CSCI 3104 Spring 2022
Instructors: Profs. Chen and Layer

Retake Token Quiz- Standard 21

Due Date .... April 28
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 21- DP: Use recurrence to solve

Problem 1. The Subset-Sum problem is defined as follows.

- Instance: We are given n items with positive weights $w_1, \ldots, w_n > 0$, as well as a target threshold $W > 0$.
- Solution: Is there a subsequence $w_{i_1}, \ldots, w_{i_k}$ such that:

$$\sum_{j=1}^{k} w_{i_j} = W.$$

That is, can we select a subsequence of items whose combined weights add to W?

For example, consider the input array $A = [4, 15, 8, 16, 23, 42]$. If $w = 31$ then the answer is "TRUE" since there is a subsequence $[15, 16]$ where the sum is equal to $15 + 16 = 31$. However, if $w = 13$ then the answer is "FALSE" since no subsequence of A has sum equal 13.

The Subset-Sum problem satisfies the following recurrence. For $0 \leq \ell \leq n$, and $0 \leq q \leq W$, define $T[\ell, q]$ be to TRUE if and only if there exists a subsequence of the first ℓ elements $[w_1, \ldots, w_\ell]$ that sum to q. Then we have:

$$T[\ell, q] = \begin{cases} TRUE & q = 0 \\ FALSE & \ell = 0 \text{ and } q > 0 \\ T[\ell-1, q] & \ell > 0 \text{ and } w_\ell > q \\ T[\ell-1, q] \text{ OR } T[\ell-1, q-w_\ell] & \ell > 0 \text{ and } w_\ell \leq q \end{cases}$$

(Note that in the final case, "OR" is the Boolean OR operation.)

Consider $A = [2, 1, 5, 3]$ and $W = 6$. Design and fill in a lookup table for this problem. Include all pointers. Clearly indicate whether a solution exists; and if so, how to recover it from the lookup table.

Answer.

W:	1	2	3	4	5	6
$\ell = 1$	False	True, w_1	False	False	False	False
$\ell = 2$	True, w_2	True, w_1	True, $\mathrm{w}_2 + T[1, 2]$	False	False	False
$\ell = 3$	True, w_2	True, w_1	True, $\mathrm{w}_2 + T[1, 2]$	False	True, w_3	True, $\mathrm{w}_3 + T[2, 1]$
$\ell = 4$	True, w_2	True, w_1	True, $\mathrm{w}_2 + T[1, 2]$	True, $\mathrm{w}_4 + T[2, 1]$	True, w_3	True, $\mathrm{w}_3 + T[2, 1]$

A solution does exist for $T[4, 6]$. We find it in the lookup table by adding w_3 to the cell that represents $T[2, 1]$ which is w_1. □