Problem Set 12

Due Date .... December 10, 2022
Name .... **Alex Ojemann**
Student ID **109722375**
Collaborators .... **None**

Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. (See this short intro to LaTeX plus other resources on Canvas.)
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

(I agree to the above, Alex Ojemann). □

3 Standard 24: Complexity Classification

For the following problem, figure out whether it is in P or NP-complete, and prove it. For P this means giving a poly-time algorithm. For NP-complete, this means three things: (1) showing it's in NP, (2) giving a reduction from an NP-complete problem we cover in class, and (3) showing that reduction runs in polynomial time. If you give a reduction, **prove** that your reduction is correct.

Problem 1. Professional badass FBI agent Sue Hoover must escape from a building, before it collapses in a cinematically short amount of time. Through her earpiece, she is told that she must disable n different power sources, each in its own room in the building. She is also told that disabling a power source in a room releases a noxious gas into that room, and she must leave the room immediately and not return.

Sue is staring at the building blueprints that show all m rooms in the building, which n of them have power sources, and which rooms have connections to each other (via a network of corridors, where a single cooridor connects exactly two rooms). Sue notes the room she is currently in, and the exit room, neither of which have power sources. She needs to decide (quickly!) whether it is possible to disable all the power sources along her way to the exit, all without exposure to the noxious gas.

Proof. This problem, escape, is NP-complete.
Escape is NP because a candidate solution can be verified in O(m) time by checking each room and seeing if there is a power source and if there is whether it has been disabled.
Escape can be reduced from Hamiltonian Path, which is known to be NP-complete. Let each vertex in Hamiltonian Path correspond to one of the M rooms in escape. Let the start vertex correspond to the room Sue is in and let the end vertex correspond to the exit room. Let each edge in the Hamiltonian Path correspond to a path between two of the rooms. If there is a solution to this Hamiltonian Path, meaning there's a path through the building that visits all m vertices exactly one time, then there must be a solution to the corresponding Escape problem because Sue must have visited all rooms to check if there is a power source and she must have only checked each room once so she can't have been exposed to the noxious gas.
The reduction runs in polynomial time because mapping the vertices to rooms takes O(n) time, mapping the edges to corridors takes O(x) time where x is the number of edges/corridors, and mapping the start and end vertices to the room Sue is in and the exit room takes O(1) time. □

4 Standards 2/3- Proofs

Problem 2. Let REG denote the class of regular languages. Let $\mathsf{DSPACE}(O(1))$ denote the class of languages L, such that L is accepted by a 3-tape Turing Machine subject to the following constraints:

- The first tape is the input tape, and is read-only.
- The second tape is the work tape, and only $O(1)$ tape cells can be used.
- The third tape is the output tape. It is write-only (in particular, you *cannot* read from the output tape).

The goal of this problem is to show that $\mathsf{REG} = \mathsf{DSPACE}(O(1))$.

(a) Suppose first that $L \in \mathsf{REG}$. Show that $L \in \mathsf{DSPACE}(O(1))$. [**Hint:** As L is regular, L is accepted by some DFA D. Can D be simulated by a $\mathsf{DSPACE}(O(1))$-TM.]

Proof. $\square$

(b) Suppose that $L \in \mathsf{DPSACE}(O(1))$. Show that $L \in \mathsf{REG}$. [**Hint:** Let M be the $\mathsf{DSPACE}(O(1))$-TM accepting L. Can a DFA simulate M?]

Proof. $\square$