

1. The natural cubic spline performs the best here because the Lagrange and Hermite interpolation methods are subject to Runge's phenomena when using equispaced points.
2. Using the Chebychev points, the Lagrange and Hermite approximations are superior because they don't exhibit Runge's phenomena anymore and the cubic splines are worse now because they're based on unequal intervals.
3. To modify the coefficients so the spline is naturally periodic, you have to enforce that the second derivatives at the endpoints are the same.