HW0 solution sheet

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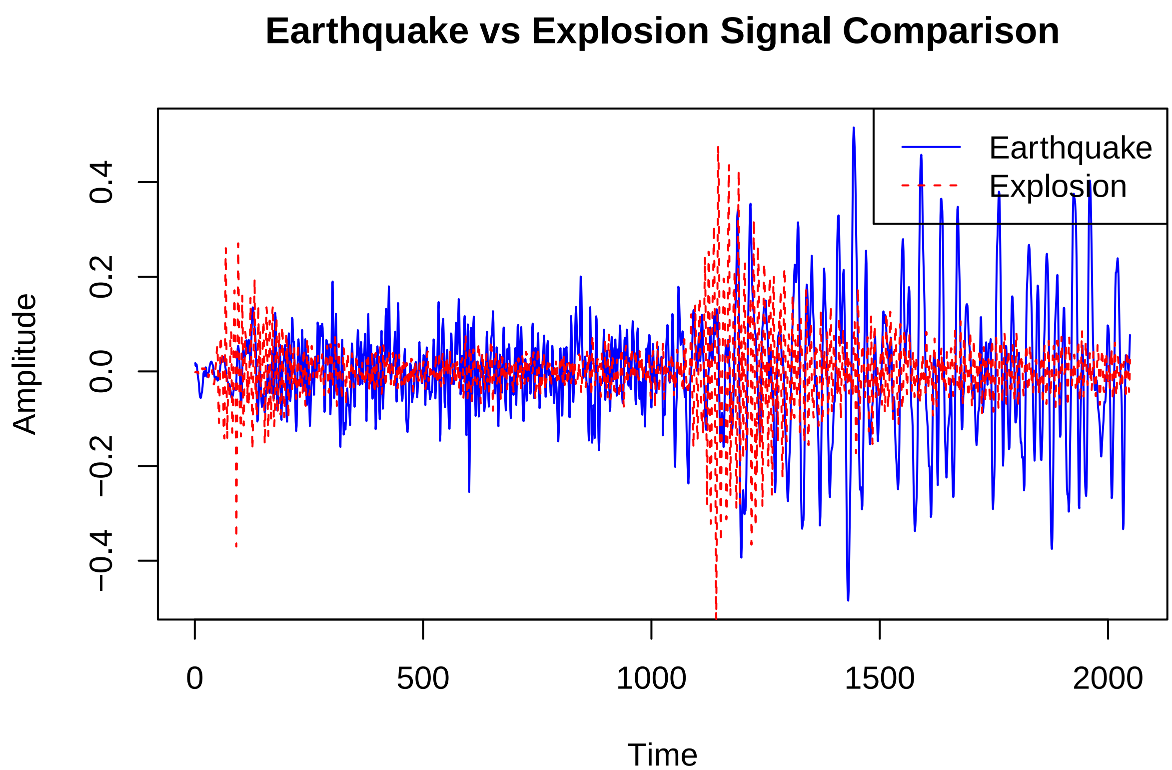
Problem 1

(a)

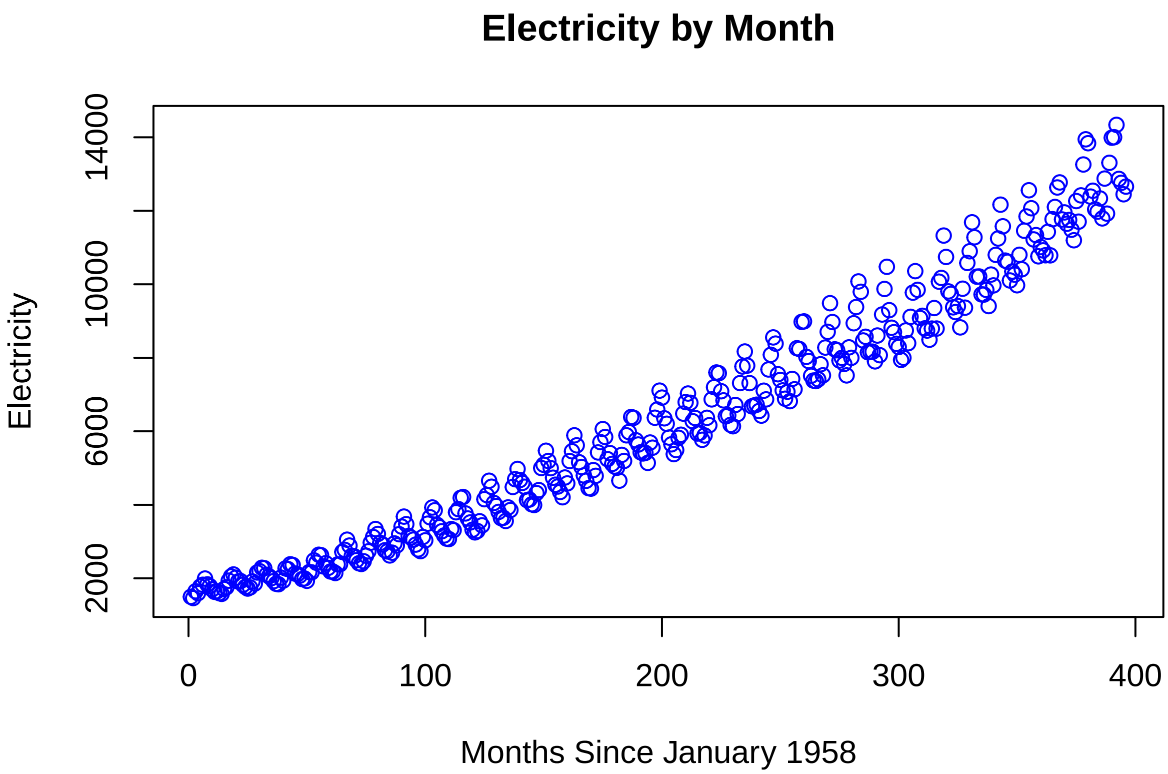
```r
plot(EQ5, type = "l", col = "blue", lty = 1, main = "Earthquake vs Explosion Signal Comparison", ylab =
lines(EXP6, col = "red", lty = 2)
legend("topright", legend = c("Earthquake", "Explosion"), col = c("blue", "red"), lty = c(1, 2))
```



When the two time series are placed on the same graph, it becomes clear that the explosion occurs when the earthquake's magnitude increases.

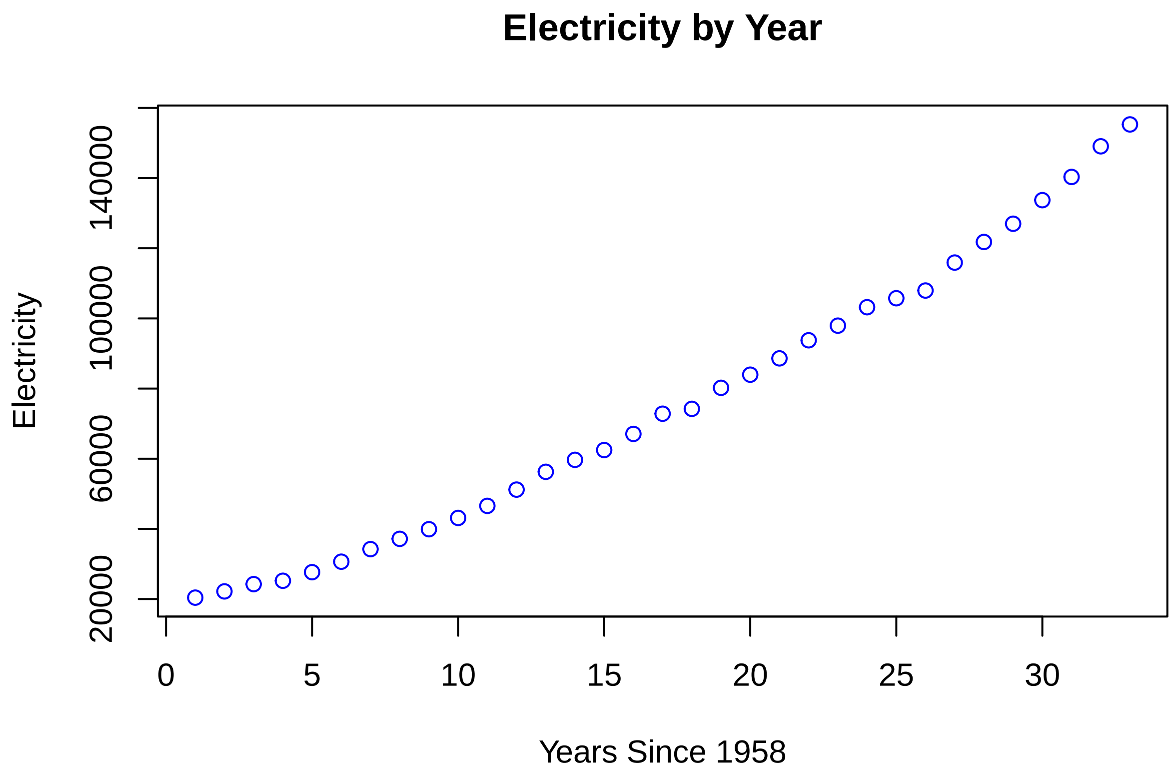
(b)

```
df = read.delim('cbe.txt')
plot(x=1:nrow(df),y=df$elec, col = "blue", lty = 1, main = "Electricity by Month", ylab = "Electricity"
```



i.

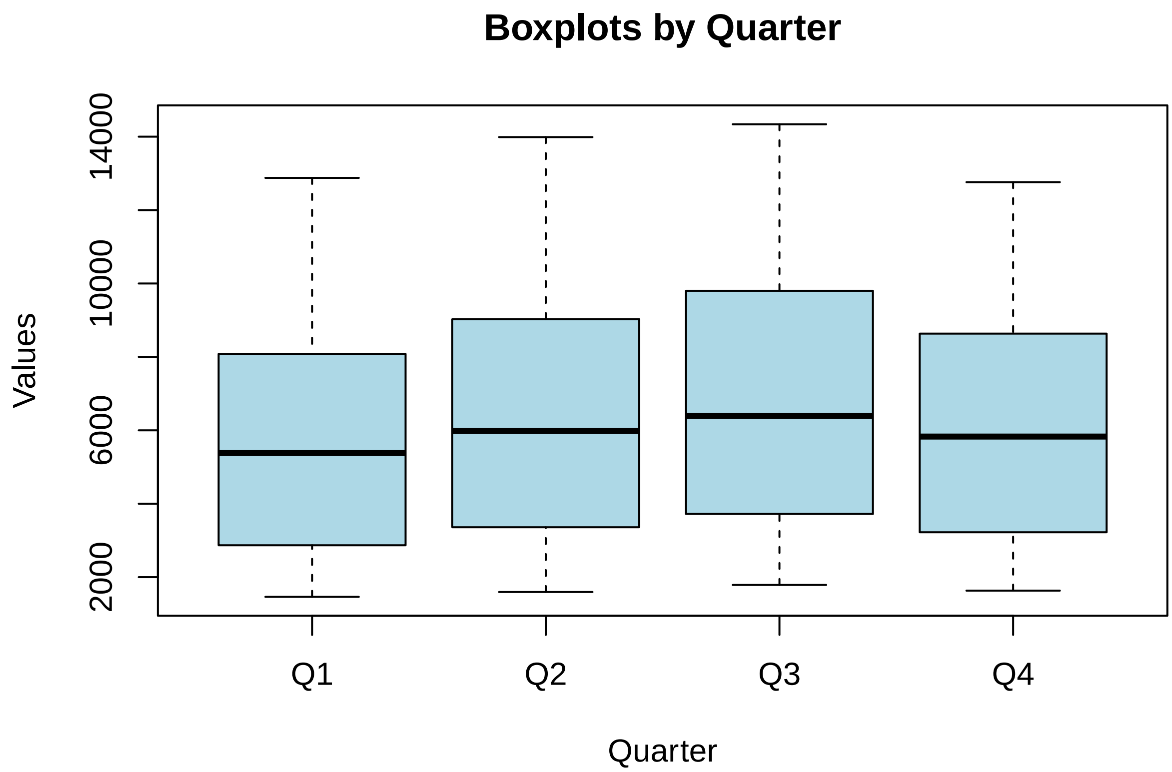
```
annual_data <- tapply(df$elec, (seq_along(df$elec) - 1) %/% 12, sum)
plot(x=1:nrow(annual_data),y=annual_data, col = "blue", lty = 1, main = "Electricity by Year", ylab = "E
```




```r
quarter_labels <- rep(c("Q1", "Q2", "Q3", "Q4"), each = 3, length.out = nrow(df))

df$Quarter <- quarter_labels

boxplot(elec ~ Quarter, data = df, col = "lightblue",
        main = "Boxplots by Quarter", xlab = "Quarter", ylab = "Values")
```



The boxplots reflect that electricity demand is much higher in Q2 and Q3 than in Q1 and Q4, possibly because the weather is colder in Australia at that time of year and houses require more heat.

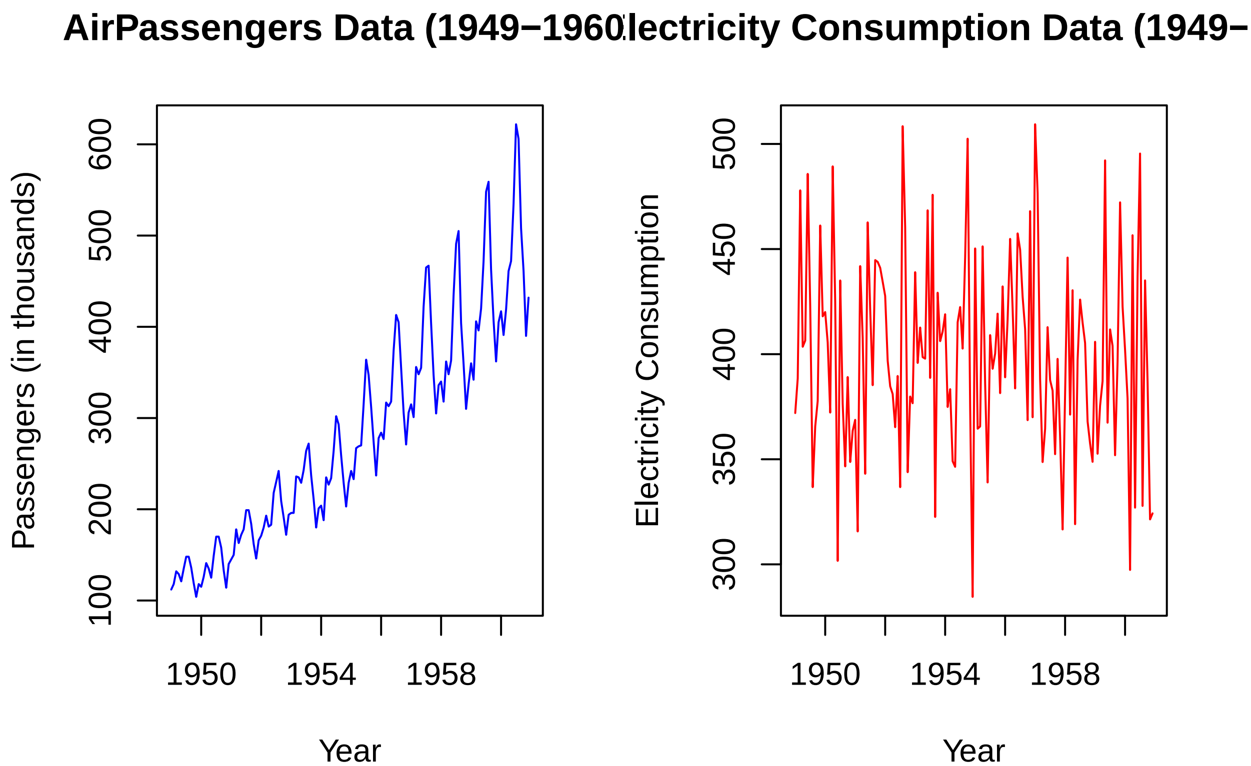
```
# Load the AirPassengers dataset
data("AirPassengers")

set.seed(123)  # For reproducibility
electricity_data <- ts(rnorm(length(AirPassengers), mean = 400, sd = 50), start = c(1949, 1), frequency

par(mfrow = c(1, 2))  # Arrange two plots, one above the other

plot(AirPassengers, type = "l", col = "blue",
     main = "AirPassengers Data (1949-1960)", ylab = "Passengers (in thousands)", xlab = "Year")

plot(electricity_data, type = "l", col = "red",
     main = "Electricity Consumption Data (1949-1960)", ylab = "Electricity Consumption", xlab = "Year"
```



ii.

The air passengers has a clear upward trend with seasonal oscillations while the electricity graph does not.

Problem 2

(a)

To find the joint pdf, you differentiate $F(x, y)$ with respect to x then with respect to y.

$$F(x,y) = xy\,[1 + \theta(1-x)(1-y)]$$

$$\frac{\partial}{\partial x}F(x,y) = y\,[1 + \theta(1-x)(1-y)] + xy \cdot \theta(-1)(1-y)$$

$$= y\,[1 + \theta(1-x)(1-y)] - \theta yx(1-y)$$

$$f(x,y) = \frac{\partial}{\partial y}\,[y\,(1 + \theta(1-x)(1-y)) - \theta yx(1-y)]$$

$$f(x,y) = 1 + \theta(1-x)(1-y)$$

(b)

To find the marginal density of x, we integrate $f(x, y)$ with respect to y.

$$f_X(x) = \int_0^1 \left[1 + \theta(1-x)(1-y)\right]\,dy$$

$$f_X(x) = \int_0^1 1\,dy + \theta(1-x)\int_0^1 (1-y)\,dy$$

$$\int_0^1 1\,dy = 1$$

$$\int_0^1 (1-y)\,dy = \left[y - \frac{y^2}{2}\right]_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$f_X(x) = 1 + \frac{\theta}{2}(1-x)$$

(c)

$$\mathbb{E}[X] = \mathbb{E}[Y] = \int_0^1 x \cdot f_X(x)\,dx = \int_0^1 x\left(1 + \frac{\theta}{2}(1-x)\right)dx$$

$$\mathbb{E}[X] = \int_0^1 x\,dx + \frac{\theta}{2}\int_0^1 x(1-x)\,dx$$

$$\int_0^1 x\,dx = \frac{1}{2}$$

$$\int_0^1 x(1-x)\,dx = \int_0^1 (x - x^2)\,dx = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$\mathbb{E}[X] = \frac{1}{2} + \frac{\theta}{2} \cdot \frac{1}{6} = \frac{1}{2} + \frac{\theta}{12}$$

$$\mathrm{Var}(X) = \mathrm{Var}(Y) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

$$\mathbb{E}[X^2] = \int_0^1 x^2 \cdot f_X(x)\,dx = \int_0^1 x^2 \left(1 + \frac{\theta}{2}(1-x)\right) dx$$

$$\mathbb{E}[X^2] = \int_0^1 x^2\,dx + \frac{\theta}{2}\int_0^1 x^2(1-x)\,dx$$

$$\int_0^1 x^2\,dx = \frac{1}{3}$$

$$\int_0^1 x^2(1-x)\,dx = \int_0^1 (x^2 - x^3)\,dx = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\mathbb{E}[X^2] = \frac{1}{3} + \frac{\theta}{2} \cdot \frac{1}{12} = \frac{1}{3} + \frac{\theta}{24}$$

$$\mathrm{Var}(X) = \left(\frac{1}{3} + \frac{\theta}{24}\right) - \left(\frac{1}{2} + \frac{\theta}{12}\right)^2 = \frac{1}{12} - \frac{\theta}{24} - \frac{\theta^2}{144}$$

(d)

$$\mathrm{Cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]^2$$

$$\mathbb{E}[XY] = \frac{1}{4} + \frac{\theta}{36}, \quad \mathbb{E}[X]^2 = \frac{1}{4} + \frac{\theta}{12} + \frac{\theta^2}{144}$$

$$\mathrm{Cov}(X,Y) = \left(\frac{1}{4} + \frac{\theta}{36}\right) - \left(\frac{1}{4} + \frac{\theta}{12} + \frac{\theta^2}{144}\right)$$

$$\mathrm{Cov}(X,Y) = \frac{-\theta}{18} - \frac{\theta^2}{144}$$

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X) \cdot \mathrm{Var}(Y)}}$$

$$\mathrm{Corr}(X,Y) = \frac{\frac{-\theta}{18} - \frac{\theta^2}{144}}{\frac{1}{12} - \frac{\theta}{24} - \frac{\theta^2}{144}}$$

(e)

$$\mathrm{Cov}(\bar{X}_n, \overline{X^2}_n) = \mathrm{Cov}\left(\frac{1}{n}\sum_{i=1}^n X_i, \frac{1}{n}\sum_{i=1}^n X_i^2\right)$$

$$\mathrm{Cov}(\bar{X}_n, \overline{X^2}_n) = \frac{1}{n}\mathrm{Cov}(X, X^2)$$

$$\mathrm{Cov}(X, X^2) = \mathbb{E}[XX^2] - \mathbb{E}[X]\mathbb{E}[X^2]$$

$$\mathbb{E}[X^3] = \int_0^1 x^3\,dx + \frac{\theta}{2}\int_0^1 x^3(1-x)\,dx$$

$$\int_0^1 x^3 \, dx = \frac{1}{4}$$
$$\int_0^1 x^3(1-x) \, dx = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$$

$$\mathbb{E}[X^3] = \frac{1}{4} + \frac{\theta}{40}$$

$$\text{Cov}(X, X^2) = \left(\frac{1}{4} + \frac{\theta}{40}\right) - \left(\frac{1}{2} + \frac{\theta}{12}\right)\left(\frac{1}{3} + \frac{\theta}{24}\right)$$
$$\left(\frac{1}{2} + \frac{\theta}{12}\right)\left(\frac{1}{3} + \frac{\theta}{24}\right) = \frac{1}{6} + \frac{\theta}{48} + \frac{\theta}{36} + \frac{\theta^2}{288}$$

$$= \frac{1}{6} + \frac{\theta}{24} + \frac{\theta^2}{288}$$

$$\text{Cov}(X, X^2) = \frac{1}{4} + \frac{\theta}{40} - \left(\frac{1}{6} + \frac{\theta}{24} + \frac{\theta^2}{288}\right)$$

$$\text{Cov}(X, X^2) = \frac{1}{12} + \left(\frac{\theta}{40} - \frac{\theta}{24}\right) - \frac{\theta^2}{288}$$
$$\frac{\theta}{40} = \frac{3\theta}{120}, \quad \frac{\theta}{24} = \frac{5\theta}{120}$$
$$\frac{\theta}{40} - \frac{\theta}{24} = \frac{-2\theta}{120} = \frac{-\theta}{60}$$
$$\text{Cov}(X, X^2) = \frac{1}{12} - \frac{\theta}{60} - \frac{\theta^2}{288}$$
$$\text{Cov}(\bar{X}_n, \overline{X^2}_n) = \frac{1}{n}\left(\frac{1}{12} - \frac{\theta}{60} - \frac{\theta^2}{288}\right)$$

(f)

The sample mean of X_n will approach the population mean which we found earlier is

$$\mathbb{E}[X] = \frac{1}{2} + \frac{\theta}{12}$$

The sample mean of X_n^2 will approach the population mean which we found earlier is

$$\mathbb{E}[X^2] = \frac{1}{3} + \frac{\theta}{24}$$

(g)

The asymptotic distribution of X_n is a normal distribution with a mean of

$$\mathbb{E}[X] = \frac{1}{2} + \frac{\theta}{12}$$

and a variance of

$$\text{Var}(X) = \frac{1}{12} - \frac{\theta}{24} - \frac{\theta^2}{144}$$

by the Central Limit Theorem.

Problem 3

(a)

i. 1 + 2i
ii. -2 - 6i
iii. 11 - 2i
iv. = (1-2i)(3+4i) / (3-4i)(3+4i) = (11-2i)/25
v. = sqrt(3) * sqrt(12) * -1 = -6

(b)

There is a solution because $sqrt(x^2 + y^2) < 1$.

$$sqrt(x^2 + y^2) = sqrt(1/4^2 + 1/4^2) = sqrt(1/8) < 1$$

$Sum = 1/(1-z) = 1/(1-(1/4-i/4)) = 1/(3/4-i/4) = (3/4+i/4)/(10/16) = 48/40+16i/40 = 6/5+2i/5$

(c)

$$\mid e^{i*\pi*t} \mid = sqrt(cos^2(\pi * t) + sin^2(\pi * t)) = sqrt(1) = 1$$

(d)

There are 10 complex roots by the fundamental theorem of algebra because the highest power of z is 10.

(e)

$$(-1 + / - sqrt(1 - 4 * 2 * 3))/(2 * 2) = -1/4 + / - sqrt(-23)/4 = -1/4 + / - i * sqrt(23)/4$$