)\right)^{2/(1-m)}\right]^{-(1-m)}.$$

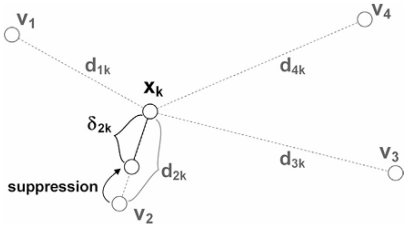
References

[1] J.L. Fan, W.Z. Zhen, and W.X. Xie. Suppressed fuzzy c-means clustering algorithm. *Pattern Recognition Letters*, 24:1607–1612, 2003.

[2] F. Höppner and F. Klawonn. Improved fuzzy partitions for fuzzy regression models. *International Journal of Approximate Reasoning*, 32:85–102, 2003.



(a) Multimodal fuzzy memberships near cluster boundaries for varying fuzzy exponent m.



(b) Suppression effect: Winner cluster ($w_k = 2$) gains increased membership while non-winners are suppressed.

Figure 1: Figures adapted from [3].

[3] László Szilágyi and Sándor M. Szilágyi. Generalization rules for the suppressed fuzzy c-means clustering algorithm. *Neurocomputing*, 139:298–309, 2014.

[4] L. Szilágyi, S.M. Szilágyi, and Z. Benyó. Analytical and numerical evaluation of the suppressed fuzzy c-means algorithm: a study on the competition in c-means clustering models. *Soft Computing*, 14:495–505, 2010.