Computability and Computational Complexity
Exercises Explained

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1 Undecidability

Exercise 1. Give informal reductions from the hello-world problem to the following:

1. Given a program and an input, does the program stop?

2. Given a program and an input, does the program ever produce an output?

3. Given two programs and an input, do the two programs produce the same output?

Solution:

1. In order to understand whether the program P stops on input I, modify P constructing a new program P' so that

   - When P would halt, P' will output `hello, world`
   - When P would output `hello, world`, P' halt

   In this way, if the original program P printed `hello, world`, the new P' will halt, whereas if the original program stopped, the new one will print `hello, world`.

2. This is simply accomplished by replacing any output statement of P by one that outputs `hello, world`.

3. If we consider P as a general program and P' as the standard `hello, world` one, then checking whether the two programs produce the same output is trivial, since you just need to replace any output of P by one that outputs `hello, world`.

2 Turing Machines

Exercise 1. Show the Instantaneous Descriptions (IDs) of the Turing Machine for $\{0^n 1^n\}$ for the following input tapes:

1. 00

2. 000111

3. 00111

Solution: