

CS 3133 Foundations of Computer Science
A term 2022

Homework 3

Due date: October 6th, 2022

Decidability, Turing Machines.

Grade is between 0 to 100. The (maximal) grade of every question is identical and the sum of grades is the final grade. Typesetting your homework is highly recommended.

1. Prove that every regular language L is decidable by providing a Turing machine that decides L . Explain your answer.
2. Give a detailed implementation of a Turing that is given a nonempty binary string w and outputs the string $w\#w$. In words, the machine on a given binary input w outputs a copy of the string w to the right of w with a $\#$ symbol between the two strings.
3. In class we gave a description of a Turing machine deciding all *binary* strings of the form $w\#w$. Extend this example to the case where the alphabet has three characters 0, 1, 2. Namely give a detailed description of a Turing machine that accepts all strings of the form $w\#w$ and $w \in \{0, 1, 2\}^*$. As in the example in class there is no need to display the rejecting state and the transitions into it in the diagram of the Turing Machine.
4. Prove that the set of all *infinite* binary strings is not countable. Hint: use Diagonalization.
5. Consider the problem of deciding whether a given regular expression R is equivalent to a given DFA A . Define formally this decision problem and prove that it is decidable.