Homework 1

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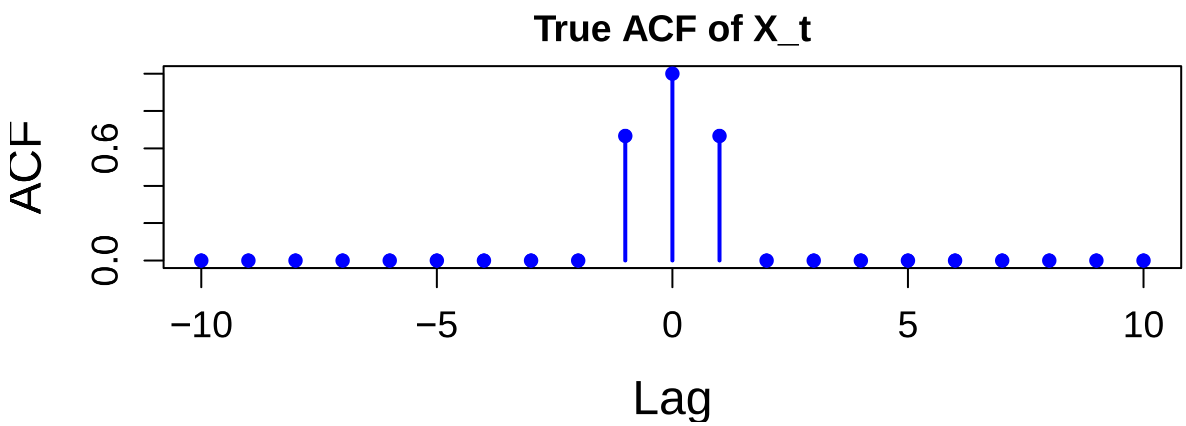
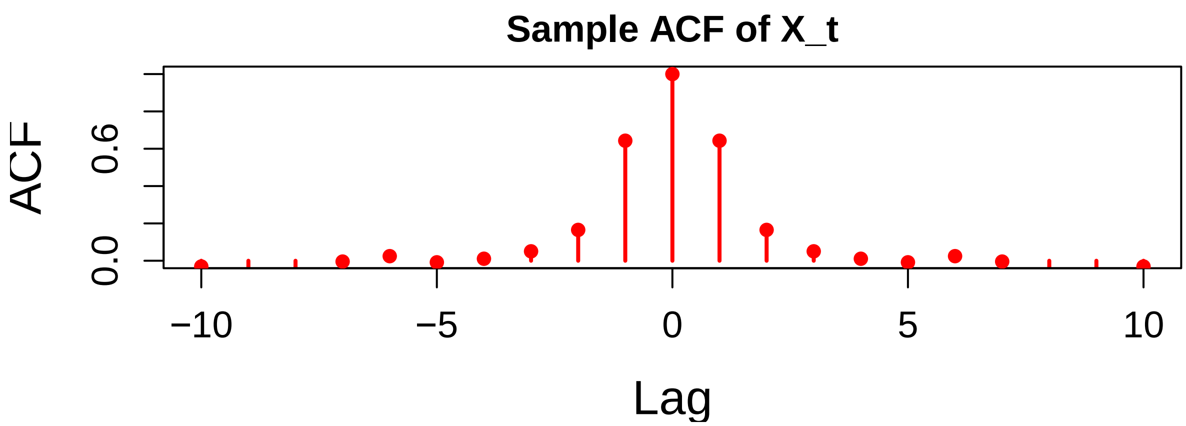
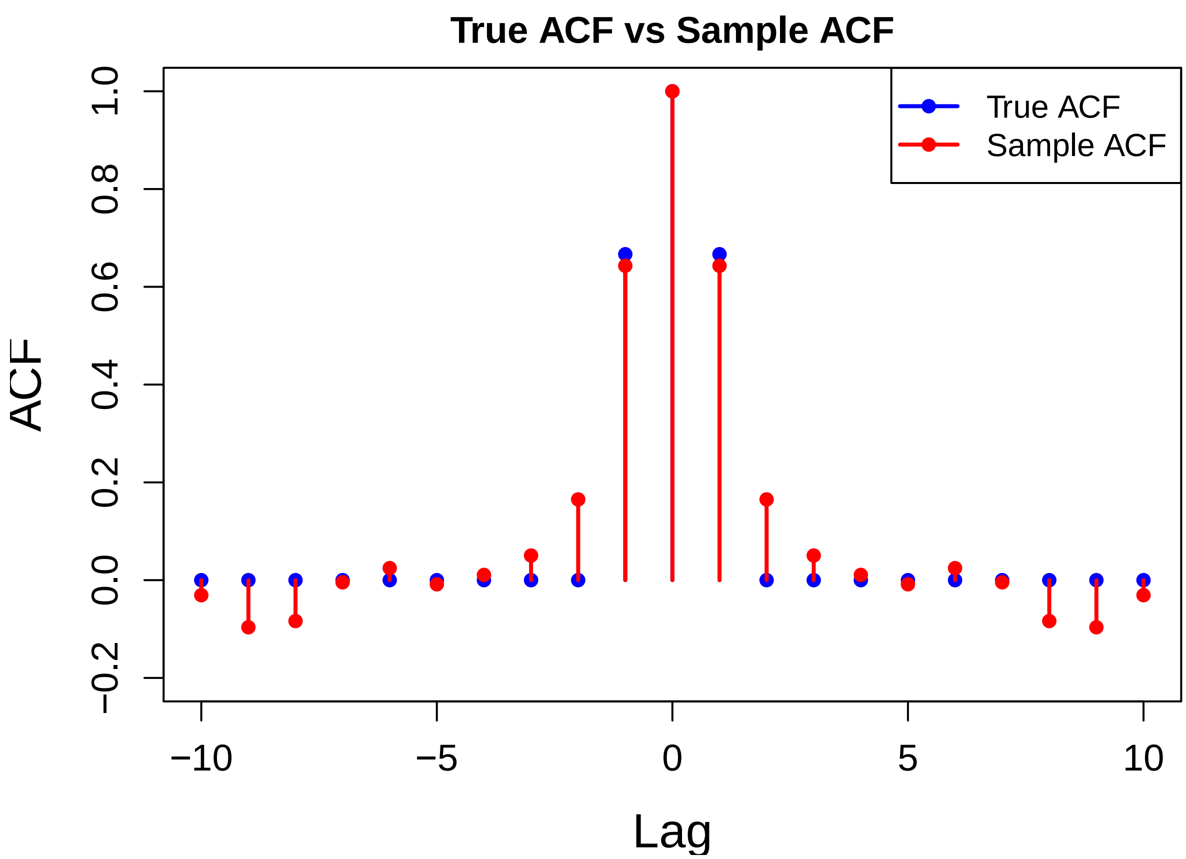
Problem 1

$$\gamma(0) = 6 * \sigma^2$$
$$\gamma(1) = \gamma(-1) = 4 * \sigma^2$$
$$\rho(0) = 6 * \sigma^2/6 * \sigma^2 = 1$$

$$\rho(1) = \rho(-1) = 4 * \sigma^2/6 * \sigma^2 = 2/3$$







Problem 2

Part a

```
##
## Call:
## lm(formula = log_jj ~ 0 + time + Q1 + Q2 + Q3 + Q4)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.29318 -0.09062 -0.01180  0.08460  0.27644
##
## Coefficients:
##        Estimate Std. Error t value Pr(>|t|)
## time  1.672e-01  2.259e-03   74.00   <2e-16 ***
## Q1   -3.283e+02  4.451e+00  -73.76   <2e-16 ***
## Q2   -3.282e+02  4.451e+00  -73.75   <2e-16 ***
## Q3   -3.282e+02  4.452e+00  -73.72   <2e-16 ***
## Q4   -3.284e+02  4.452e+00  -73.77   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1254 on 79 degrees of freedom
## Multiple R-squared:  0.9935, Adjusted R-squared:  0.9931
## F-statistic:  2407 on 5 and 79 DF,  p-value: < 2.2e-16
```

Part b

The average annual increase in the logged earnings per share if the model is correct is $0.1672 * 4 = 0.6688$ as shown in the summary of the regression model in part a multiplied by 4 to reflect the average yearly increase instead of the average quarterly increase.

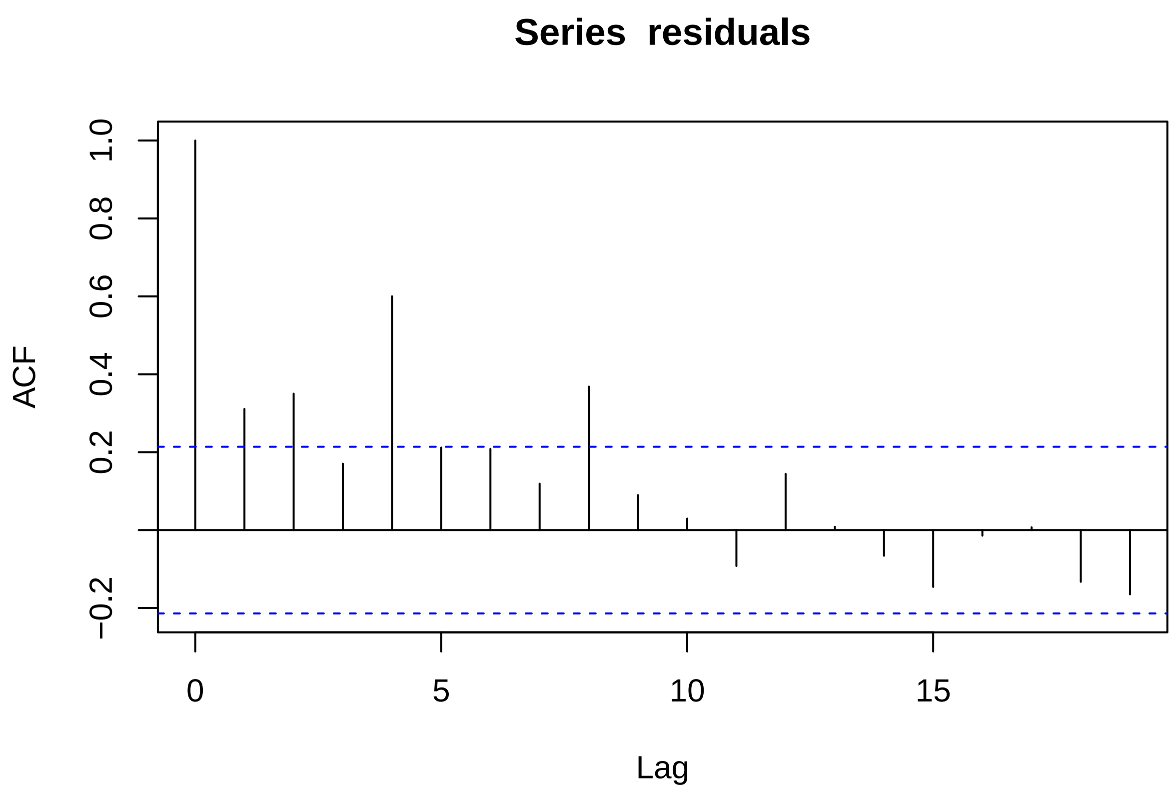
Part c

The logged earnings per share decreases from the third to the fourth quarter if the model is correct because the estimate for the Q4 indicator variable is lower than that of the Q3 indicator variable.

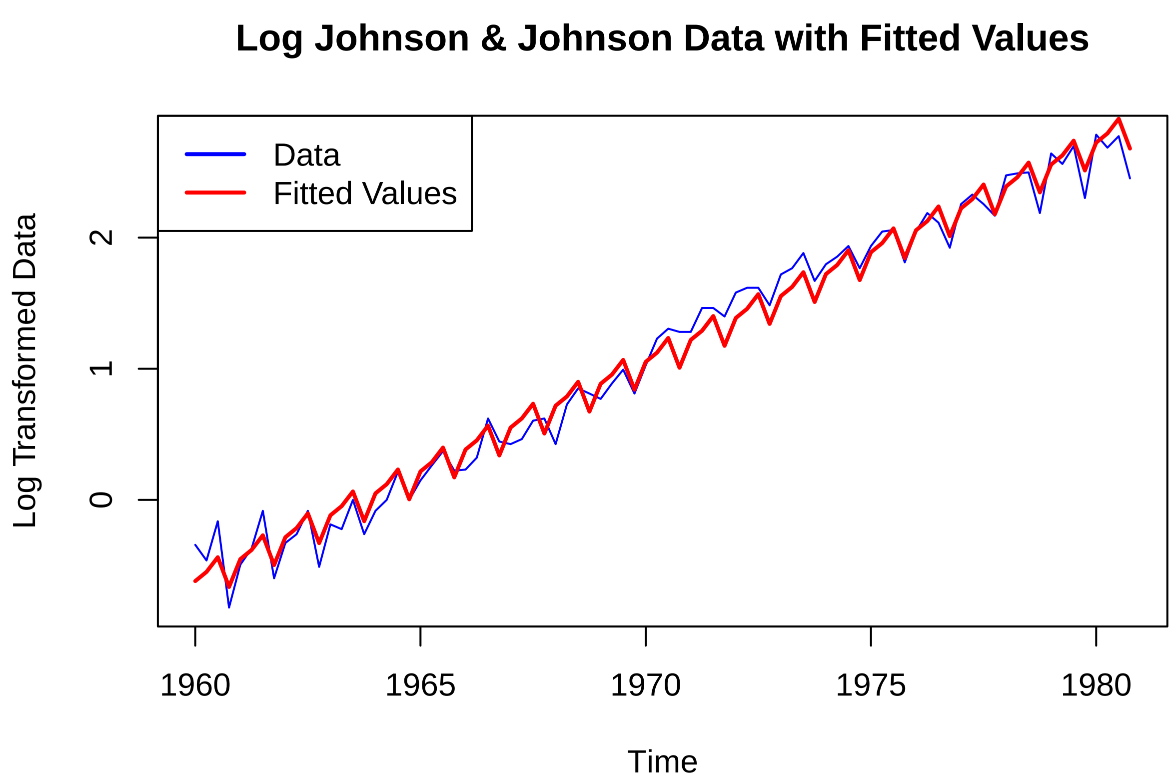
Part d

If you attempt to include an intercept term in the model from part a, you can no longer use all of Q1, Q2, Q3, and Q4 as indicator variables. One of these will be redundant when fitting the model because one of them must be the baseline quarter from which the other quarter variables reflect the average relative difference from the baseline quarter in logged earnings per share.

Part e



Series residuals
ACF
1.0
0.8
0.6
0.4
0.2
−0.2
0
5
10
15
Lag



Log Johnson & Johnson Data with Fitted Values
Data
Fitted Values
Log Transformed Data
2
1
0
1960
1965
1970
1975
1980
Time

Problem 3

Part a

Yes, this is stationary because the mean is constant and the autocovariance only depends on the lag.

$\mu = 0$

$\gamma(h) = 2 * \sigma^2$ when h = 0, $\gamma(h) = -\sigma^2$ when h = 3, and $\gamma(h) = 0$ otherwise.

Part b

This is not stationary because the autocovariance depends on the time rather than the lag.

Part c

This is not stationary because the mean is not constant.

Part d

Yes, this is stationary because the mean is constant and the autocovariance only depends on the lag.

$\mu = E[W_t^2] = Var(W_t) + E[W_t]^2 = Var(W_t) + 0 = \sigma^2$

$\gamma(h) = E[W_t^4] - E[W_t^2]^2 = E[W_t^4] - \sigma^4$ when h = 0 and 0 otherwise.

Part e

Yes, this is stationary because the mean is constant and the autocovariance only depends on the lag.

$\mu = E[W_t W_{t-2}] = E[W_t]E[W_{t-2}] = 0$

$\gamma(h) = E[W_t^2 W_{t-2}^2] - E[W_t W_{t-2}]^2 = E[W_t^2 W_{t-2}^2] - 0 = E[W_t^2 W_{t-2}^2]$ when h = 0 and $E[W_t W_{t-2} W_{t+2} W_t] = E[W_t]^2 E[W_{t-2} W_{t+2}] = \sigma^2 * 0 = 0$ when h = 2 and 0 otherwise.

Problem 4

X is white noise because:

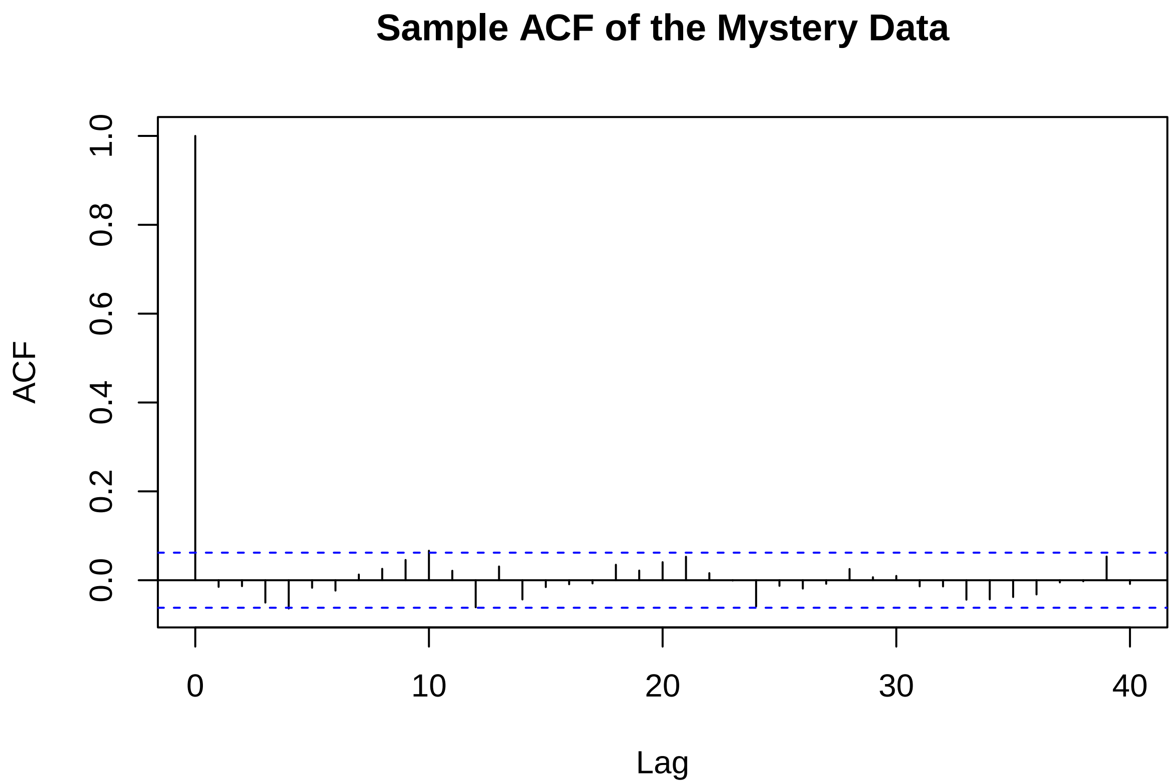
$\mu = E[X] = 1/2 * 0 + 1/2 * E[(1 - W_{t-1})Z_t] = 1/2 * 0 + 1/4 * 0 + 1/4 * Z_t = 0$

$Var(X_t) = E[X_t^2] - E[X_t]^2 = E[X_t^2] = 1/4 * E[Z_t^2] = 1/4$

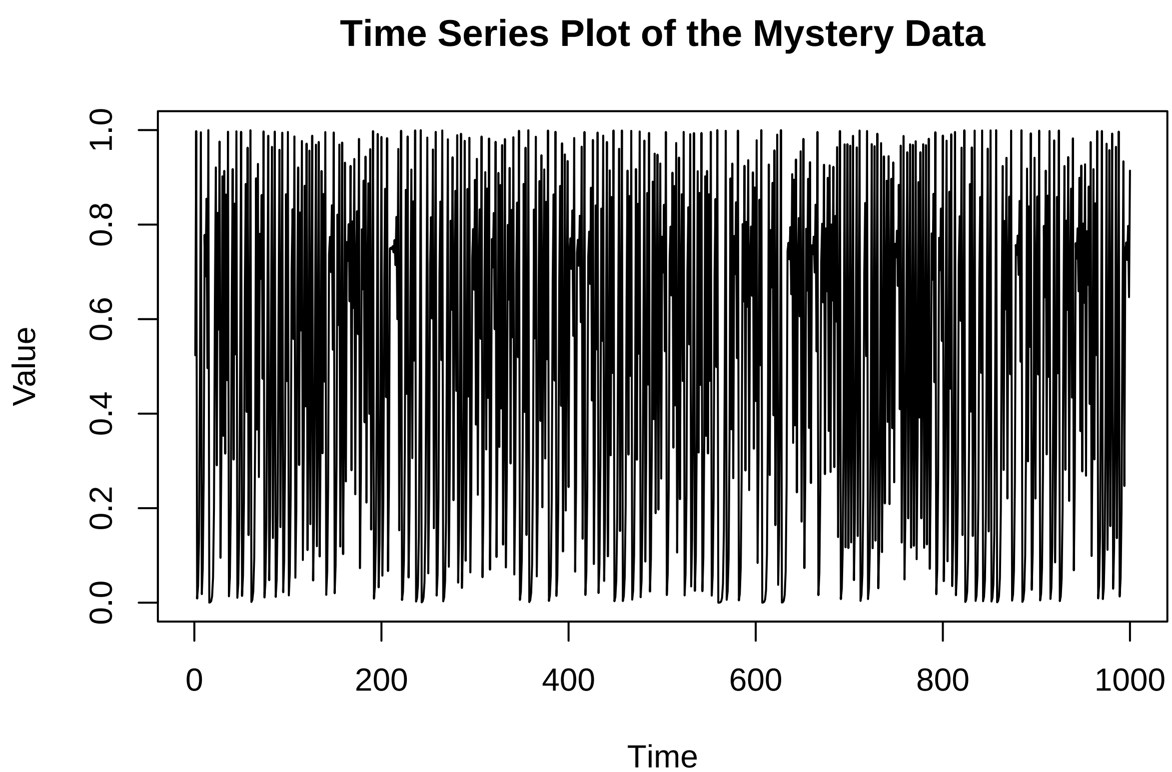
$\gamma(h) = 0$ for all $h \neq 0$ because $\gamma(1) = E[W_t W_{t-1}(1 - W_t)(1 - W_{t-1})Z_t Z_{t+1}] = E[W_t W_{t-1}(1 - W_t)(1 - W_{t-1})]E[Z_t]E[Z_{t+1}] = 0$

X is not iid because it depends in values from the past $(W_{t-1}\})$

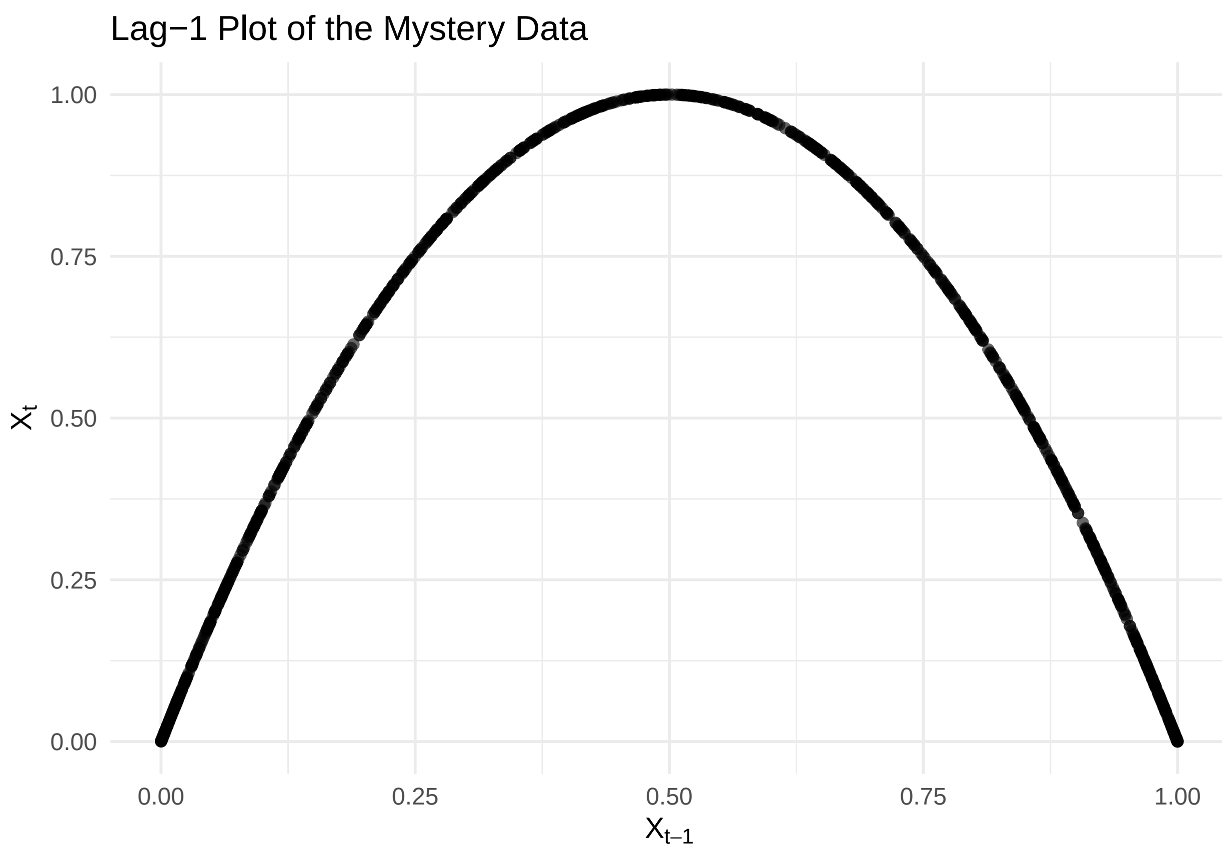
Problem 5



Sample ACF of the Mystery Data
ACF
Lag
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40



Time Series Plot of the Mystery Data
Value
Time
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000



Part a

Looking at the ACF and the series itself, it looks like it could be modeled as white noise.

Part b

Seeing the lag 1 plot, it is clear that there is a relationship between X_t and X_{t-1}, specifically a negative quadratic, so that rules out the possibility that it's white noise.