CS 3133 Foundations of Computer Science
A term 2022

Homework 1

Due date: September 12th, 2022

Finite Automata and Regular Languages.

Every homework will receive a grade between 0 to 100. The (maximal) grade of every question is identical and the sum of grades is the final grade. Typesetting your homework is highly recommended.

1. Let L be a **finite** set of binary strings. Prove that L is regular.

2. Let $s = \sigma_1 \dots \sigma_k$ be a binary string of length $k > 0$. We say that a binary string $w = w_1 \dots w_n$ contains s as a subsequence if there are k indices $1 \leq i_1 < i_2, \dots < i_k \leq n$ such that $w_{i_r} = s_r$ for every $1 \leq r \leq k$. For example, if $s = 11$ then $10001, 1010$ and 110 contain s as a subsequence whereas 000 and 1000 do not. Prove that the language of all binary strings containing a fixed binary string s of length k as a subsequence is a regular language.

3. Exercise 1.6 from the book: Solve items b. c. d.

4. Exercise 1.20 from the book: Solve items b. c. d. and e.

5. Let Σ be an alphabet with $n > 1$ symbols. Give an NFA that recognizes the language L of all strings w for which there exists a letter $\sigma \in \Sigma$ such that $w = u\sigma$ with $u \in (\Sigma \setminus \sigma)^*$. In words, L is the set of all strings ending a letter σ that only occurs in the last position in the string. For example if $\Sigma = \{a, b, c\}$ then $ababab c$ and cba belong to L but $ababa, abca$ do not belong to L.