Homework 4

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2.3: 3. a. Yes; $\begin{bmatrix}1\\-2\\-3\end{bmatrix} = \begin{bmatrix}1\\1\\0\end{bmatrix} - 3\begin{bmatrix}0\\1\\1\end{bmatrix}$

c. $\begin{bmatrix}x_1 & 0 & 2x_3\\ 2x_1 & -x_2 & 0\\ 0 & 3x_2 & x_3\\ x_1 & 0 & -x_2\end{bmatrix}$ +2R4 $= \begin{bmatrix}\cancel{3}\\0\\-1\\-2\end{bmatrix}$

$\left[\begin{array}{ccc|c}3x_1 & 0 & 0 & -1\\ 2x_1 & -x_2 & 0 & 0\\ 0 & 3x_2 & x_3 & -1\\ x_1 & 0 & -x_3 & 2\end{array}\right]$ $-2/3R1$

$\left[\begin{array}{ccc|c}3x_1 & 0 & 0 & -1\\ 0 & -x_2 & 0 & 2/3\\ 0 & 3x_2 & x_3 & -1\\ x_1 & 0 & -x_3 & 2\end{array}\right]$ $+3R2$

$\left[\begin{array}{ccc|c}3x_1 & 0 & 0 & -1\\ 0 & -x_2 & 0 & 2/3\\ 0 & 0 & x_3 & 1\\ x_1 & 0 & -x_2 & 2\end{array}\right]$

$x_1 = -1/3$

$x_2 = -2/3$

$x_3 = 1$

$x_1 - x_3 \neq 2$

$\boxed{\text{Does not span}}$

4. a. No b. No c. No d. Yes e. Yes f. No

16. True because when $c_1 \dots c_n = 0$, it's the zero vector

21. a. Independent e. Independent
g. Dependent, $\begin{bmatrix}-6\\3\\0\\9\end{bmatrix} = -3/2\begin{bmatrix}4\\2\\0\\-6\end{bmatrix}$

31. a. The subset must only have vectors from the original set, and you don't have a linear combo in the original set so you can't have one in the subset.

2.3. 31. b. No; Set = $\left\{ \begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}2\\3\end{pmatrix}, \begin{pmatrix}3\\5\end{pmatrix} \right\}$ Subset = $\left\{ \begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}3\\5\end{pmatrix} \right\}$

2.4. 2. b. No; doesn't span
c. Yes

3. a. Yes because every vector in R_3 can be formed with linear combinations of them.
b. No because $\begin{bmatrix}1\\0\\2\end{bmatrix} + \begin{bmatrix}3\\-1\\1\end{bmatrix} = \begin{bmatrix}4\\-1\\3\end{bmatrix}$
c. No because they're not linearly independent
d. 3 because there are four vectors but one is a linear combination of the others.

14. a. Basis: $\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&1\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\1&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}$
4 vectors in the basis so M_{2x2} is 4 dimensional
b. The basis of M_{mxn} would include the vectors with all 0s and a 1 at (m,n) for each of the m rows and n columns so there are mn vectors in the basis so the dimension is mn.

17. a. Basis: $\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&1\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}$ Dimension = 3
b. Basis: nxn matricies with all 0s and one 1 in each spot on or above the main diagonal.
Dimension: $n(n+1)/2$

2.5. 1.6. Image: $\begin{bmatrix} x \\ -2x \end{bmatrix}$ kernel: RREF $= \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$x = y - 2z$

$$\begin{bmatrix} y-2z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

c. RREF: $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 1 \\ 4 & 0 & 5 \end{bmatrix} +2R1$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 8 & 7 \\ 4 & 0 & 5 \end{bmatrix} -4R1$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 8 & 7 \\ 0 & -8 & -7 \end{bmatrix} +R2$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 8 & 7 \\ 0 & 0 & 0 \end{bmatrix} /8$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 7/8 \\ 0 & 0 & 0 \end{bmatrix} -2R2$

$\begin{bmatrix} 1 & 0 & 5/4 \\ 0 & 1 & 7/8 \\ 0 & 0 & 0 \end{bmatrix}$

Image:

$$\begin{bmatrix} 1 & 0 & 5/4 \\ 0 & 1 & 7/8 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s \\ t \\ u \end{bmatrix} = \begin{bmatrix} s + \frac{5u}{4} \\ t + \frac{7u}{8} \\ 0 \end{bmatrix}$$

kernel: $x = -5/4\, z$

$y = -7/8\, z$

$$z \begin{bmatrix} -5/4 \\ -7/8 \\ 1 \end{bmatrix}$$

2.5 2.b. RREF: $\begin{bmatrix} 1 & 0 & -1/4 \\ 0 & 1 & -3/8 \end{bmatrix}$ Kernel = $\begin{bmatrix} 1/4 \\ 3/8 \\ 1 \end{bmatrix}$ It's a line

6. True

7.f. RREF: $\begin{bmatrix} 1 & 0 & 13/2 & 3/2 \\ 0 & 1 & 3/2 & 1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Kernel: $x^* = y \begin{bmatrix} -13/2 \\ -3/2 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -3/2 \\ -1/2 \\ 0 \\ 1 \end{bmatrix}$

9.a. Since $P^2 = P$, $Pw = w$ if $w \in \text{img}(P)$

13. $2x_1 + x_2 = 2\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} + \begin{bmatrix} -11 \\ 5 \\ -1 \end{bmatrix} = \begin{bmatrix} -9 \\ 11 \\ 11 \end{bmatrix}$

14. True

21.a. Image: $\begin{bmatrix} x \\ 2x \end{bmatrix}$ Kernel: $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

Coimage: $\begin{bmatrix} x \\ -3x \end{bmatrix}$ Cokernel: $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

30.a. If A is symmetric, the coimage = the image and the cokernel = the kernel because the rows and columns are the same and the coimage and cokernel are found by taking the image and kernel of a matrix with the rows and columns flipped.