CS 3133 Foundations of Computer Science
A term 2022

Homework 2

Due date: September 26th, 2022

Regular Languages. Context free Grammars.

Every homework will receive a grade between 0 to 100. The (maximal) grade of every question is identical and the sum of grades is the final grade. Typesetting your homework is highly recommended.

1. Prove that the set of regular languages is closed under taking complements. Namely if L is regular then so is the set of all strings that **do not** belong to L, $\Sigma^* \setminus L$. Use this and DeMorgan laws to prove that the set of regular languages is closed under unions: If A and B are regular languages then so is their union $A \cap B$.

2. Prove that the set of context-free languages is closed under concatenation. Namely, if both A and B are context free, then so is their concatenation $A \circ B$.

3. Professor Brainy claims that there is a trivial proof that the set of context-free languages are closed under the star operation. Namely, if L is context-free then so is L^*. To prove this simply take the grammar G generating L and add to it the rule SS where S is the start variable. Prove by providing a simple counter example that this grammar might not generate L^*.

4. Give a context-free grammar that generates the language of all well parenthesized expressions over $(, \{,), \}$. Namely the set of all strings where the number of left or right paranthesis of each of the two types is the same and that the for any prefix the number of left parenthesis (of each type) is no smaller than the number of right parenthesis. Give a parse tree for the derivation of the string $((\{\}))$.

5. Prove that the following languages over binary alphabet are context free:

   (a) 0^*1^*.

   (b) All binary strings of the form 1^n0^n or 0^n1^n where n is a natural number.

   (c) All binary strings that are of the from ww^R for a binary string w. Here w^R is simply the string w written in reverse order. For example, for $w = 1000, w^R$ is 0001.