

# Chapter 9 Decision Analysis

- 9.1 Context and basic steps (most important part)
- 9.2 Example
- 9.3 Multistage decision analysis (example)
- 9.4 Hierarchical decision analysis (example)
- 9.5 Personal vs. institutional decision analysis

# Bayesian decision theory

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- Expected utility  $E[U(x) | d] = \int U(x)p(x | d)dx$
- Choose decision  $d^*$ , which maximizes the expected utility

$$d^* = \arg \max_d E[U(x) | d]$$

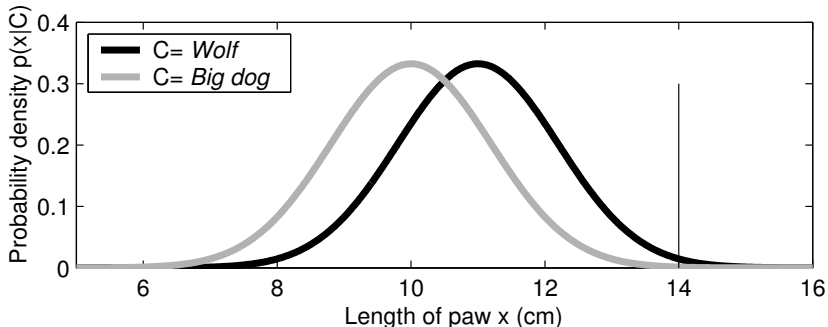
## Example of decision making: 2 choices

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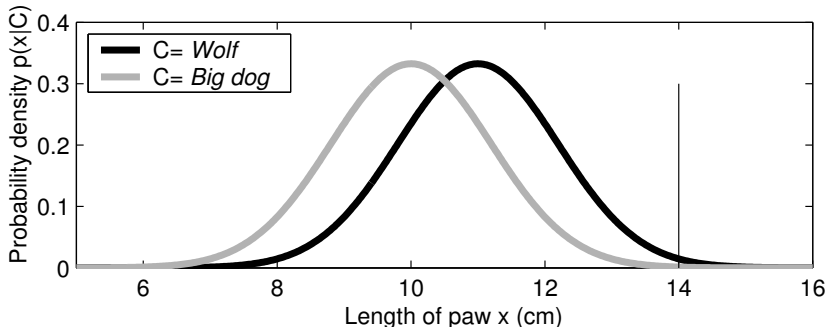
- Helen is going to pick mushrooms in a forest, while she notices a paw print which could made by a dog or a wolf
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- Likelihood of wolf is 0.92 (alternative being dog)

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- Posterior probability of wolf is 10%

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Utility matrix  $U(x)$

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Utility matrix  $U(x)$

Action $d$	Expected utility $E[U(x) \mid d]$
Stay home	0
Go to the forest	-100+0.9

Utilities for different actions

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- Example illustrates that the uncertainties (probabilities) related to all consequences need to be carried on until final decision making

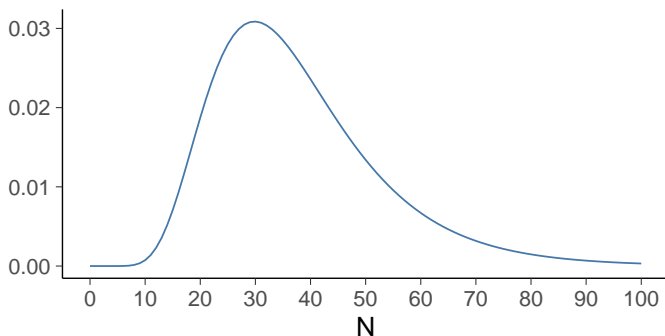
## Example of decision making: several choices

- You decide to earn money by selling a seasonal product
  - You pay 7€ per each, and sell them 10€ each
  - You need to decide how many ( $N$ ) items to buy

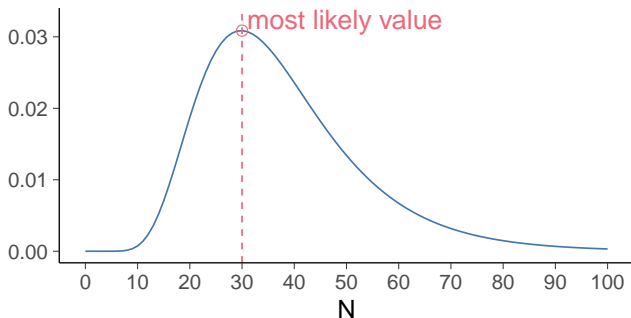


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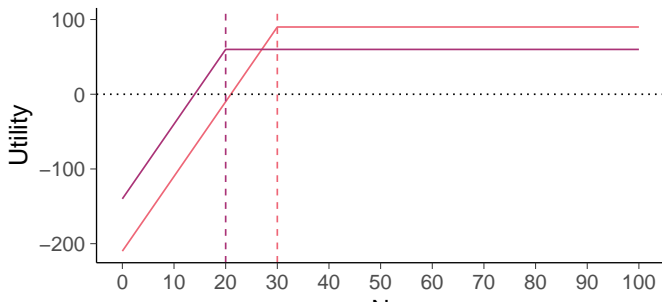
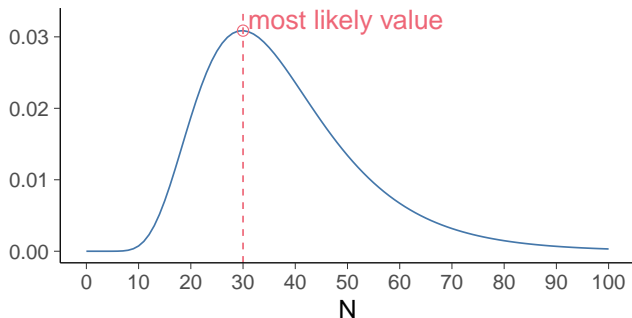
- You decide to earn money by selling a seasonal product
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  - You ask your friends how many they used to sell and estimate a distribution for how many you might sell



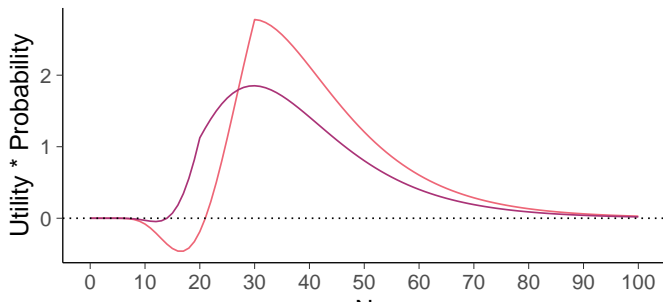
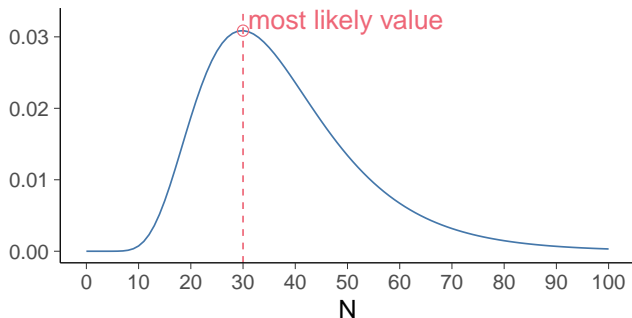
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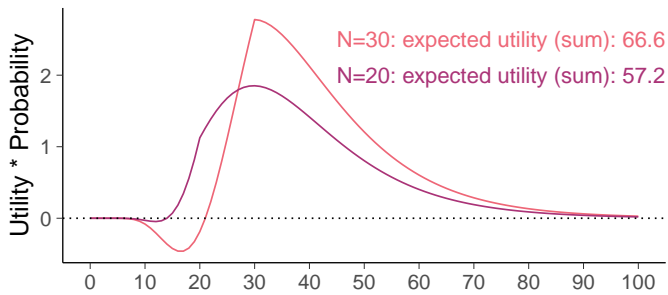
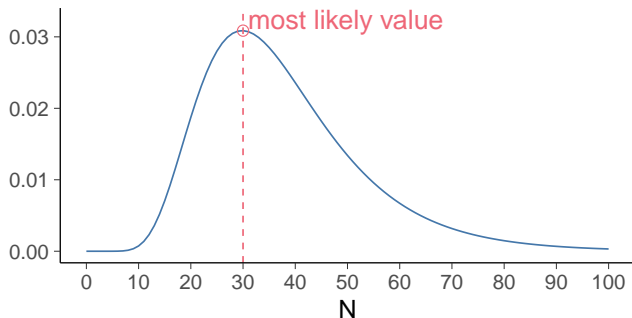
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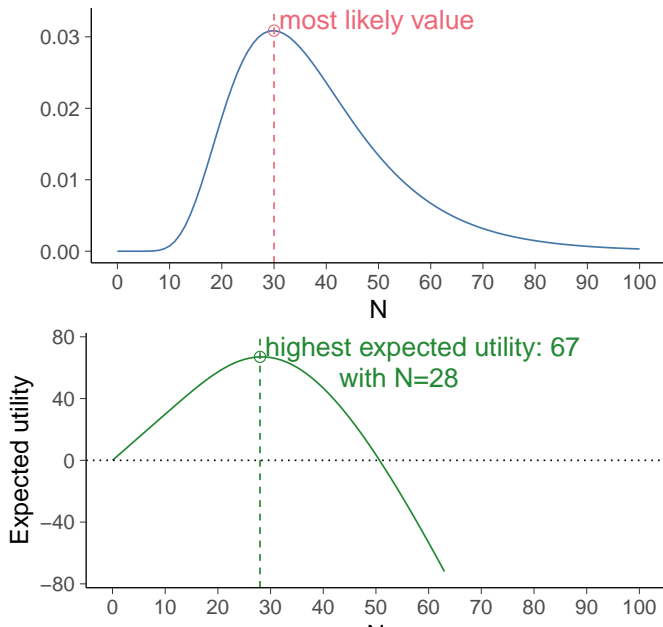
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# Decision making in sales

- Common task in commerce and restaurants

# Challenges in decision making

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- What is the cost of human life?
- Multiple parties having different utilities

# Model selection as decision problem

- Choose the model that maximizes the expected utility of using the model to make predictions / decisions in the future

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- Quality adjusted life time
  - See the book for the multi-stage decision making

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- Which experiment would give most additional information
  - decide values  $x_{n+1}$  for the next experiment
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- Example 3
  - optimal paper helicopter wing length

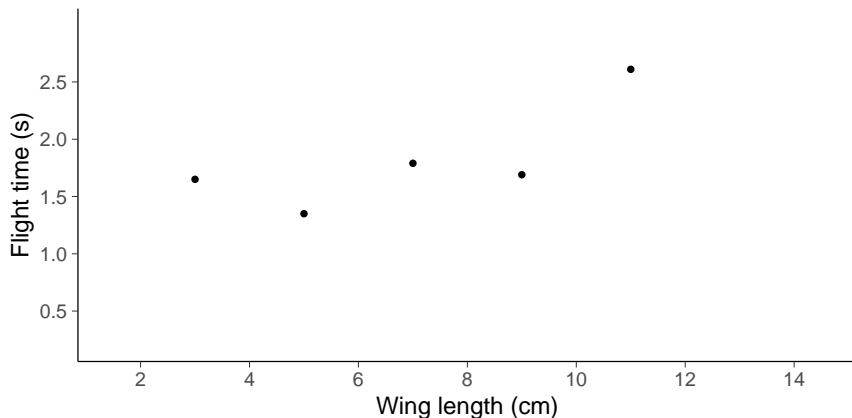


# Bayesian optimization

- Design of experiment
- Used to optimize, for example,
  - machine learning / deep learning model structures, regularization, and learning algorithm parameters
  - material science
  - engines
  - drug testing
  - part of Bayesian inference for stochastic simulators

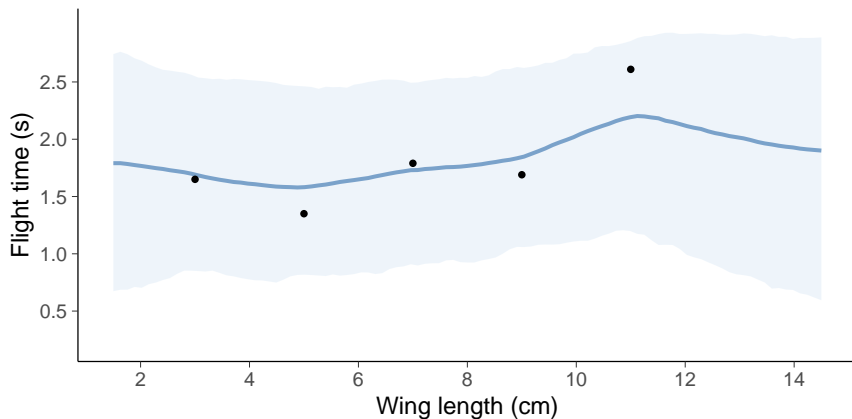
# Bayesian optimization of wing length

Start with a small number of experiments



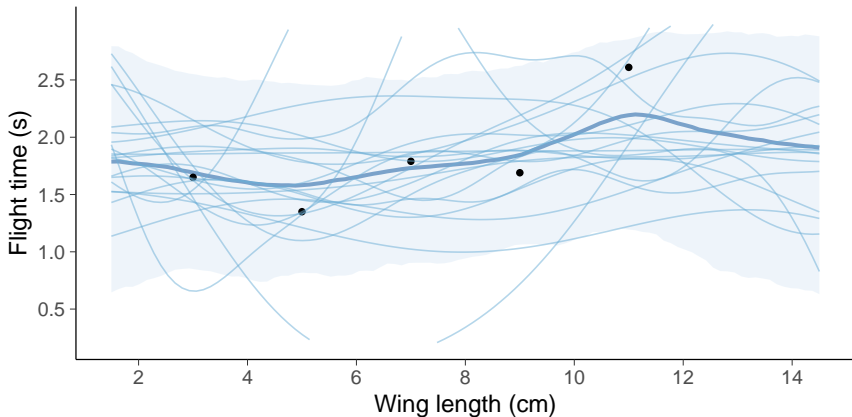
# Bayesian optimization of wing length

Gaussian process model



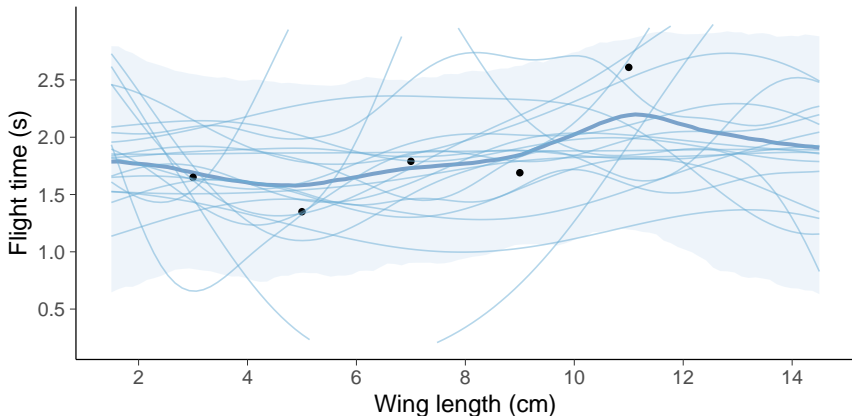
# Bayesian optimization of wing length

Gaussian process model – posterior draws



# Bayesian optimization of wing length

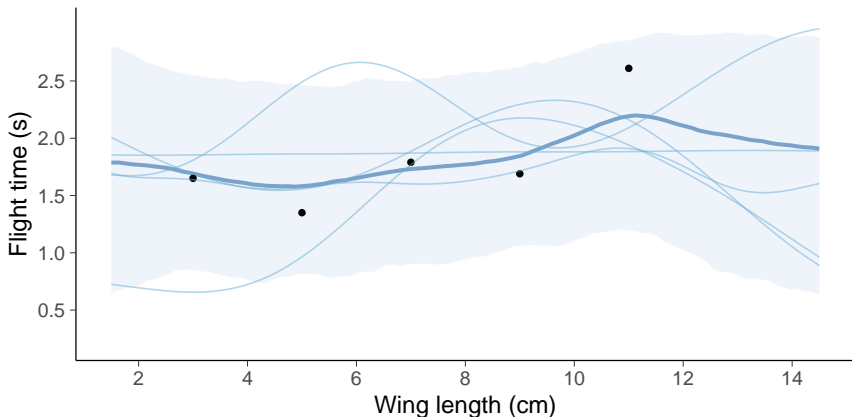
Gaussian process model – posterior draws



- Thompson sampling:
  - pick one posterior draw (function)
  - find the wing length corresponding to the max. of that draw
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# Bayesian optimization of wing length

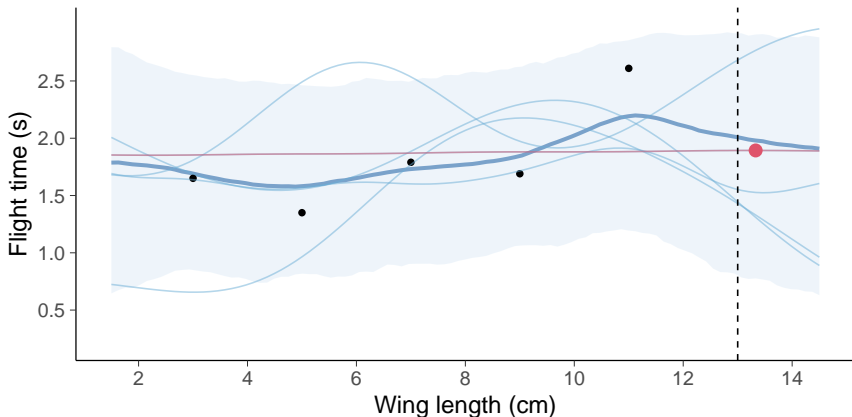
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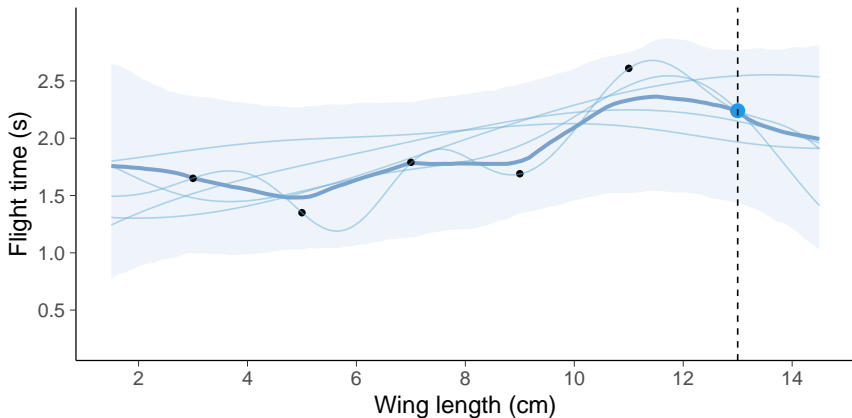
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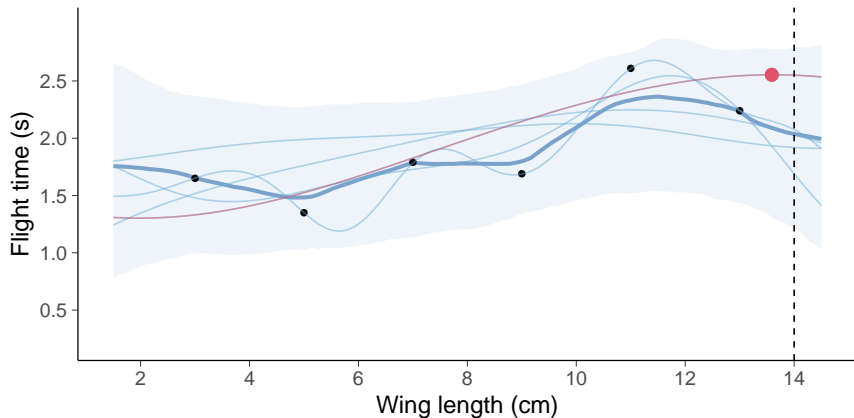
Gaussian process model – Thompson sampling





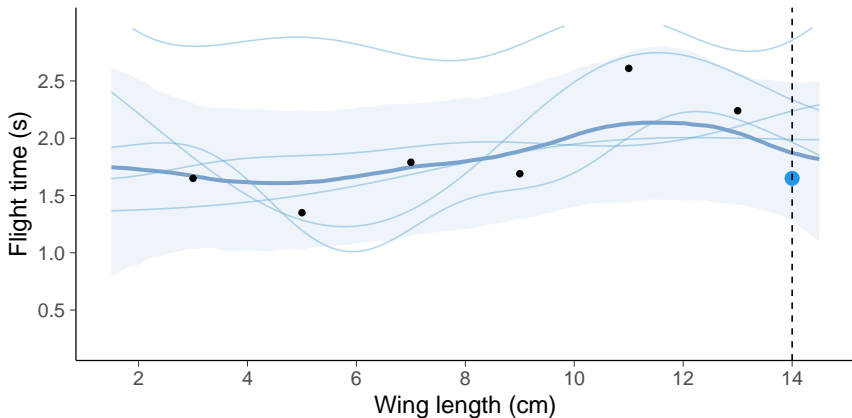
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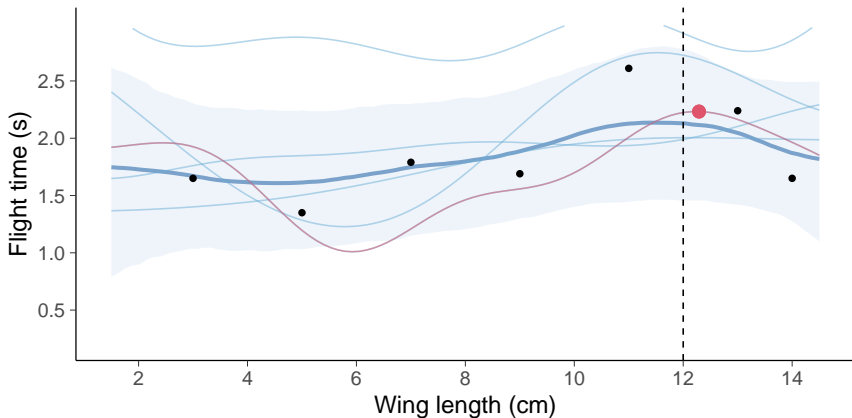
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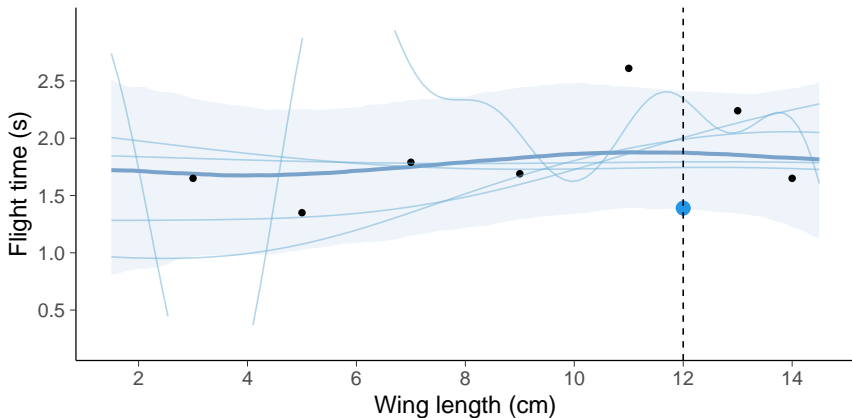
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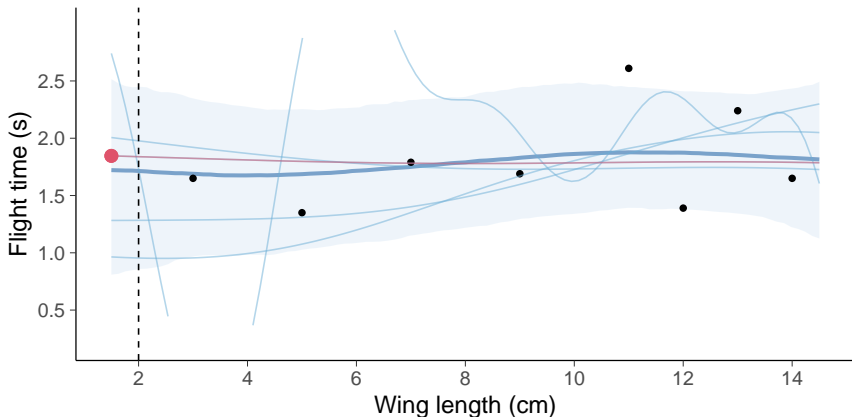
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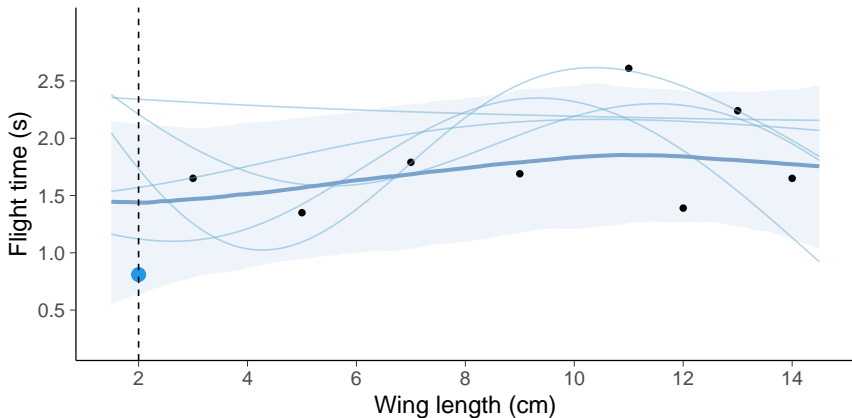
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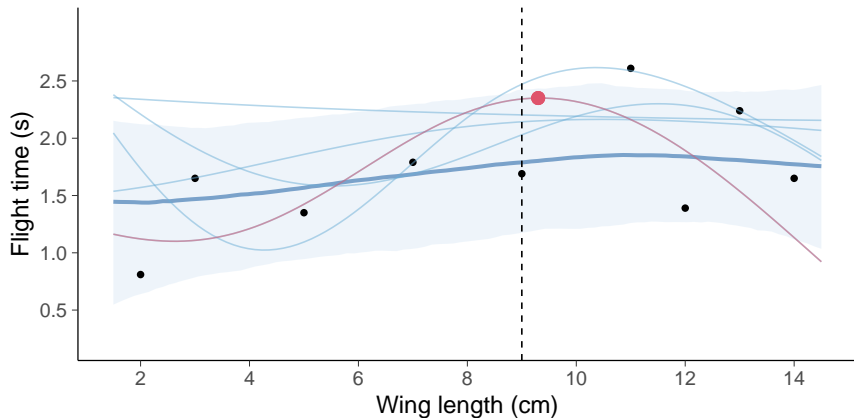
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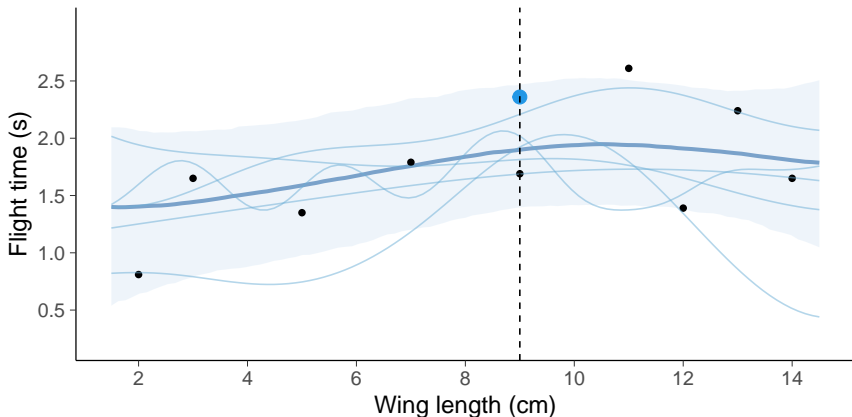
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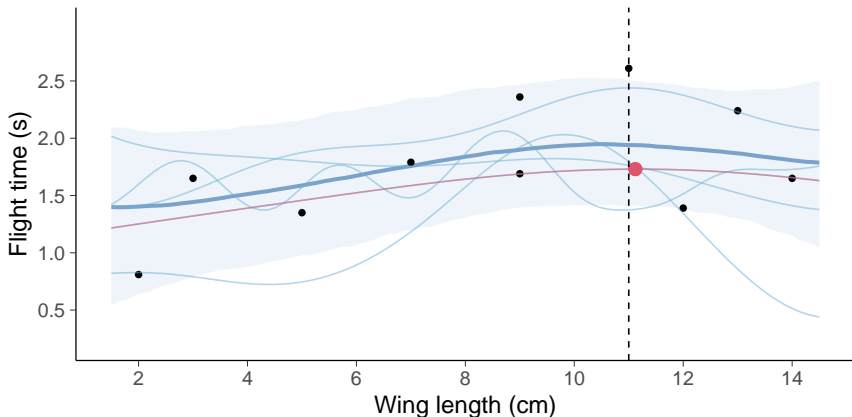
Gaussian process model – Thompson sampling





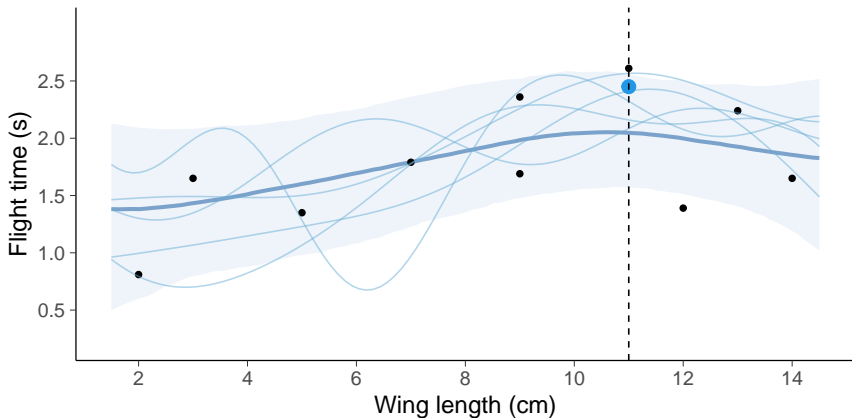
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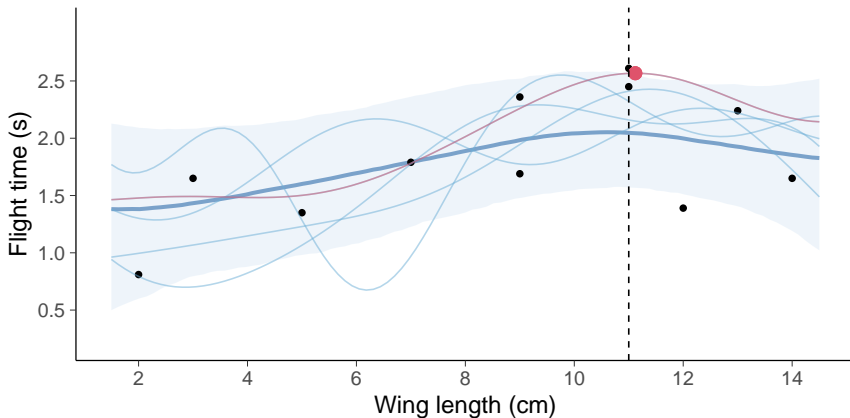
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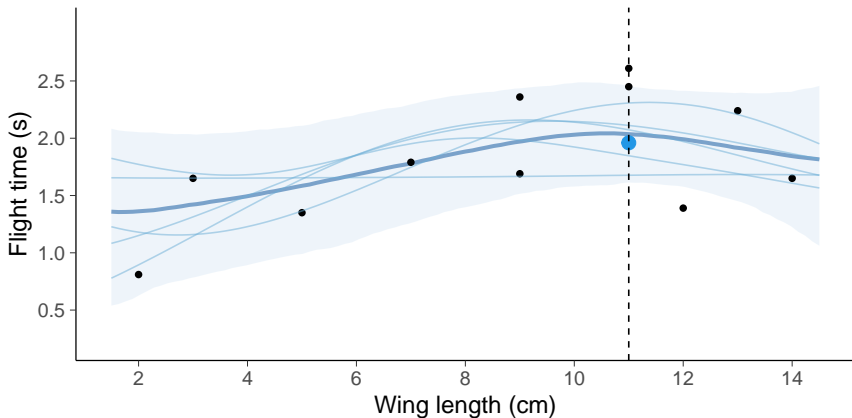
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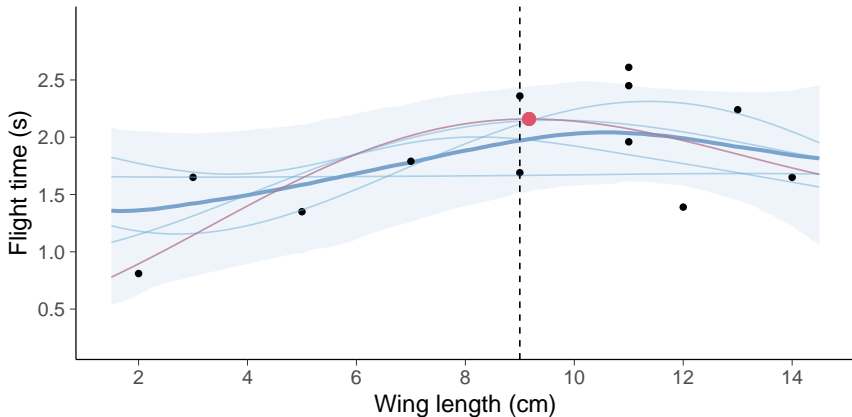
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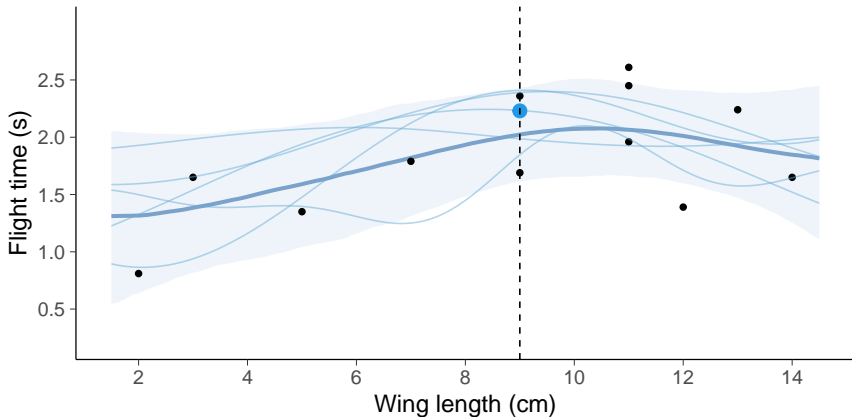
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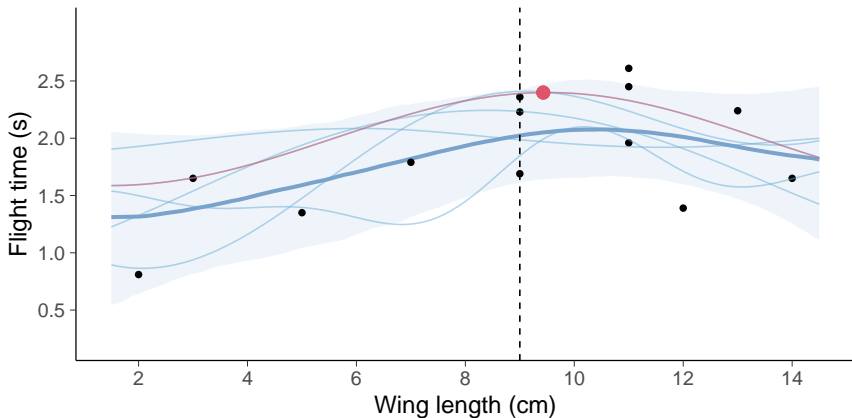
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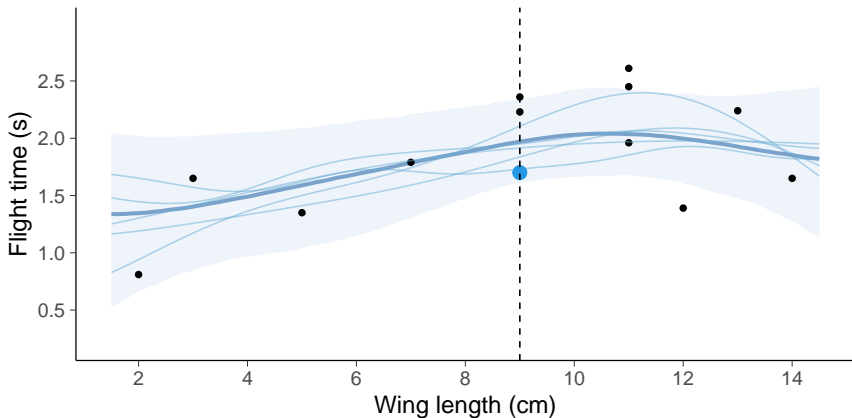
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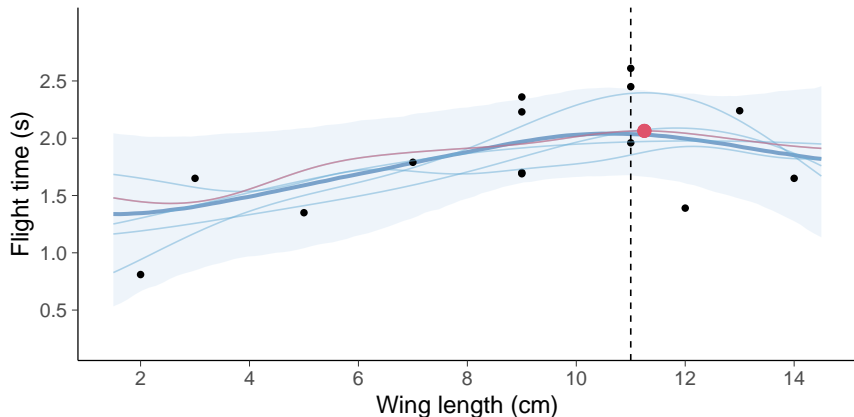
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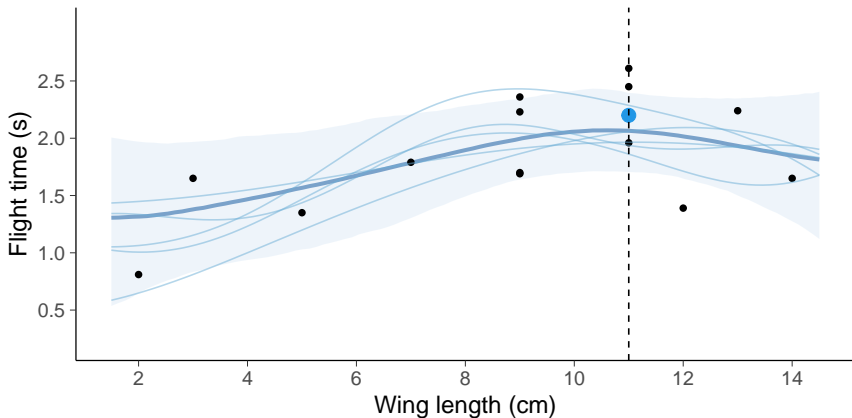
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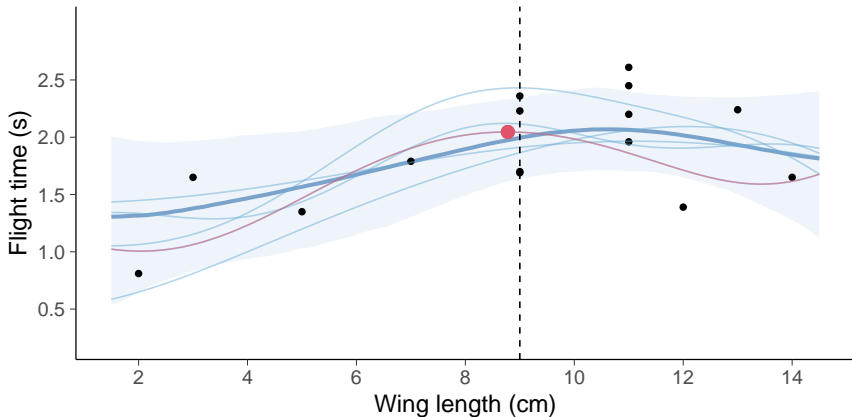
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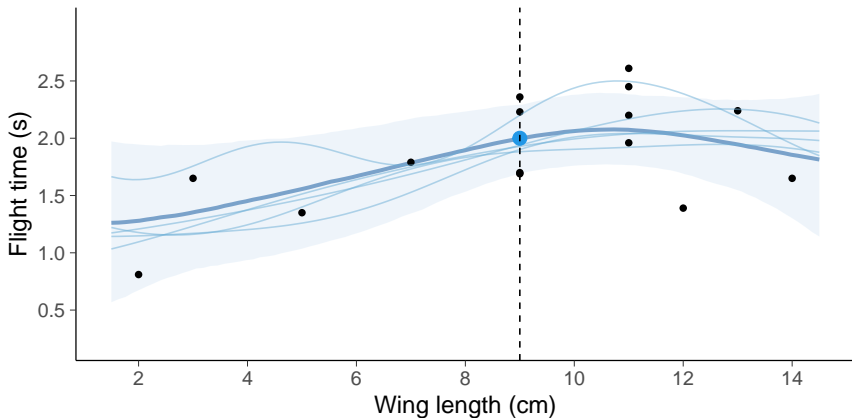
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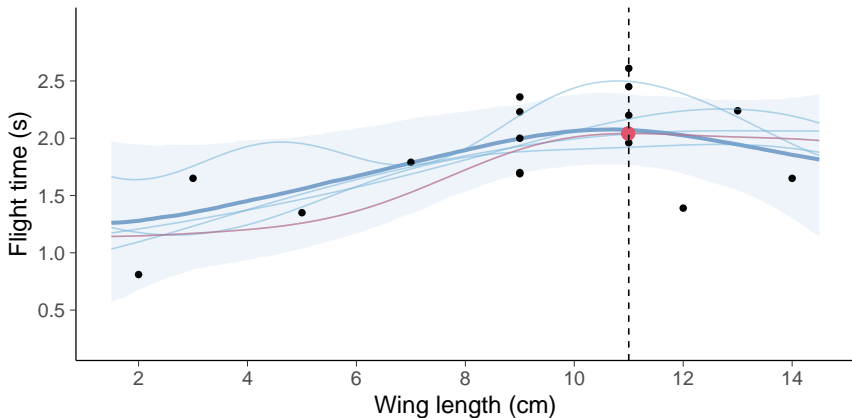
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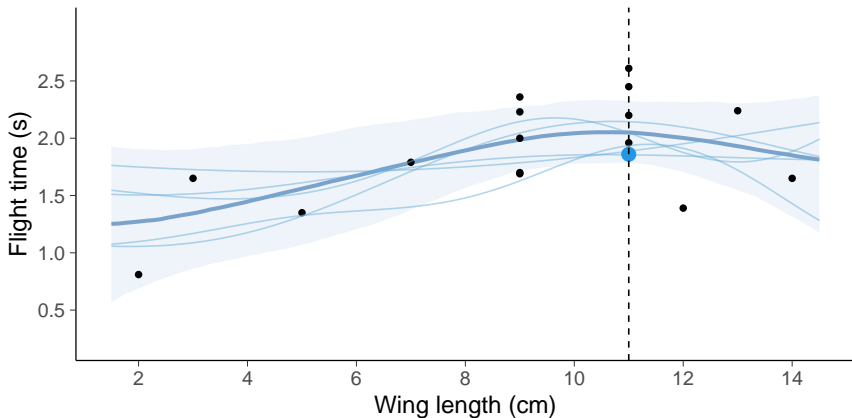
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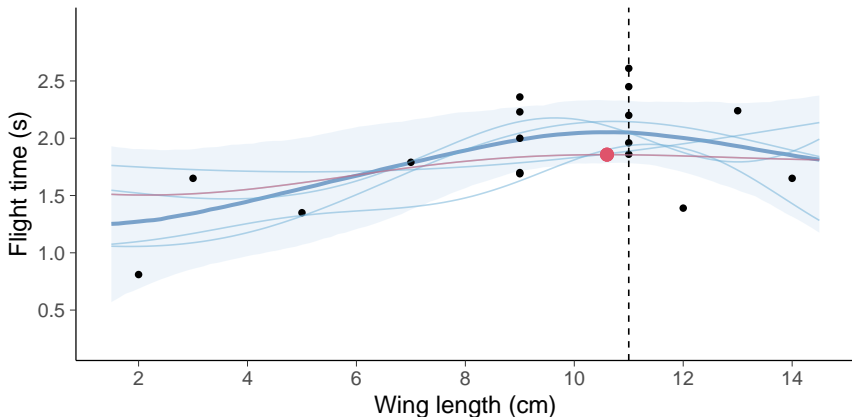
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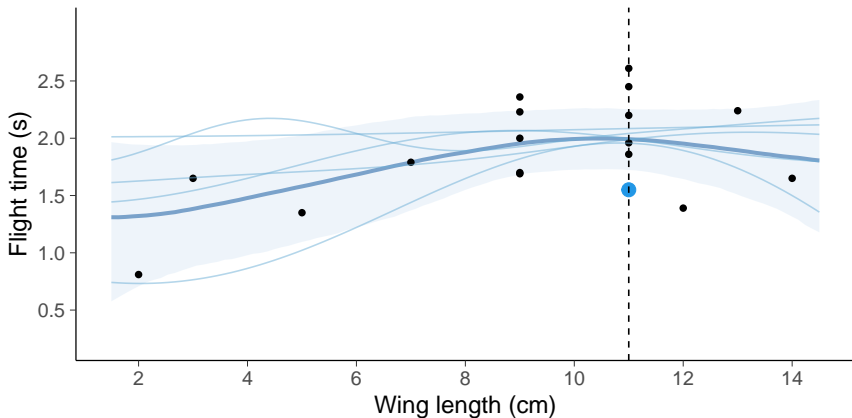
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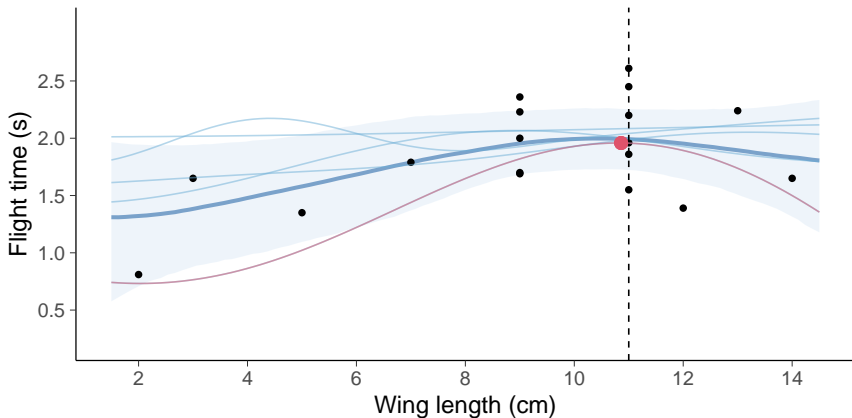
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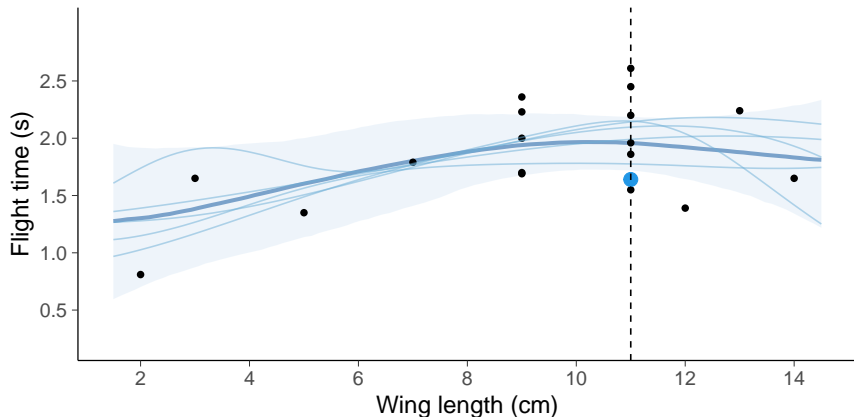
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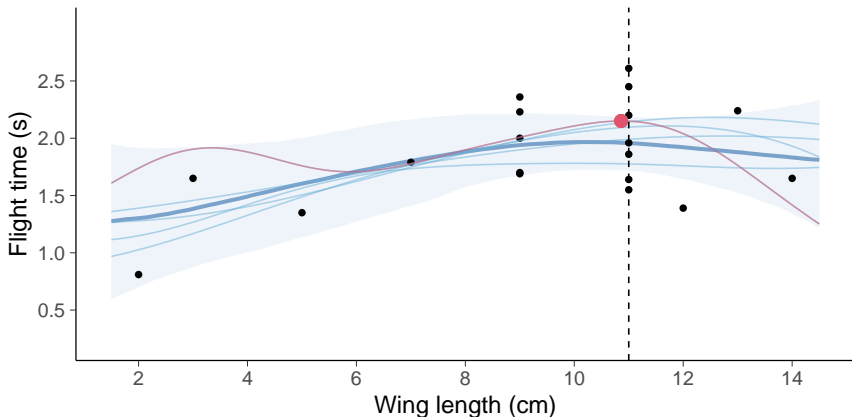
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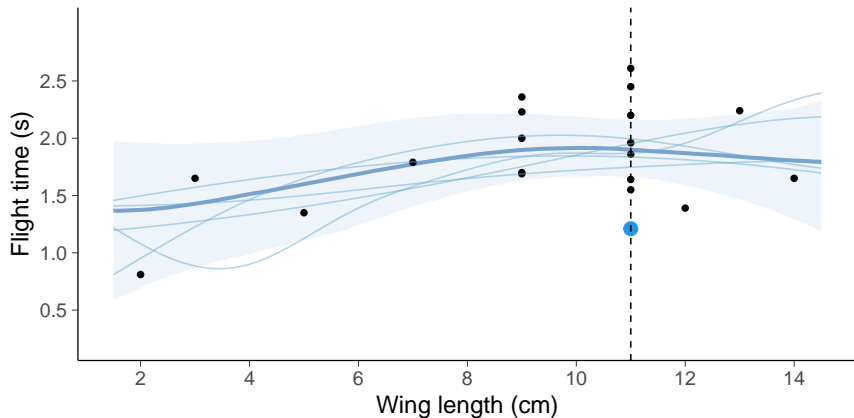
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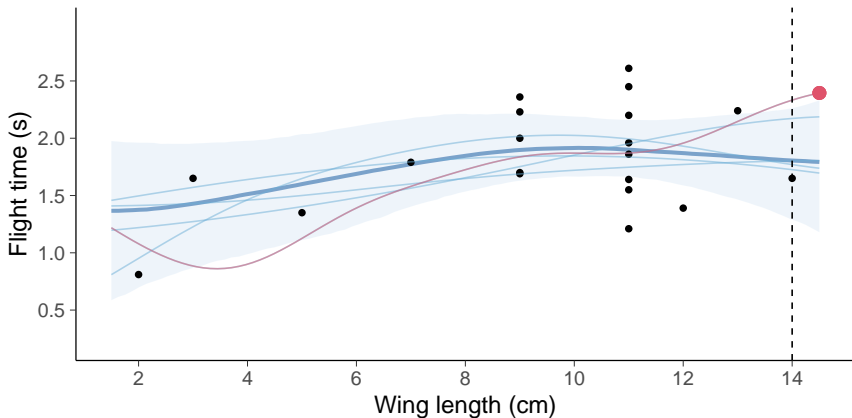
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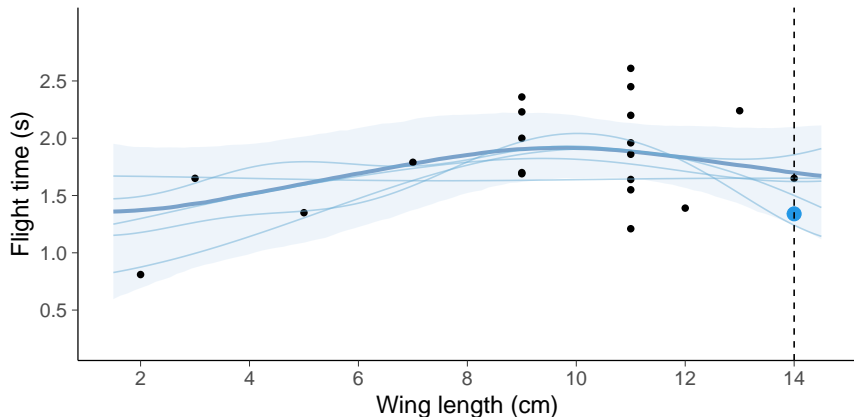
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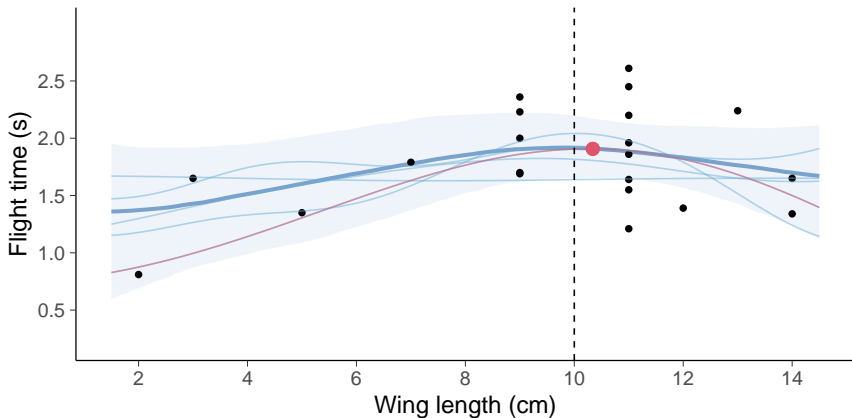
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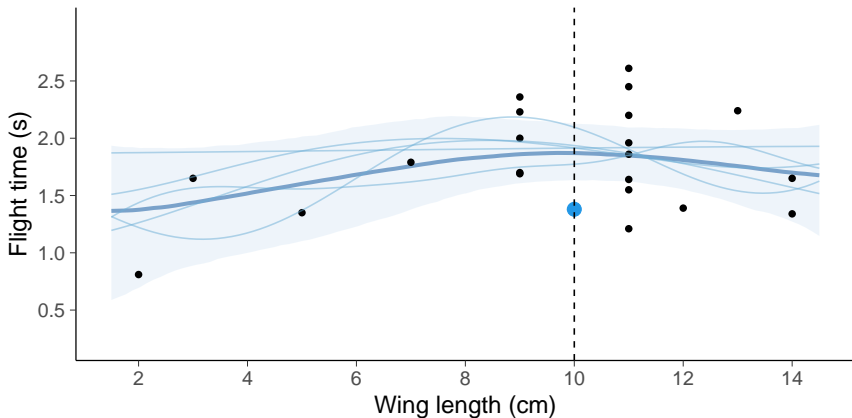
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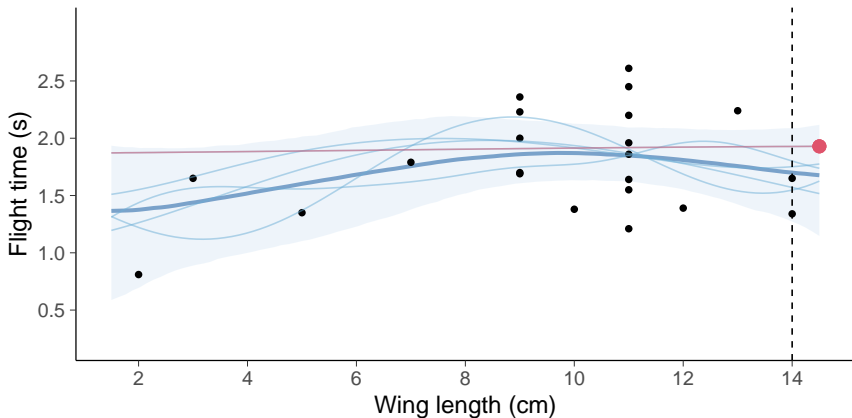
Gaussian process model – Thompson sampling





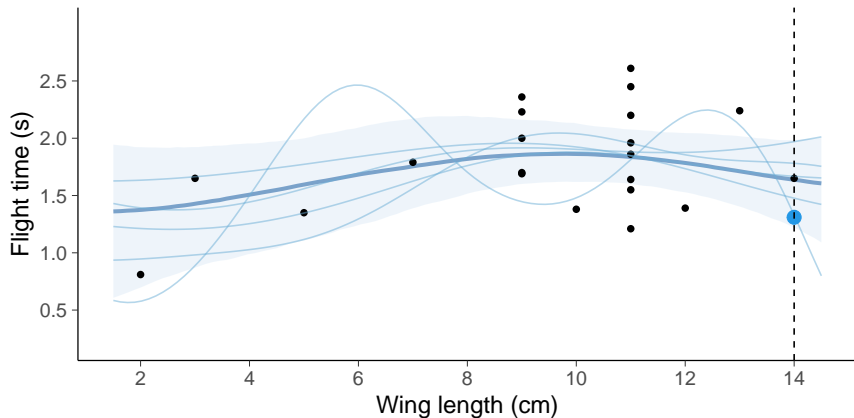
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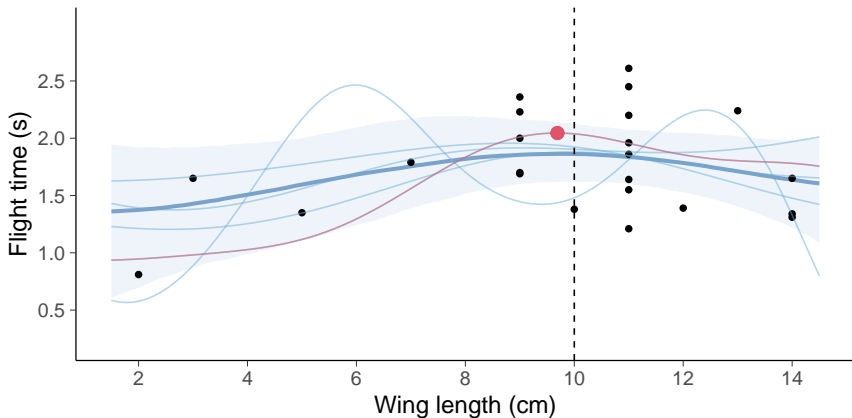
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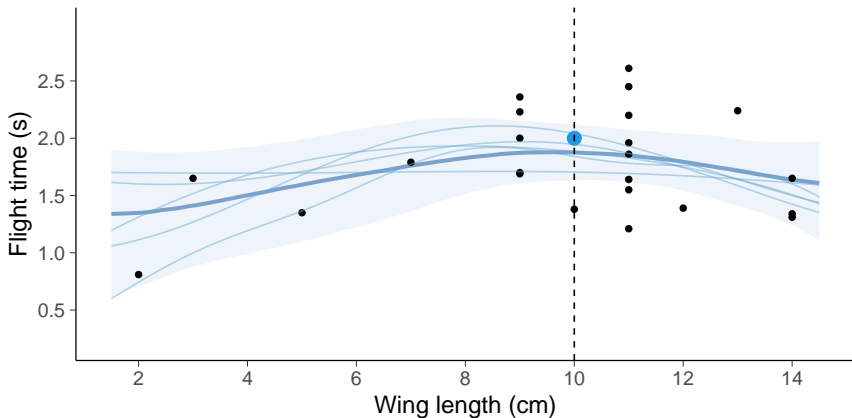
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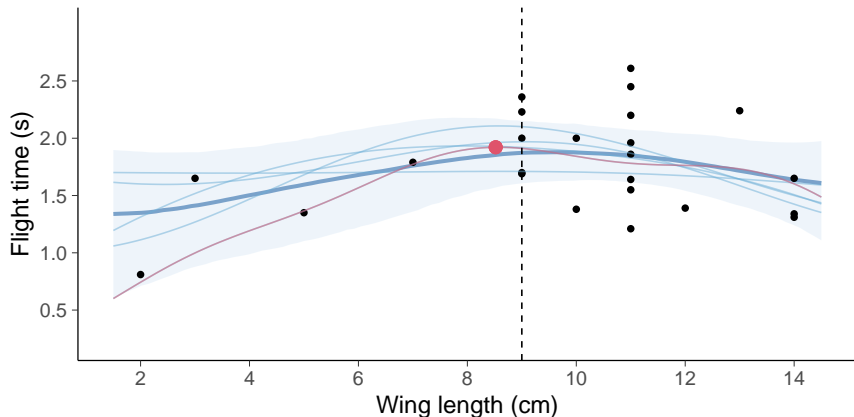
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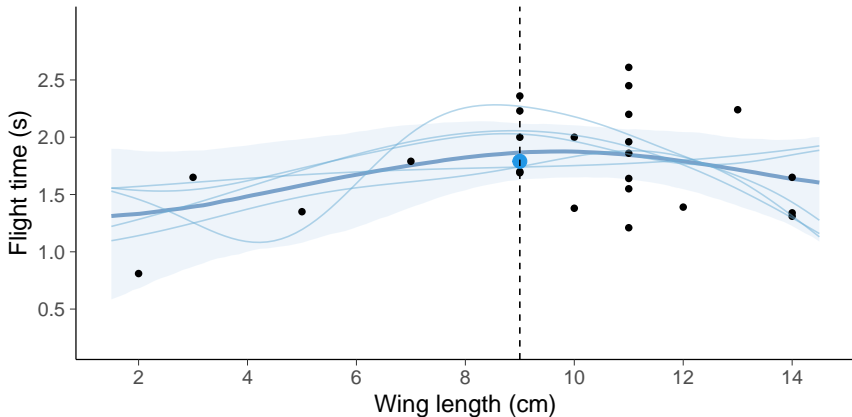
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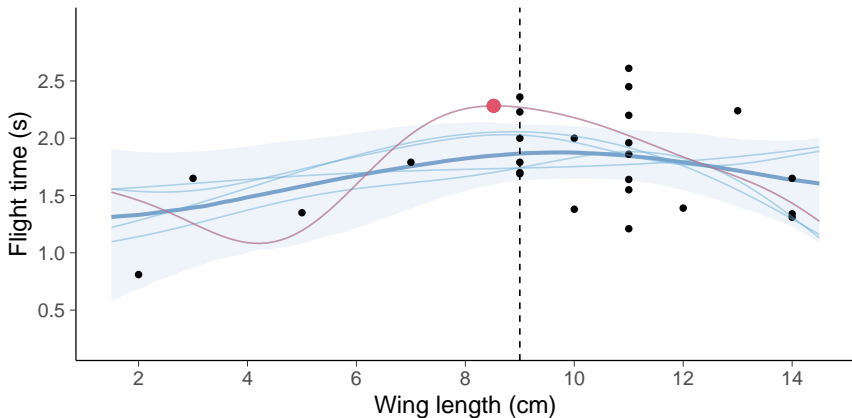
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## Gaussian process model – Thompson sampling



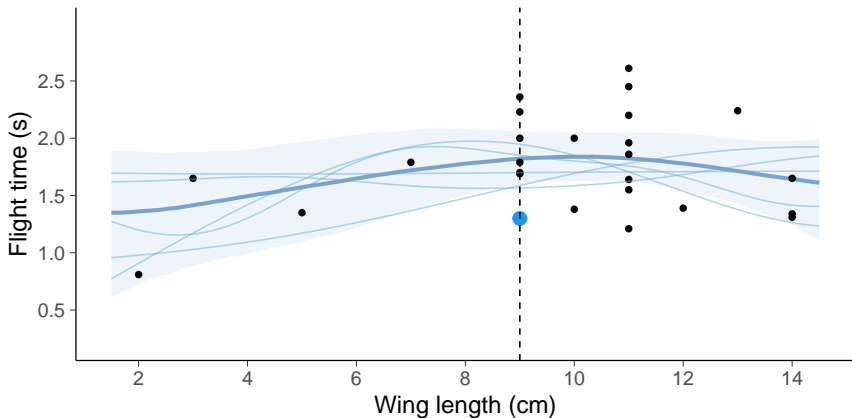
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# Bayesian optimization of wing length

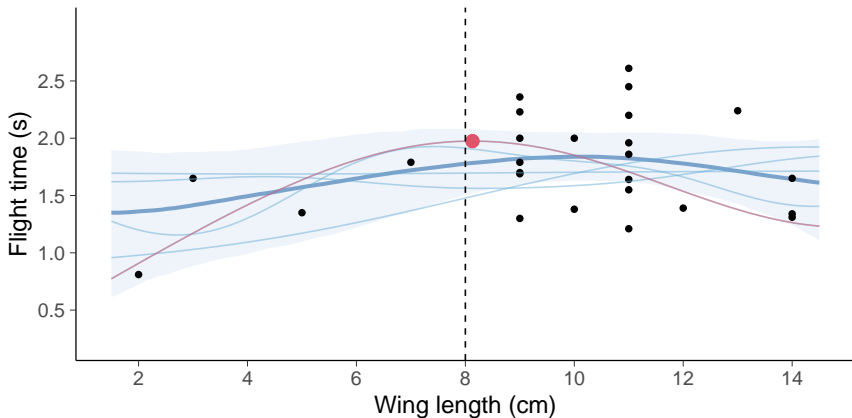
Gaussian process model – Thompson sampling





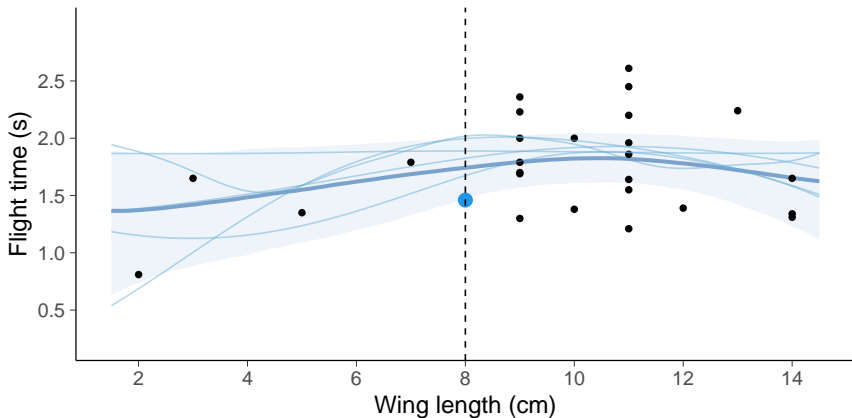
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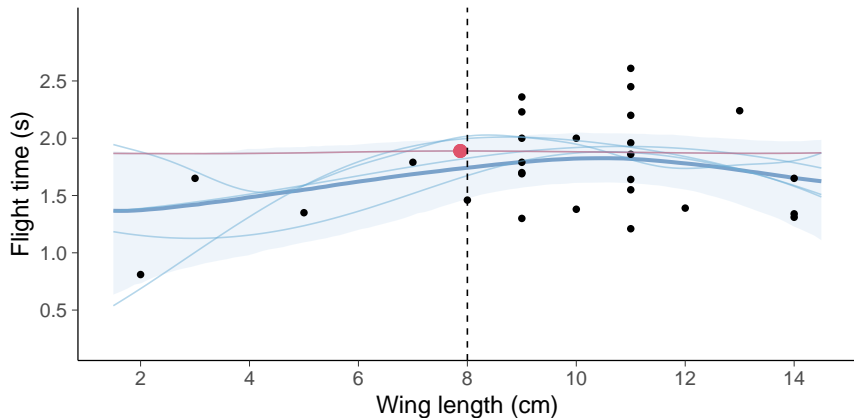
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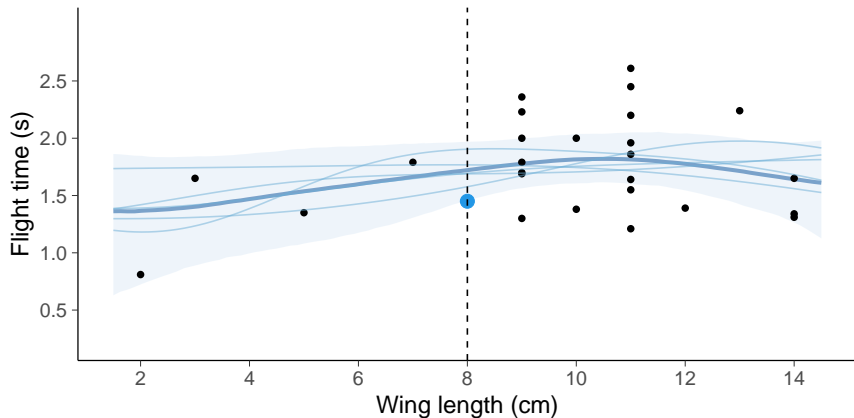
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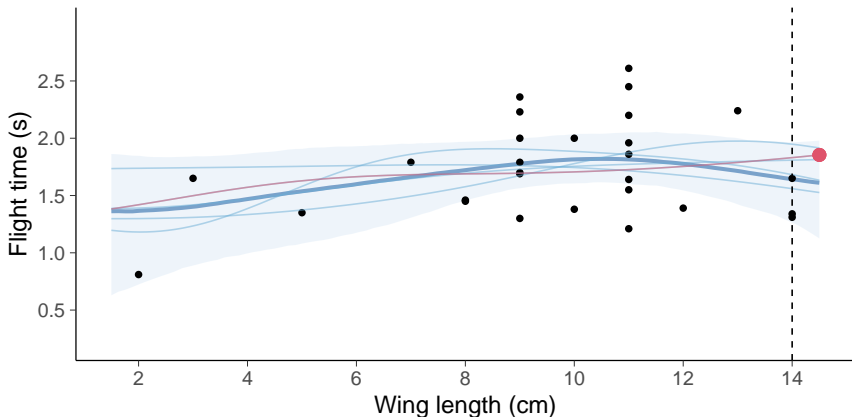
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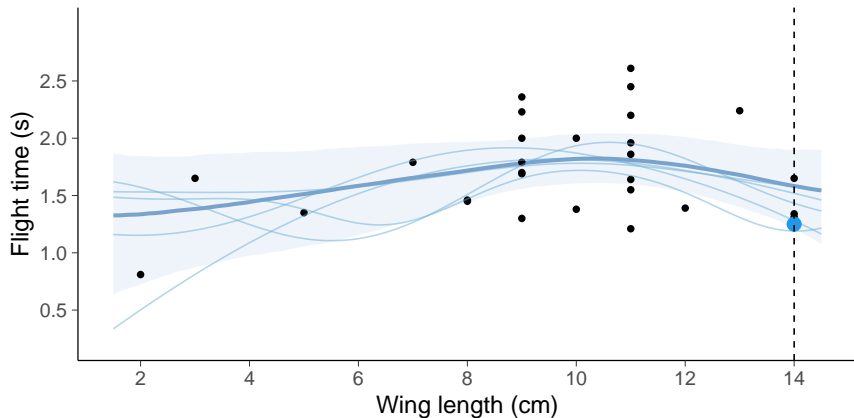
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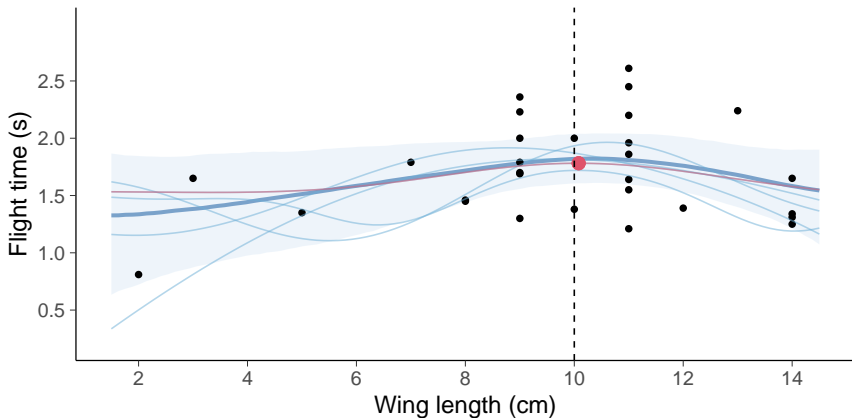
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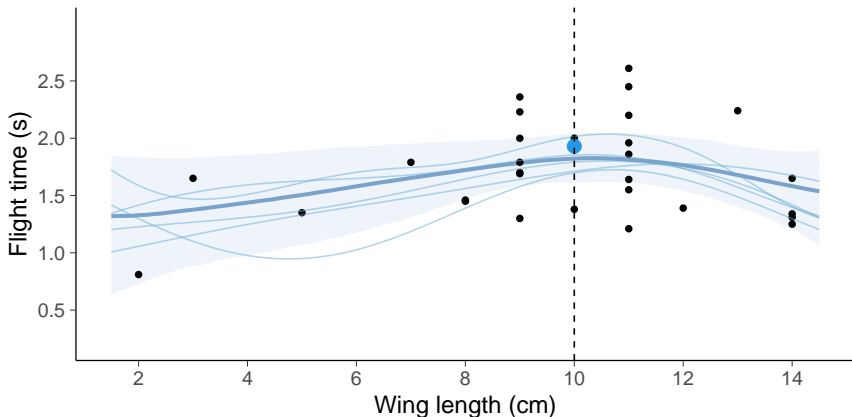
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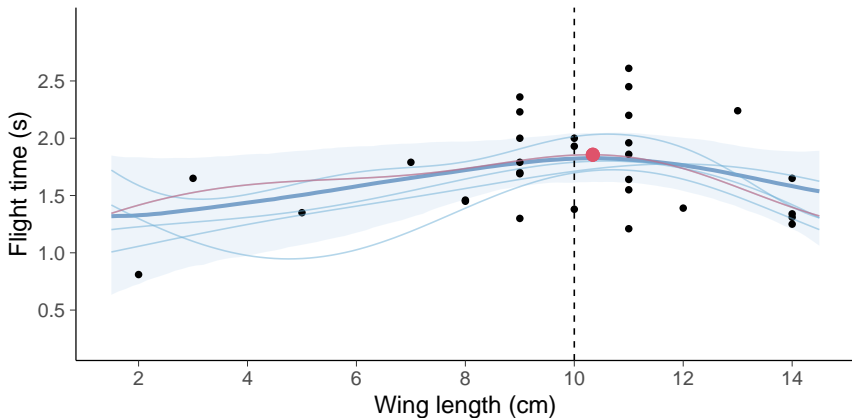
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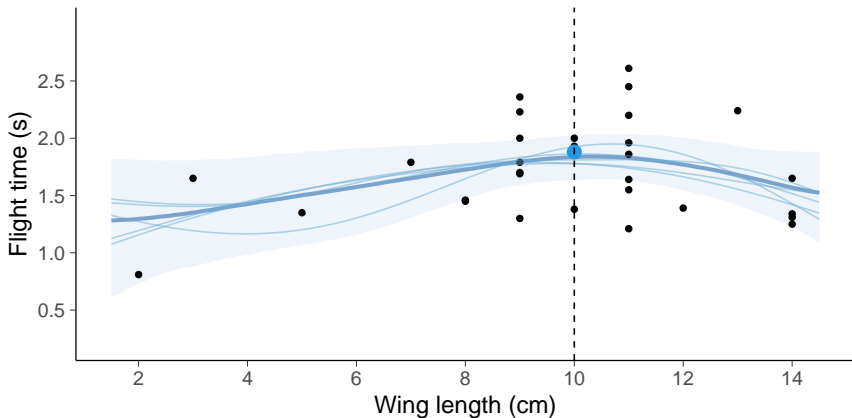
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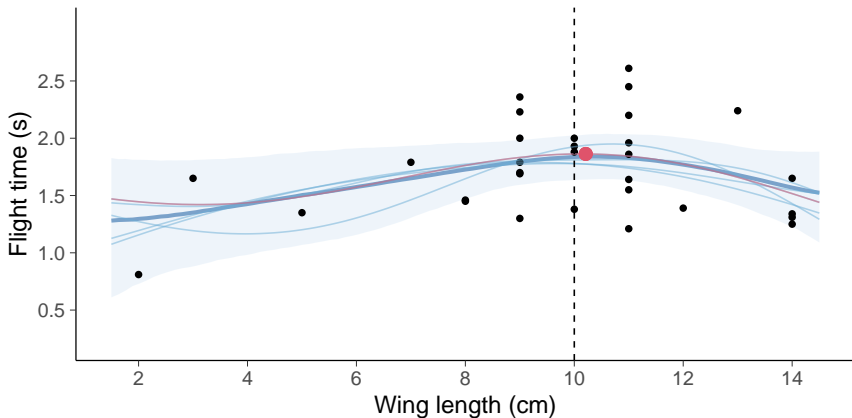
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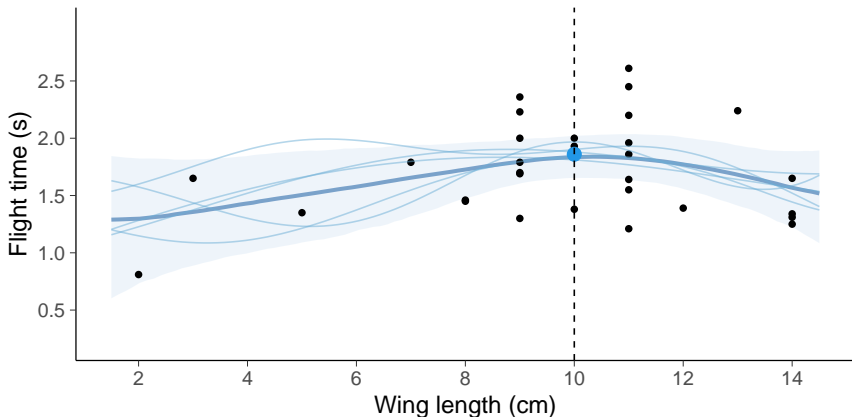
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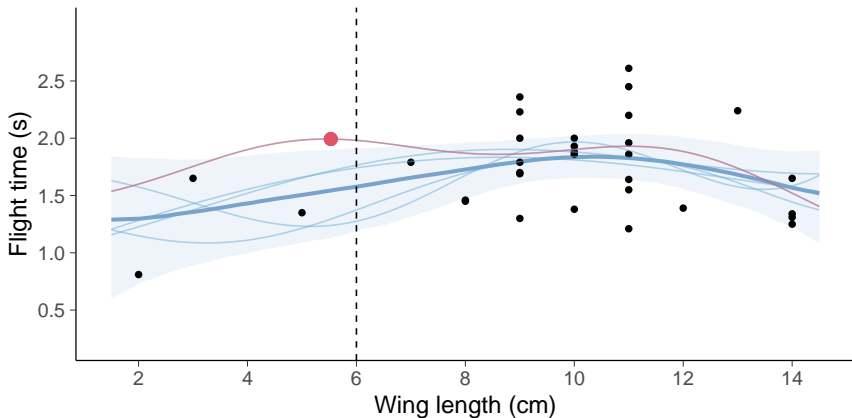
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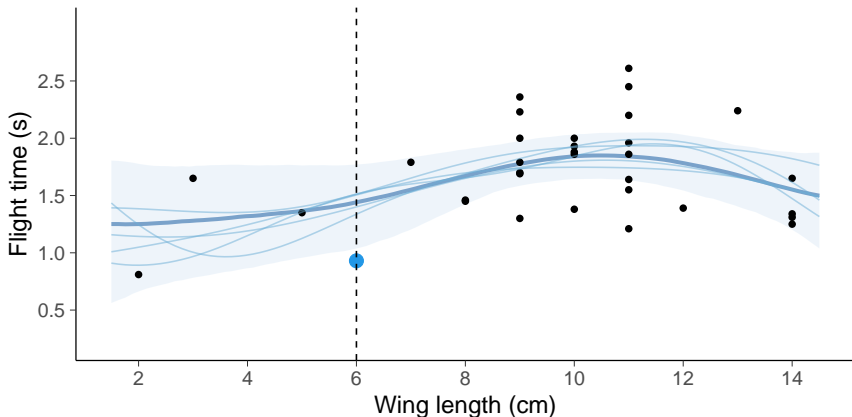
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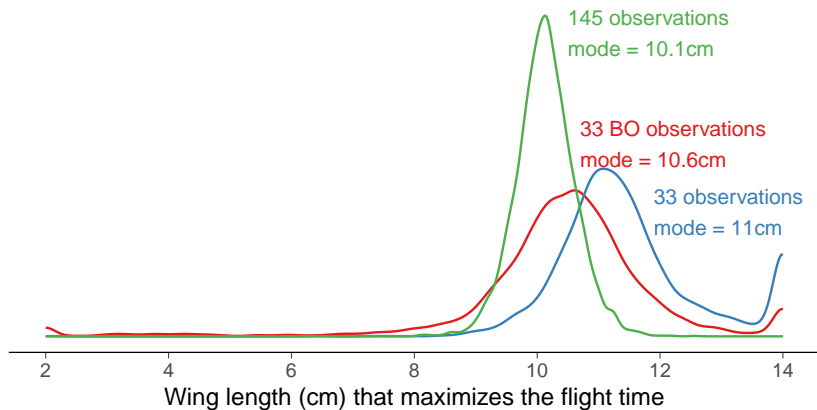


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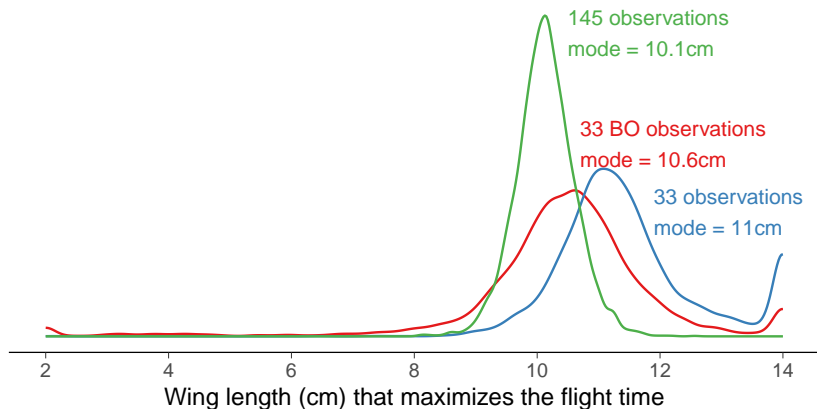
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# Bayesian optimization of wing length



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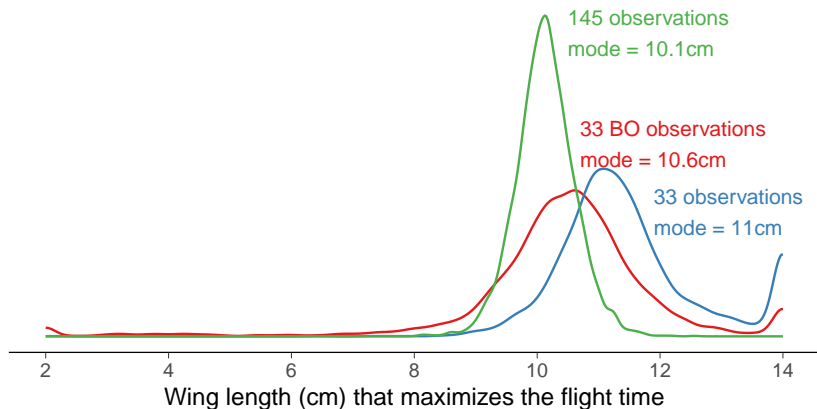


33 BO obs. post. Wasserstein-1 distance  $\approx 0.77$

33 first obs. post. Wasserstein-1 distance  $\approx 1.36$



# Bayesian optimization of wing length



33 BO obs. post. Wasserstein-1 distance  $\approx 0.77$

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We obtain about 50% increase in efficiency

# Examples of big Bayesian decision making success stories

- Bayesian optimization of ML algorithms
- Bayesian optimization of new medical molecules
- Bayesian optimization of new materials
- A/B testing
- Customer retention / satisfaction
- Marketing