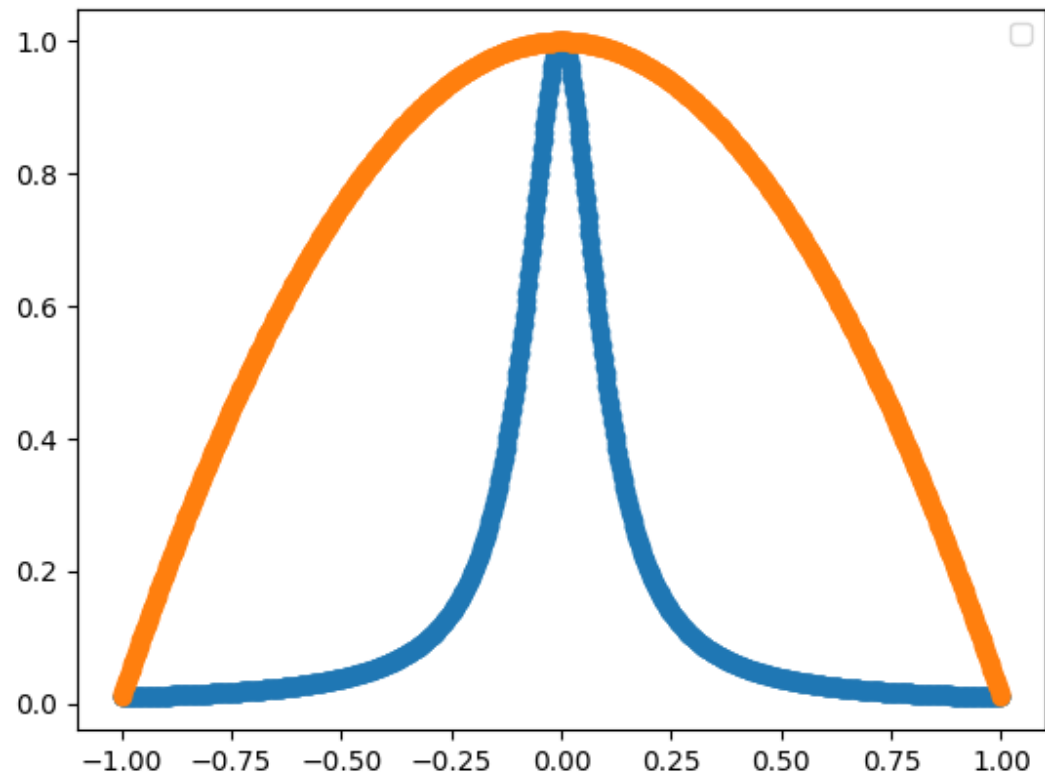


1. a.

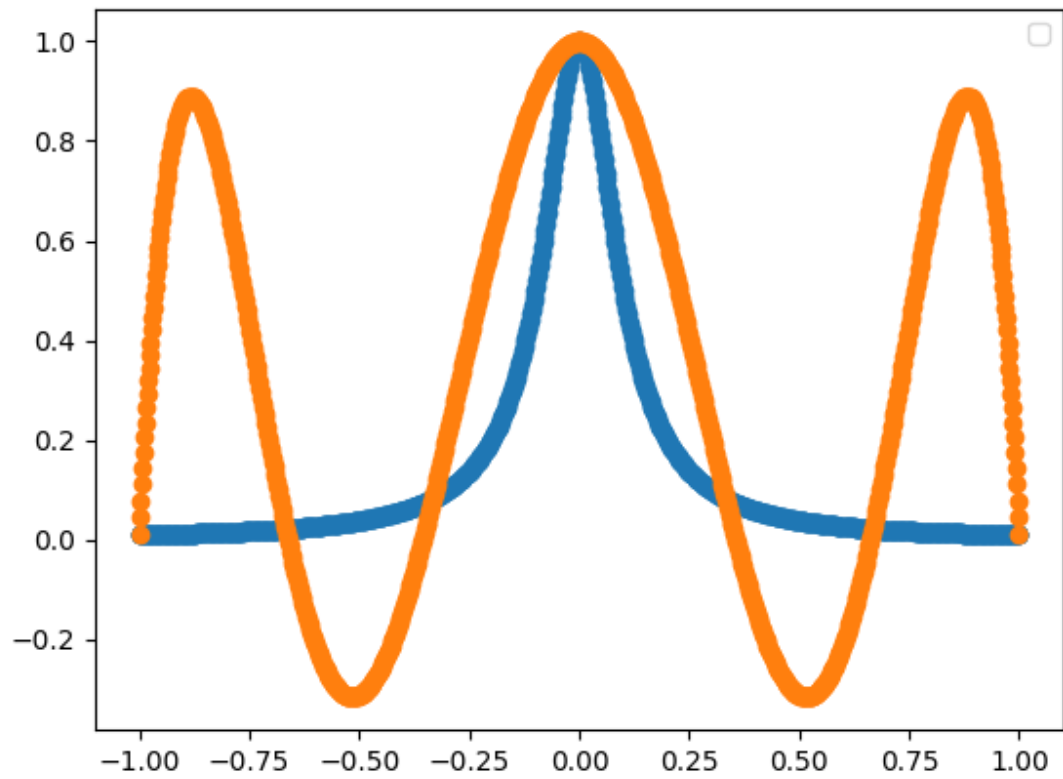
1. a. Given $p(x) = c_n + c_{n-1}x + c_{n-2}x^2 + \dots + c_1x^{n-1}$
 To set up a matrix to solve for $c = [c_1, c_2, \dots, c_n]$
 each row j should look like $[x_j^{n-1}, x_j^{n-2}, \dots, x_j, 1]$
 thus we get the system:

$$\begin{bmatrix} x_1^{n-1} & x_1^{n-2} & \dots & x_1 & 1 \\ x_2^{n-1} & x_2^{n-2} & \dots & x_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_n^{n-1} & x_n^{n-2} & \dots & x_n & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

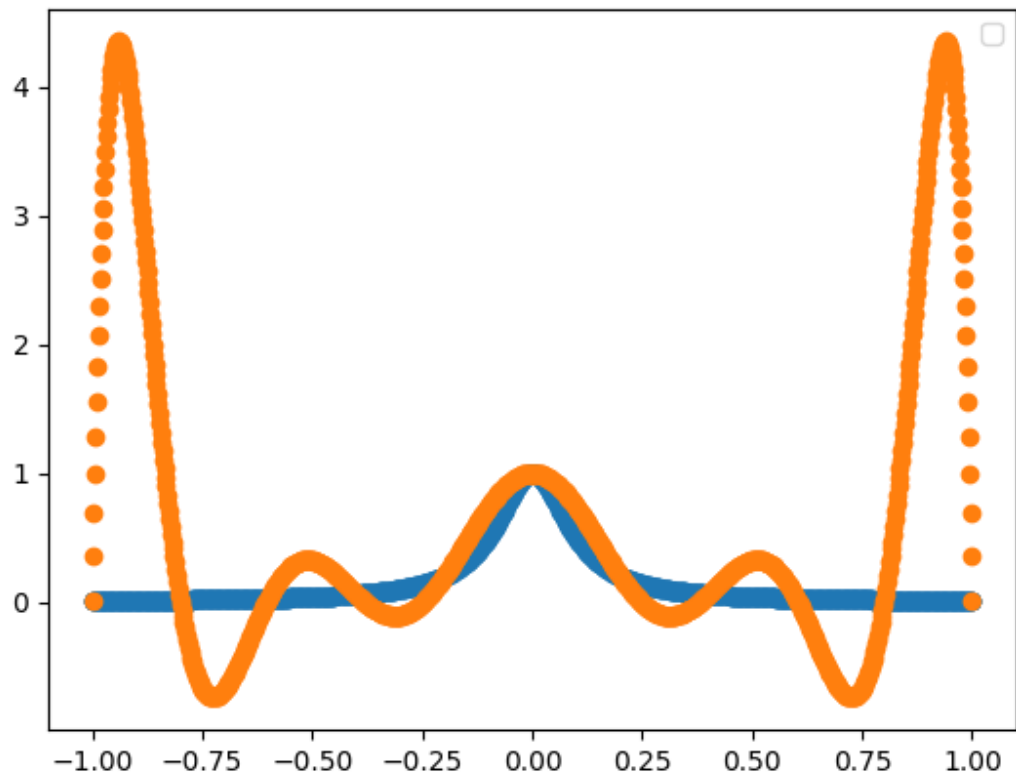
b. N = 2: Coefficients of the interpolating polynomial: [1. 0. -0.99009901]



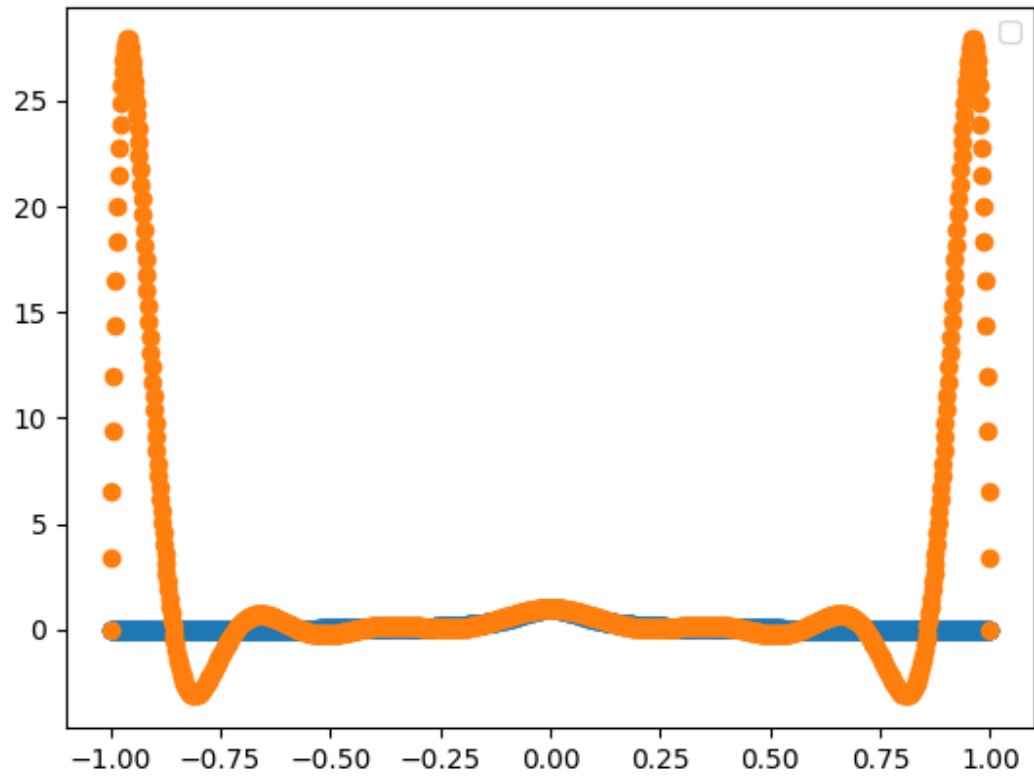
N = 6: Coefficients of the interpolating polynomial: [1.00000000e+00 -4.57134330e-15
-1.11640376e+01 1.91846539e-14
2.81632210e+01 -1.46133106e-14 -1.79892824e+01]



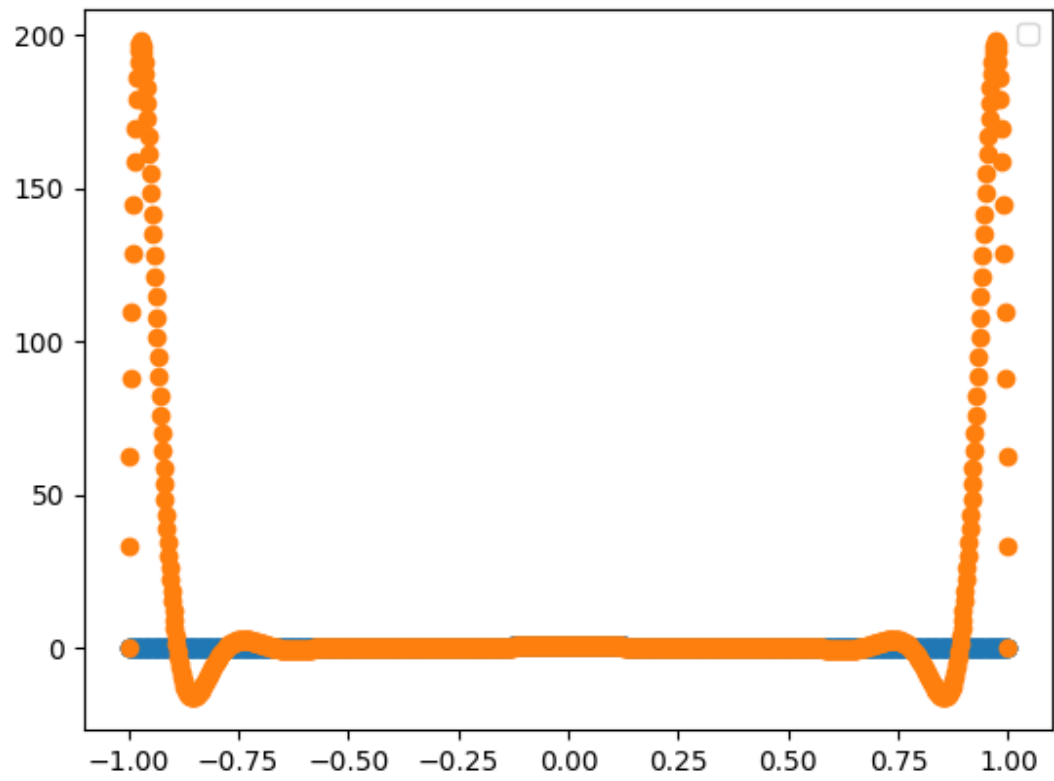
N = 10: Coefficients of the interpolating polynomial: [1.00000000e+00 -9.33756214e-14
-2.85821012e+01 9.99200722e-13
2.45009366e+02 -7.31337389e-12 -8.03461048e+02 1.33236400e-11
1.07037731e+03 -6.91609124e-12 -4.84333624e+02]



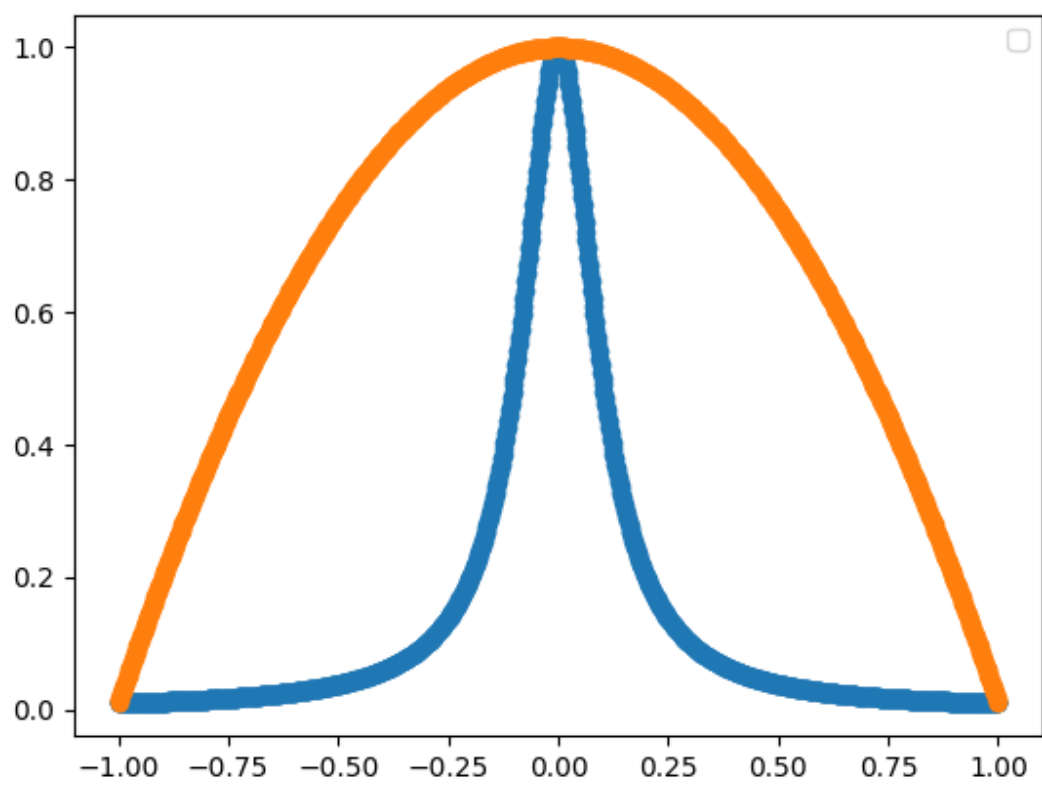
N = 14: Coefficients of the interpolating polynomial: [1.00000000e+00 1.08083597e-11
-4.72915260e+01 -4.49397825e-10
8.24611336e+02 5.67764344e-09 -6.27681370e+03 -2.62339517e-08
2.33629897e+04 5.45256926e-08 -4.41411046e+04 -5.20669147e-08
4.03497885e+04 1.85361198e-08 -1.40731699e+04]



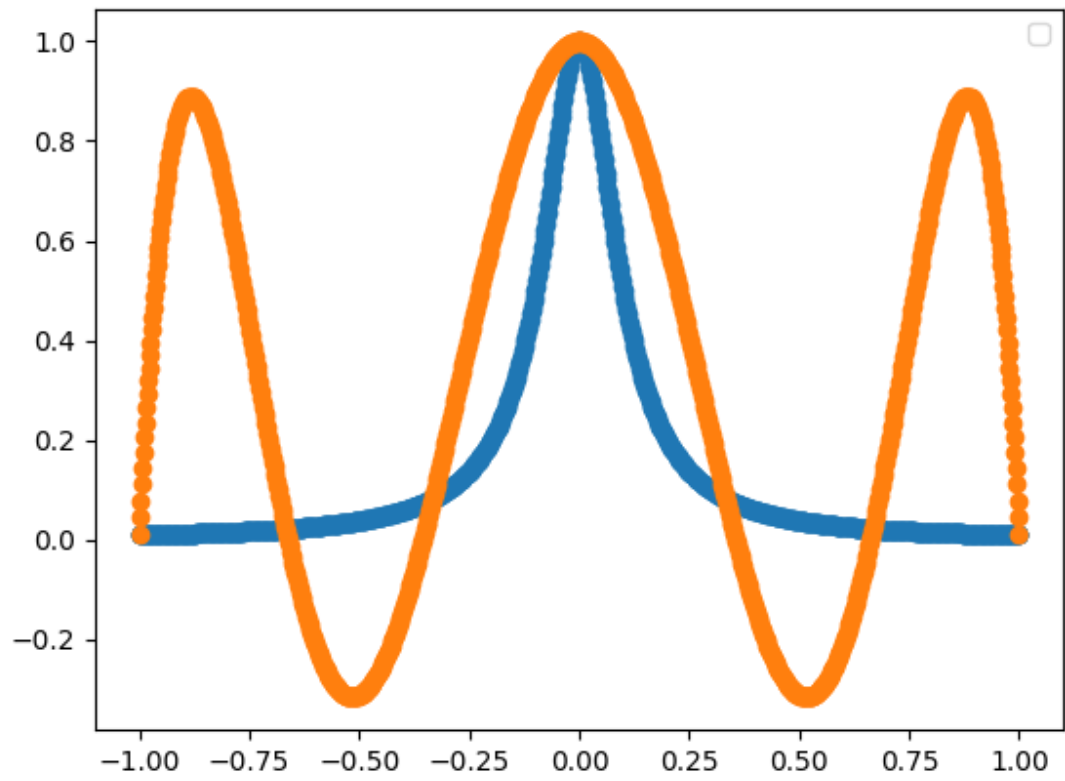
N = 18: Coefficients of the interpolating polynomial: [1.00000000e+00 -5.52447811e-10
-6.34449893e+01 3.12456347e-08
1.78531462e+03 -5.71258487e-07 -2.39624675e+04 5.00612208e-06
1.70256904e+05 -2.25343083e-05 -6.83213028e+05 5.53808676e-05
1.58862963e+06 -7.51227823e-05 -2.10697664e+06 5.27049718e-05
1.47020678e+06 -1.48943057e-05 -4.16664039e+05]



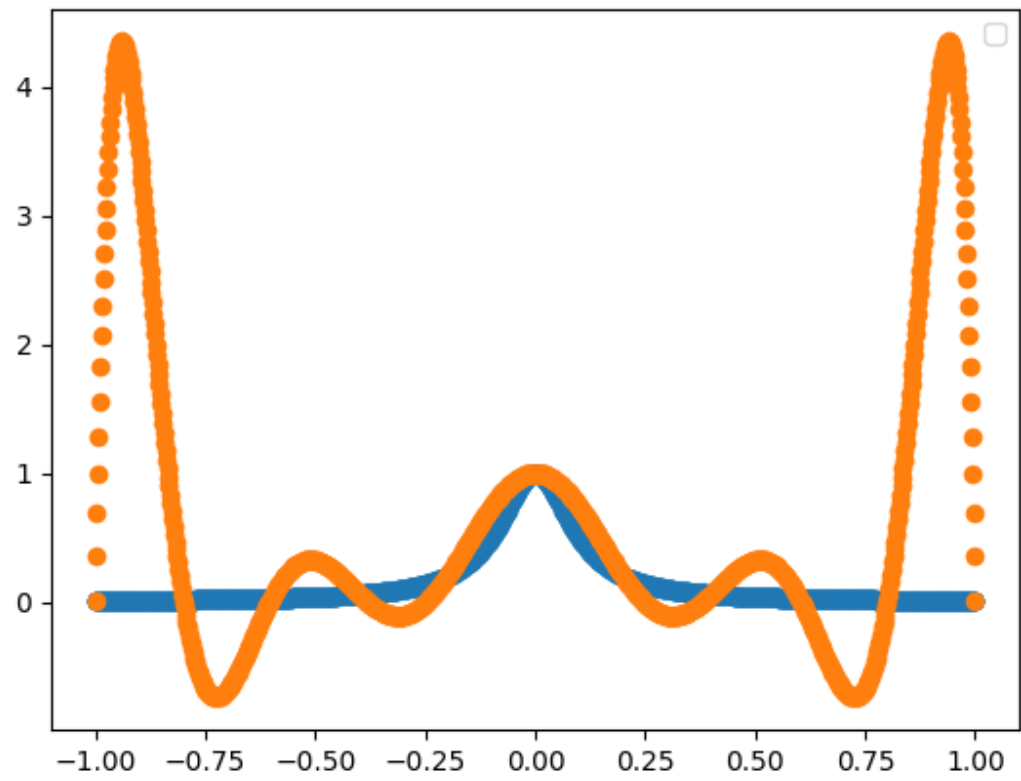
2. Using Lagrange Polynomial:
N = 2:



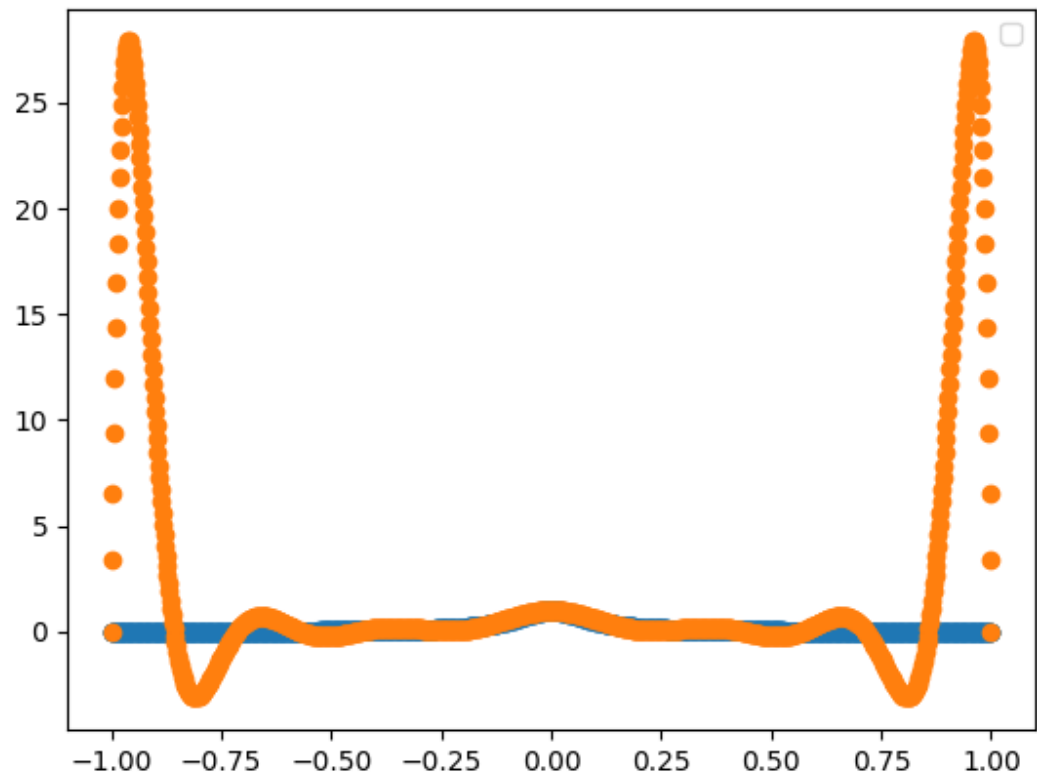
N = 6:



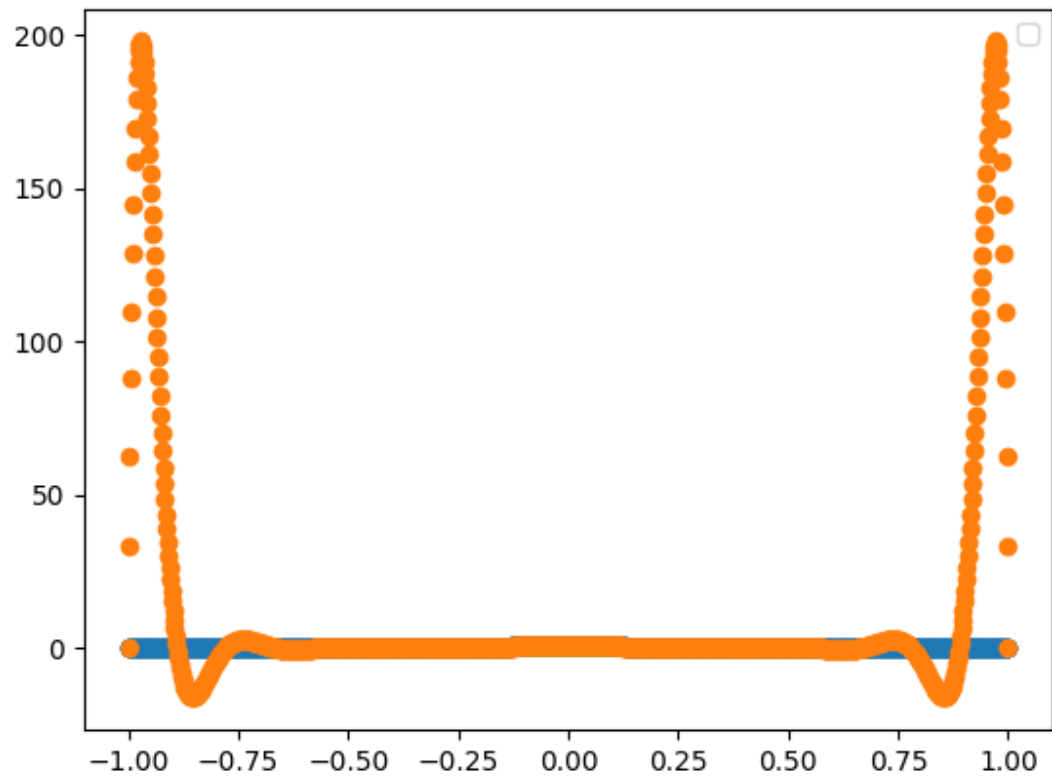
N = 10:



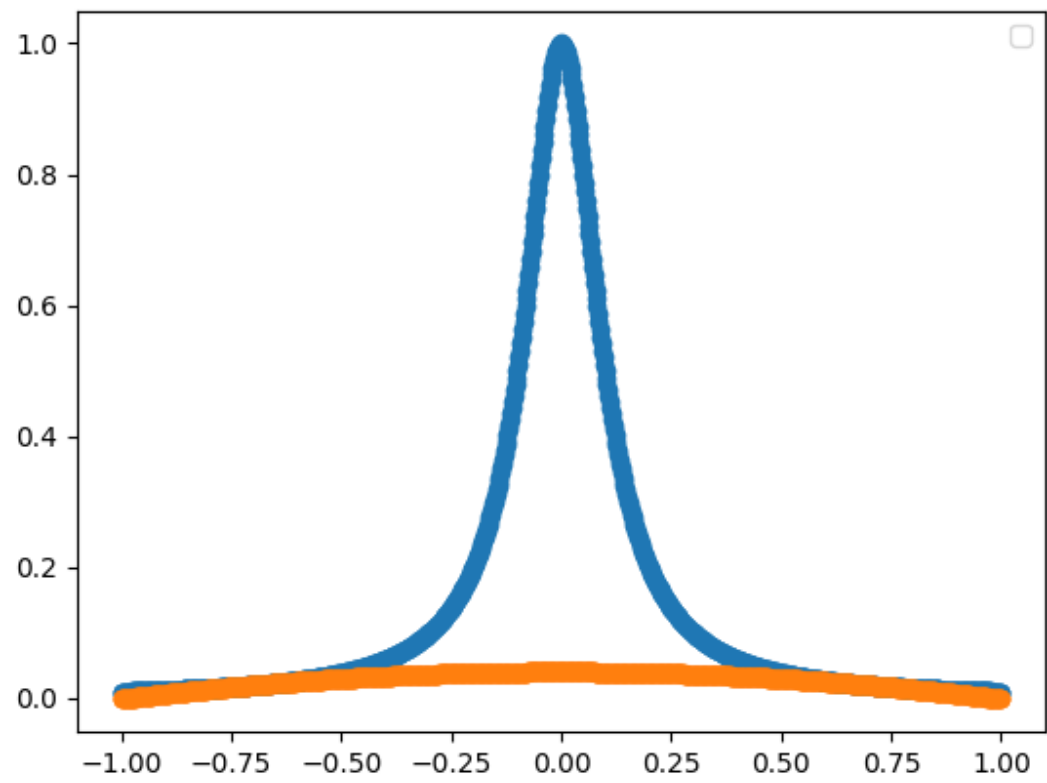
N = 14:



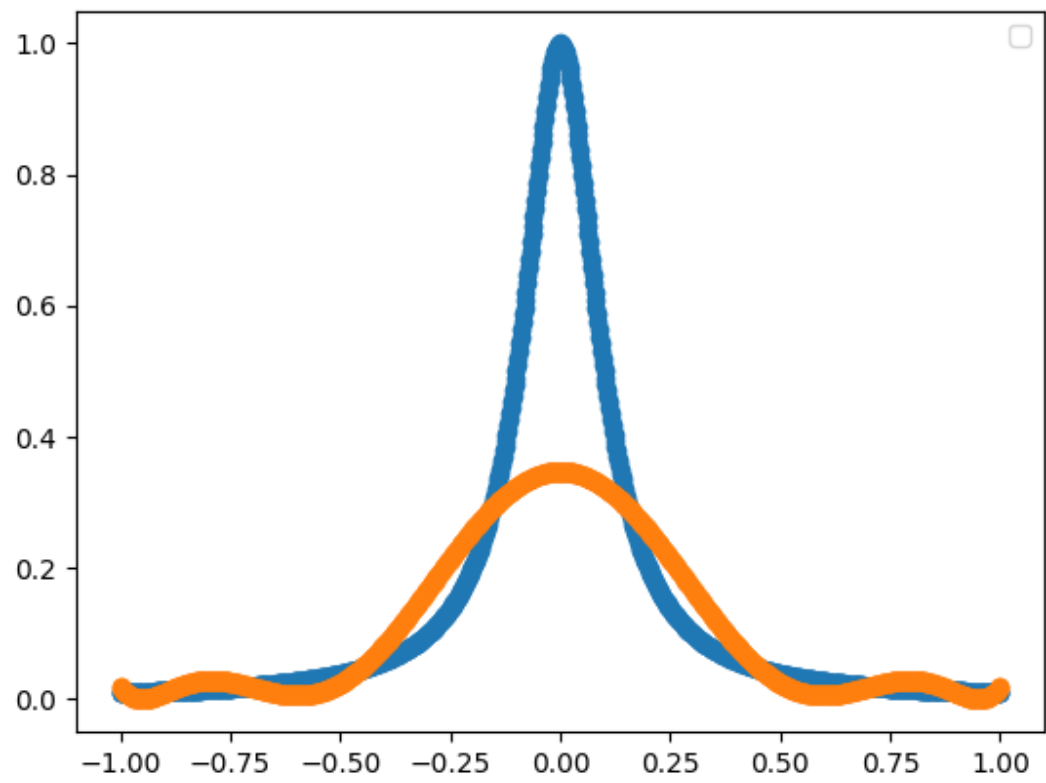
N = 18:



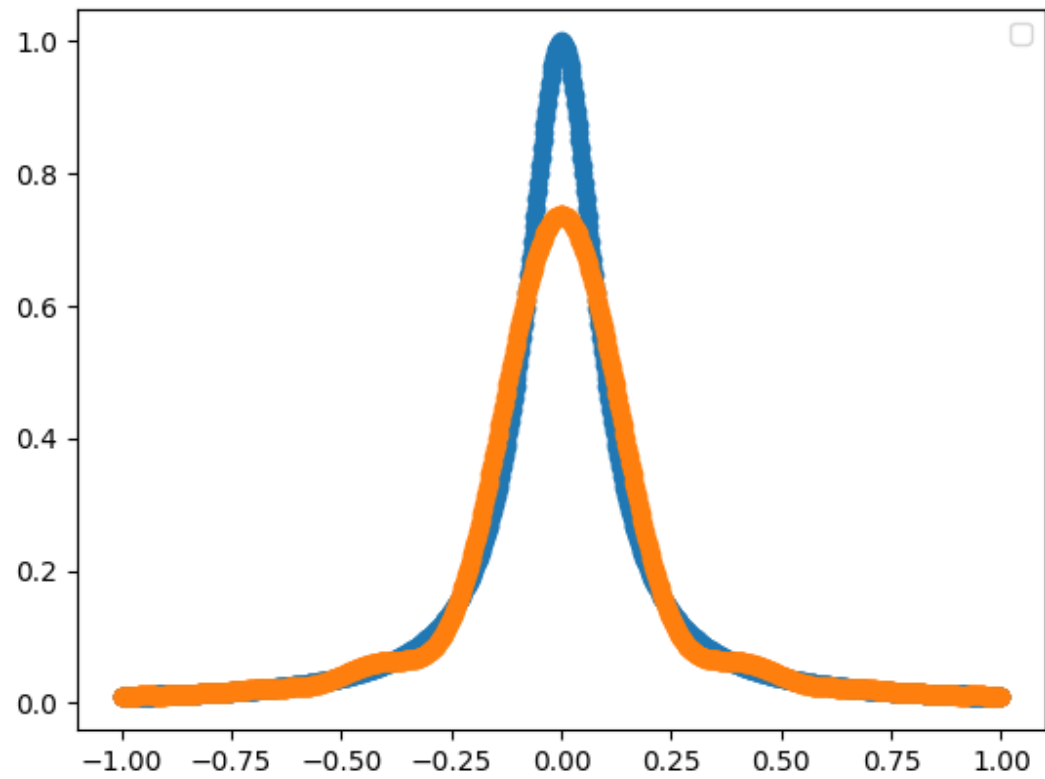
3. Using the monomial basis with the Chebyshev points
N = 2:



N = 10:



N = 20:



The interpolation no longer exhibits Runge's phenomena. However, the interpolation can still fail for some values of N because the Vandermonde matrix is singular.