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Exam 2

1. a. True
   b. False
   c. True
   d. False
   e. False
   f. True
   g. False
   h. False
   i. False
   j. True

2. $\begin{bmatrix}1\\0\\6\end{bmatrix} = -\begin{bmatrix}-1\\2\\0\end{bmatrix} + 2\begin{bmatrix}0\\1\\3\end{bmatrix}$

thus $\begin{bmatrix}x\\y\\z\end{bmatrix} = -\begin{bmatrix}1\\0\\1\end{bmatrix} + 2\begin{bmatrix}1\\-1\\2\end{bmatrix} = \boxed{\begin{bmatrix}1\\-3\\3\end{bmatrix}}$

3. Biliniearity: $\langle cu+dv, w\rangle = 2(cu_1+dv_1)(w_1-2w_2) + (cu_2+dv_2)(-4w_1+9w_2+w_3) + (cu_3+dw_3)(w_2+4w_3)$
$= c(2u_1(w_1-2w_2) + u_2(-4w_1+9w_2+w_3) + u_3(w_2+4w_3)) + d(2v_1(w_1-2w_2) + v_2(-4w_1+9w_2+w_3) + v_3(w_2+4w_3))$
$= c\langle u,w\rangle + d\langle v,w\rangle$ ✓

Symmetry: $\langle u,v\rangle = 2u_1v_1 - 4u_1v_2 - 4u_2v_1 + 9u_2v_2 + u_2v_3 + u_3v_2 + 4u_3v_3 = 2v_1u_1 - 4v_2u_1 - 4v_1u_2 + 9v_2u_2 + v_3u_2 + v_2u_3 + 4v_3u_3 = \langle v,u\rangle$ ✓

Positivity: If $u=v=0$, $\langle u,v\rangle = 0$
Otherwise $\langle u,v\rangle = (u_1+v_1)^2 - u_1^2 - v_1^2 + (-\sqrt{2}u_1 + \sqrt{2}v_2)^2 - 2u_1^2 - 2v_2^2 + (-\sqrt{2}u_2 + \sqrt{2}v_1)^2 - 2u_2^2 - 2v_1^2 + (\frac{3}{\sqrt{2}}u_2 + \frac{3}{\sqrt{2}}v_2)^2 - \frac{9}{2}u_2^2 - \frac{9}{2}v_2^2 + (\frac{u_3}{2} + \frac{v_2}{2})^2 - \frac{u_3^2}{4} - \frac{v_2^2}{4} + (\frac{u_2}{2} + \frac{v_3}{2})^2 - \frac{u_2^2}{4} - \frac{v_3^2}{4} + (\sqrt{2}u_3 + \sqrt{2}v_3)^2 - 2u_3^2 - 2v_3^2$

3(continued). Each of the positive parts of the previous expression is greater than the negative part that follows

so $\langle u,v \rangle > 0$ when $u \neq 0$ or $v \neq 0$ ✓

4. $x_1 = \langle 1,1,0 \rangle$ $x_2 = \langle -2,-2,1 \rangle$ $x_3 = \langle -1,1,1 \rangle$

$y_1 = x_1 \quad \langle 1,1,0 \rangle$

$y_2 = x_2 - \left(\frac{x_2 \cdot y_1}{\|y_1\|^2}\right) y_1 = \langle -2,-2,1 \rangle - \frac{\langle -2,-2,1 \rangle \cdot \langle 1,1,0 \rangle}{(\sqrt{2})^2} \langle 1,1,0 \rangle$

$= \langle -2-(-2)(1), -2-(-2)(1), 1-0 \rangle = \langle 0,0,1 \rangle$

$y_3 = x_3 - \left(\frac{x_3 \cdot y_1}{\|y_1\|^2}\right) y_1 - \left(\frac{x_3 \cdot y_2}{\|y_2\|^2}\right) y_2$

$= \langle -1,1,1 \rangle - \frac{\langle -1,1,1 \rangle \cdot \langle 1,1,0 \rangle}{(\sqrt{2})^2} \langle 1,1,0 \rangle - \frac{\langle -1,1,1 \rangle \cdot \langle 0,0,1 \rangle}{(1)^2} \langle 0,0,1 \rangle$

$= \langle -1-0-0, 1-0-0, 1-0-1 \rangle = \langle -1,1,0 \rangle$

$y_1 / \|y_1\| = \langle 1/\sqrt{2}, 1/\sqrt{2}, 0 \rangle$

$y_2 / \|y_2\| = \langle 0,0,1 \rangle$

$y_3 / \|y_3\| = \langle -1/\sqrt{2}, 1/\sqrt{2}, 0 \rangle$

$$Q = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix}$$

$$R = Q^T A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 & -1 \\ 1 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & -2\sqrt{2} & 0 \\ 0 & 1 & 1 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

$$A = QR = \boxed{\begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & -2\sqrt{2} & 0 \\ 0 & 1 & 1 \\ 0 & 0 & \sqrt{2} \end{bmatrix}}$$

5.

$$q(x,y,z) = [x, y\ z]\begin{bmatrix} 2 & -4 & 0 \\ -4 & 9 & 1 \\ 0 & 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} - 2[x, y\ z]\begin{bmatrix} -2 \\ 6 \\ 5 \end{bmatrix} - \sqrt{581}$$

$$K = \begin{bmatrix} 2 & -4 & 0 \\ -4 & 9 & 1 \\ 0 & 1 & 4 \end{bmatrix} \quad f = \begin{bmatrix} -2 \\ 6 \\ 5 \end{bmatrix}$$

Critical pt at $Kx = f$

$$\left[\begin{array}{ccc|c} 2 & -4 & 0 & -2 \\ -4 & 9 & 1 & 6 \\ 0 & 1 & 4 & 5 \end{array}\right] +2R1$$

$$\left[\begin{array}{ccc|c} 2 & -4 & 0 & -2 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 4 & 5 \end{array}\right] -R2$$

$$\left[\begin{array}{ccc|c} 2 & -4 & 0 & -2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 3 & 3 \end{array}\right]$$

Using backsubstitution: $3z = 3$ $z = 1$ min at $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$y + z = 2$ $y + 1 = 2$ $y = 1$

$2x - 4y = -2$ $2x - 4 = -2$ $x = 1$

$$q(1,1,1) = 2(1)^2 - 8(1)(1) + 9(1)^2 + 2(1)(1) + 4(1)^2 + 4(1) - 12(1) - 10(1) - \sqrt{581}$$
$$= -9 - \sqrt{581}$$

This is a minimizer because for all other values of x, y, and z, q is larger than $-9-\sqrt{581}$ because the coefficients of x^2, y^2 and z^2 are greater than 0