Homework 11

Alex Ojemann
Dr. James Cuny

5.3: 1. $y_1 = x_1 = \langle 1,2,-1 \rangle$

$y_2 = \langle 0,-1,3 \rangle - \frac{\langle 0,-1,3 \rangle \cdot \langle 1,2,-1 \rangle}{(\sqrt{6})^2} \langle 1,2,-1 \rangle$

$= \langle 5/6, 2/3, 13/6 \rangle$

$W = \langle 1,1,1 \rangle \times \langle 1,2,-1 \rangle \times \langle 5/6, 2/3, 13/6 \rangle$

$= \langle 6/7, 114/105, 216/210 \rangle$

4. a. $\text{Proj}_u v = \frac{\langle 3,2,1 \rangle \cdot \langle 1,1,1 \rangle}{(\sqrt{4})^2} \langle 1,1,1 \rangle = \langle 7/4, 7/4, 7/4, 7/4 \rangle$

Distance from b: $\sqrt{(3-7/4)^2 + (1-7/4)^2 + (2-7/4)^2 + (1-7/4)^2} = \sqrt{44/16} = \sqrt{11}/2$

b. i. $W = \langle 3,2,1 \rangle \times \langle 1,0,0 \rangle \langle 1,0,0 \rangle + \frac{\langle 3,2,1 \rangle \cdot \langle 0,0,1 \rangle}{(\sqrt{2})^2} \langle 0,0,1 \rangle$

$= \langle 2,2,3/2,3/2 \rangle$

Distance $= \sqrt{(3-2)^2 + (1-2)^2 + (2-3/2)^2 + (1-3/2)^2} = \sqrt{3}$

c. $W = \langle 3,2,1 \rangle \times \langle 1,0,0 \rangle \langle 1,0,0 \rangle + \langle 3,2,1 \rangle \times \langle 0,0,1 \rangle \langle 0,1,0 \rangle$

$+ \langle 3,2,1 \rangle \times \langle 0,0,0 \rangle \langle 0,0,0 \rangle = \langle 3,2,0 \rangle$

Distance $= \sqrt{(3-3)^2 + (1-1)^2 + (2-2)^2 + (1-0)^2} = 1$

d. Spanned by $\langle 1,1,-1,-1 \rangle$, $\langle 1,-1,1,-1 \rangle$, and $\langle 1,-1,-1,1 \rangle$

$W = \langle 3,2,1 \rangle \times \langle 1,-1,-1 \rangle \langle 1,1,-1,-1 \rangle + \frac{\langle 3,2,1 \rangle \times \langle 1,-1,1,-1 \rangle}{(\sqrt{4})^2} \langle 1,-1,1,-1 \rangle$

$\perp \frac{\langle 3,2,1 \rangle \times \langle 1,-1,-1,1 \rangle}{(\sqrt{4})^2} \langle 1,-1,-1,1 \rangle = \langle 5/4, -3/4, 1/4, -3/4 \rangle$

Distance $= \sqrt{(3-5/4)^2 + (1-(-3/4))^2 + (2-1/4)^2 + (1-(-3/4))^2} = \sqrt{49/4} = 7/2$

5.3; 5.W $= \frac{\langle 1,1,2\rangle}{(\sqrt{6})^2}\langle 1,2,-1\rangle + \frac{\langle 1,1,2\rangle \cdot \langle 0,1,2,1\rangle}{(\sqrt{6})^2}\langle 0,1,-1,3\rangle$

$+ \frac{\langle 1,1,2,2\rangle \cdot \langle 1,0,3,3\rangle}{(\sqrt{14})^2}\langle 1,0,3,2\rangle = \langle 1/6 + 1/4,\ 1/3 - 5/6,\ -1/6 + 1/8 + 33/14,\ 5/6 + 23/14\rangle$

$= \langle 20/21, -1/2, 27/7, 31/42\rangle$

SF: 2.a $\begin{bmatrix} 1 & 2 \\ 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

$A^TA\begin{bmatrix} x \\ y \end{bmatrix} = A^Tb$

$\begin{bmatrix} 3 & -1 \\ -1 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 9 \end{bmatrix}^{-1}\begin{bmatrix} -2 \\ 8 \end{bmatrix} = \begin{bmatrix} 9/26 & 1/26 \\ 1/26 & 3/26 \end{bmatrix}\begin{bmatrix} -2 \\ 8 \end{bmatrix} = \begin{bmatrix} -3/13 \\ 4/13 \end{bmatrix}$

b. $\begin{bmatrix} 4 & -2 \\ 2 & 3 \\ 1 & -2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ -1 \\ 2 \end{bmatrix}$

$A^TA\begin{bmatrix} x \\ y \end{bmatrix} = A^Tb$

$\begin{bmatrix} 25 & 0 \\ 0 & 21 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -8 \end{bmatrix} \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/25 \\ -8/21 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 1 & -2 \\ 3 & 0 & -2 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 5 & 1 & 0 & 1 \\ 3 & 0 & 2 & 0 \\ 1 & -1 & 3 & 2 \end{array}\right]$ $\left[\begin{array}{ccc|c} 5 & 1 & 0 & 1 \\ 3 & 0 & 2 & 0 \\ 6 & 0 & 3 & 3 \end{array}\right]$ $\left[\begin{array}{ccc|c} 5 & 1 & 0 & 1 \\ 3 & 0 & 2 & 0 \\ 0 & 0 & -1 & 3 \end{array}\right]$ $\left[\begin{array}{ccc|c} 5 & 1 & 0 & 1 \\ 0 & -3/5 & 2 & -3/5 \\ 0 & 0 & -1 & 3 \end{array}\right]$

$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ -9 \\ -3 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 0 & -1 & -3 \\ -5 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 3 \end{bmatrix}$

$A^T A x = A^T b$

$\begin{bmatrix} 30 & -12 & 0 \\ -12 & 6 & -4 \\ 0 & -4 & 20 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -14 \\ 5 \\ 7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} -14 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 13/30 & 1 & 1/5 \\ 1 & 5/2 & 1/2 \\ 1/5 & 1/2 & 3/20 \end{bmatrix} \begin{bmatrix} -14 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 2 \\ -3/4 \end{bmatrix}$

S.A.2.e. $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

$A^TAx = A^Tb$

$\begin{bmatrix} 3 & 1 & 1 & -1 \\ 1 & 3 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 0 \\ 2 \end{bmatrix}$

$x = A^{-1} \begin{bmatrix} 4 \\ 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 3/4 & -1/4 & -3/4 & 1/2 \\ -1/4 & 5/12 & 1/4 & -1/2 \\ -3/4 & 1/4 & 1/4 & -1/2 \\ 1/4 & -1/12 & -1/4 & 5/12 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 5/6 \\ -3/2 \\ -1/6 \end{bmatrix}$

S. $A^TA = \begin{bmatrix} 11 & 4 & 5 \\ 4 & 34 & -12 \\ 5 & -12 & 12 \end{bmatrix}$ $A^Tb = \begin{bmatrix} -18 \\ 28 \\ 17 \end{bmatrix}$

$(A^TA)^{-1} = \frac{1}{2512} \begin{bmatrix} 268 & 12 & 122 \\ 12 & 107 & 112 \\ 122 & 112 & 350 \end{bmatrix}$

$x = \frac{1}{2512} \begin{bmatrix} 268 & 12 & 122 \\ 12 & 107 & 112 \\ 122 & 112 & 350 \end{bmatrix} \begin{bmatrix} -18 \\ 28 \\ 17 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

a. $y = 30.65 + 2.96x$

5.5:2.b.

x	y	x^2	xy
12	60	144	720
14	70	196	980
17	90	289	1530
21	100	441	2100
26	100	676	2600
30	120	900	3600

c. $30.65 + 2.96(50) = 178.65$

d. $30.65 + 2.96(100) = 326.65$