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1a)

$$R(A, B, C, D, E) \quad S = \{ AB \to D,\ \ AB \to E,\ \ BC \to D,\ \ C \to A,\ \ CD \to B \}$$

A^+= A	AB^+= ABDE	ABC^+= ABCDE	$ABCD^+$= ABCDE
B^+= B	AC^+ = AC	ABD^+= ABDE	$BCDE^+$= ABCDE
C^+= AC	AD^+= AD	ACD^+= ABCDE	
D^+= D	AE^+= AE	ABE^+= ABDE	
E^+= E	BC^+= ABCDE	ACE^+= ACE	
	CD^+= ABCDE	ADE^+= ADE	
	BD^+= BD	BCD^+= ABCDE	
	BE^+= BE	BCE^+= ABCDE	
	CE^+= ACE	BDE^+= BDE	
	DE^+= DE	CDE^+= ABCDE	

trivial super

BC, CD are candidate keys

1b)
CD->B is in BCNF because CD is a candidate key, C->A violates BCNF because C is not a candidate key, BC->D is in BCNF because BC is a candidate key, AB->E and AB->D violate BCNF because AB is not a candidate key

1c)

CD->B is already in BCNF, move to the left

C->A is not in BCNF, decompose:
C^+ = AC
Key is {C}
R_{11}(A,C), with dependencies C->A, key is C so R_{11} is in BCNF. Also R_{11}(A,C) only has 2 attributes, so by definition it is in BCNF.

A^+ = A (trivial)
C^+ = AC (candidate key)
AC^+ = AC (trivial)
C is the candidate key of this relation

ABCDE - AC + C = BCDE
Key is {BC,CD}

R_{12}(B,C,D,E), with dependencies BC-> D, BC-> E, CD->B, CD->E, this is in BCNF because candidate keys BC or CD are on the left of every FD

B^+= B (trivial)
C^+ = C (trivial)
D^+= D (trivial)
E^+=E (trivial)
BC^+ =BCDE
BD^+= BD (trivial)
BE^+= BE (trivial)
CD^+=BCDE
CE^+= CE (trivial)
DE^+= DE (trivial)

BC, CD are the candidate keys

AB->E, not in BCNF because AB is not a super key
AB^+ = ABDE
Key{AB}
R_{21}(A,B,D,E), with dependencies AB->D, AB->E, this is in BCNF because each dependency contains the candidate key AB on the left

A^+=trivial
B^+= trivial
C^+ = AC
D^+= trivial
E^+=trivial
AB^+ = ABDE
AC^+=trivial
AD^+=trivial
AE^+=trivial
BC^+ =ABDE
BD^+= trivial
BE^+= trivial
CD^+=ABDE
CE^+= trivial
DE^+=trivial

C is not in R_{21}, so AB is the only candidate key

ABCDE - ABDE + AB = ABC
Key{BC}
R_{22}(A,B,C), BC->A, this is in BCNF because all FDs contain a candidate key
A^+=trivial

B^+= trivial
C^+ = AC
D^+= trivial
E^+=trivial
AB^+ = trivial
AC^+=trivial
AD^+=trivial
AE^+=trivial
BC^+ =ABC
BD^+= trivial
BE^+= trivial
CD^+=ABC
CE^+= ACE
DE^+=trivial

D is not in R_{22}, so BC is the only candidate key

Final Answer:

R_{11}(A,C) {C->A}
R_{12}(B,C,D,E) {BC-> D, BC-> E, CD->B, CD->E}
R_{21}(A,B,D,E) {AB->D, AB->E}
R_{22}(A,B,C) {BC->A}

1d)
Combine AB->D, AB->E to make AB->DE
BC->D is redundant

Minimal basis for S = S' =
{AB->DE, C->A, CD->B}

1e) FD in S' that violate 3NF
Relation in 3NF if the left side has a candidate key or the right side has attribute that is part of candidate key
Candidate keys are BC, CD

AB->DE can be rewritten as:
AB->D, does not violate 3NF because the FD is prime
AB->E, violates 3NF because AB is not a primary key and FD is not prime

C->A violates 3NF because C is not a super key and FD is not prime
CD->B does not violate 3NF because CD is a candidate key

1f)
R1(A,B,D,E), with FD AB->DE
R2(A,C), with FD C->A
R3(B,C,D), with FD CD->B
BC is a candidate key in R3, so no need to rewrite

2a)

R(A, B, C, D, E) S = { BE → A, CD → E, AD → C }

A^+= A	AB^+= AB	ABC^+= ABC	$ABCD^+$= ABCDE
B^+= B	AC^+ = AC	ABD^+= ABCDE	$BCDE^+$= ABCDE
C^+= C	AD^+= ACDE	ACD^+= ACDE	
D^+= D	AE^+= AE	ABE^+= ABE	
E^+= E	BC^+= BC	ACE^+= ACE	
	CD^+= CDE	ADE^+= ACDE	
	BD^+= BD	BCD^+= ABCDE	
	BE^+= ABE	BCE^+= ABCE	
	CE^+= CE	BDE^+= ABCDE	
	DE^+= DE	CDE^+= CDE	

trivial super

Candidate keys are ABD, BCD, BDE

2b) BE->A , CD->E , AD->C
None of these FD meet BCNF because they do not contain candidate keys on the left side

2c)
Dependencies not in BCNF: BE->A , CD->E , AD->C

Start with left-most violation: BE^+ = ABE
Key {BE}
R1(A,B,E) using dependency BE->A
ABE is in BCNF because BE in this FD is the candidate key

A^+=A (triv)
B^+=B (triv)
E^+=E (triv)
AB^+ = AB (triv)
AE^+ = AE (Triv)
BE^+ = ABE

ABCDE - ABE + BE = BCDE

Key {BDE,BCD}
R2(B,C,D,E) using dependency BDE->C, BCD->E, CD -> E
CD -> E violates BCNF, as CD is not a key
B^+=B (triv)
C^+=(triv)
D^+=D (triv)
E^+=E (triv)
BC^+=BC (triv)
BD^+ = BD (triv)
BE^+ = BE (triv)
CD^+=CDE
CE^+ = CE
DE^+ = DE
BCE^+ = BCE
BDE^+=BCDE
BCD^+=BCDE
CDE^+ = CDE

CD->E
CD^+=CDE
Key{CD}
R3(C,D,E), using dependency CD->E, in BCNF because every dependency has a candidate key

C^+=(triv)
D^+=(triv)
E^+=(triv)
CD^+=CDE
CE^+ = CE
DE^+ = DE

ABCDE - CDE + CD = ABCD
Key {ABD, BCD}
R4(A,B,C,D), using dependencies ABD->C, BCD->A, AD -> C
AD -> C violates BCNF, as AD is not a candidate key

A^+=(triv)
B^+=B (triv)
C^+=(triv)
D^+=D (triv)
AB^+ = AB
AC^+ = AC
AD^+ = ACD
BC^+=BC (triv)

CD^+=CD
ABC^+ = ABC (triv)
ABD^+ = ABCD
BCD^+=ABCD

AD->C
AD^+=ACDE
Key{AD}
R5(A,C,D,E), using dependency AD->C, AD->E, in BCNF because every dependency has a candidate key

A^+=(triv)
B^+=(triv)
C^+=(triv)
D^+=D (triv)
E^+=E (triv)
AB^+ = A
AC^+ = AC (triv)
AD^+ = ACDE
AE^+ = AE (triv)
BC^+=C
CD^+=CDE
BE^+ = AE
CE^+ = CE (triv)
DE^+ = DE (triv)
ABC^+ = AC
ABD^+ = ACDE
BDE^+=ACDE
BCD^+=ACDE

ABCDE - ACDE + AD = ABD
Key {ABD}
R6(A,B,D),{}

A^+=(triv)
B^+=B (triv)
D^+=D (triv)
AB^+ = AB (triv)
AD^+ = AD (triv)
BD^+ = BD (triv)
ABD^+ = ABD

Final answer:

R1(A,B,E) using dependency BE->A
R2(B,C,D,E) {BDE->C, BCD->E}
R3(C,D,E), {CD->E}
R4(A,B,C,D), {ABD->C, BCD->A}
R5(A,C,D,E), {AD->C, AD->E}
R6(A,B,D)

2d)
S is already a minimal basis, can’t cancel out attributes or combine dependencies
Minimal basis S’ = S = BE->A, CD->E, AD->C

2e)
If the candidate keys are ABD, BCD, BDE, all functional dependencies of S’ (BE->A, CD->E, AD->C) satisfy 3NF because the attributes on the right are all parts of candidate keys