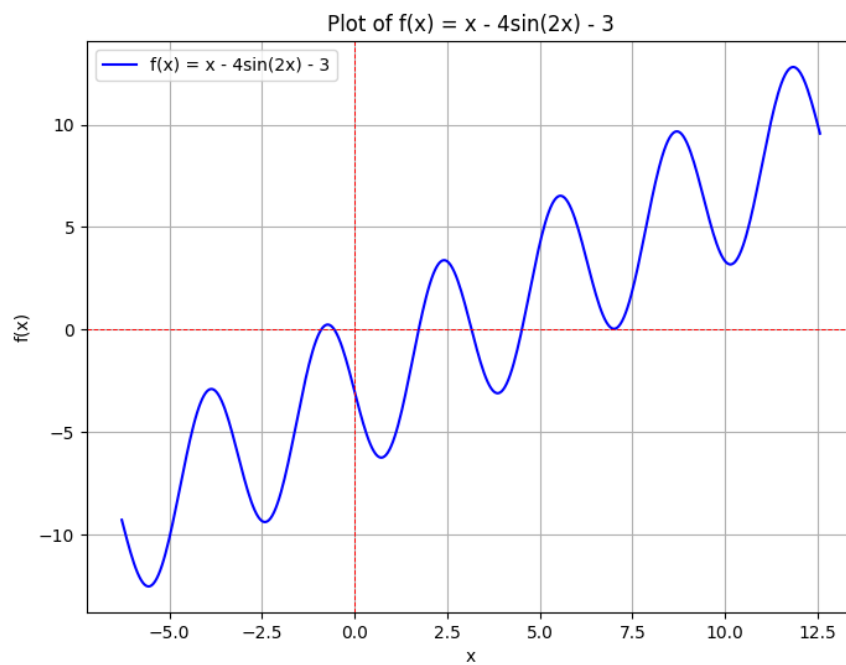


1. a. Let  $f(x) = 2x - 1 - \sin(x)$   
Interval =  $[0, 3.14]$   
 $f(0) = -1$  and  $f(\pi) = 2.14$   
Since  $f$  is less than 0 at one end of the interval and greater than 0 at the other and the function is continuous and differentiable in the interval, there must be a value  $c$  where  $f(c) = 0$  in the interval.  
b.  $f'(x) = 2 - \cos(x)$   
 $\cos(x)$  is bounded by  $[-1, 1]$  so  $f'(x)$  is always positive  
Since  $f'(x)$  is always positive there must only be one root  
c. Root: 0.8878622118756174  
Call: `print(bisection(lambda x: 2*x - 1 - np.sin(x), 0, 3.14, 10**-8))`  
Iterations = 28  
Code pushed to Git
2. a. and b. pushed to Git  
c. In part a, bisection evaluates as expected. In part b, the subtraction of large numbers and the result being close to 0 which results in many significant digits being lost. Thus the evaluation of  $f(5.2)$  evaluates to be a negative number when it should be positive, and bisection fails because  $f(a)f(b) > 0$ .
3. a. The upper bound of the number of iterations is  $\log_2((b-a)/2*\text{tol})/\log_2(2)$  where the interval is  $[a, b]$ . For the given values  $a = 1$ ,  $b = 4$ , and  $\text{tol} = 10^{-3}$ , that gives us an upper bound of 10.55, which rounds up to the nearest whole number of 11 iterations.  
b. The root is 1.378662109375 and it took 11 iterations, the same as the upper bound from part a.
4. a.  $\lim_{n \rightarrow \infty} (-16 + 12 + 6 - p)/(2 - p) = (2-p)/(2-p) = 1$   
First order convergence, rate = 1  
b.  $\lim_{n \rightarrow \infty} (\frac{2}{3} * 3^{1/3} + 3^{-2/3} - p)/(3^{1/3} - p)$   
First order convergence, rate =  $(\frac{2}{3} * 3^{1/3} + 3^{-2/3} - p)/(3^{1/3} - p)$   
c.  $\lim_{n \rightarrow \infty} (3 - p)/(3 - p) = (3-p)/(3-p) = 1$   
First order convergence, rate = 1



5. a.

There appear to be six roots.

b. The roots that can be found using fixed point iteration are the ones where the derivative of  $f(x) + x$  is less than 1.