HW0 solution sheet

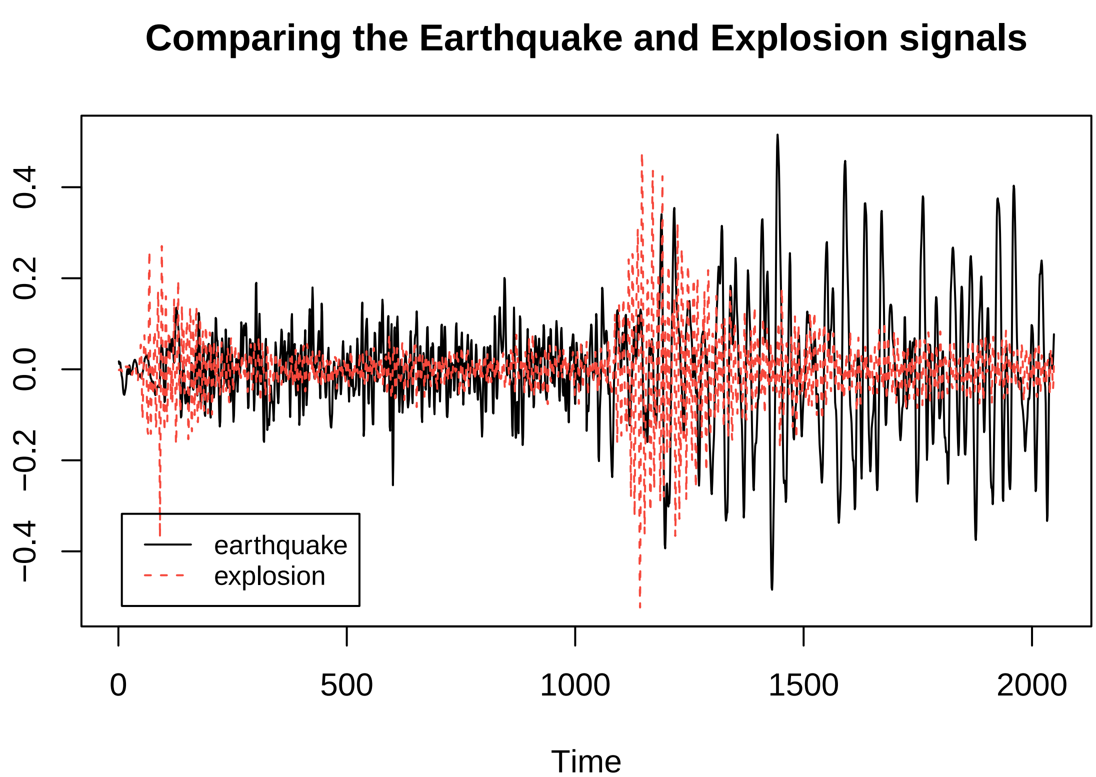
Your name

Some comments

For coding problems, suppress your code and provide only informative output/results in the main document. You can do this by setting the chunk option **echo=FALSE**, which will display the results but hide the codes. If there are non-informative messages/warnings, you can also use **message** or **warning** options to mute them. **All your codes should be provided by order in the Appendix section.** And codes for each problems should be included in their own individual chunk with the chunk option **eval=FALSE** (so the code will not be evaluated). Some examples are provided in the corresponding Rmarkdown file, and you are also welcome to ask Difan for help. For math/stats problems, to get full credits, you need to provide necessary steps of derivation so that we know you fully understand the statistical concept.

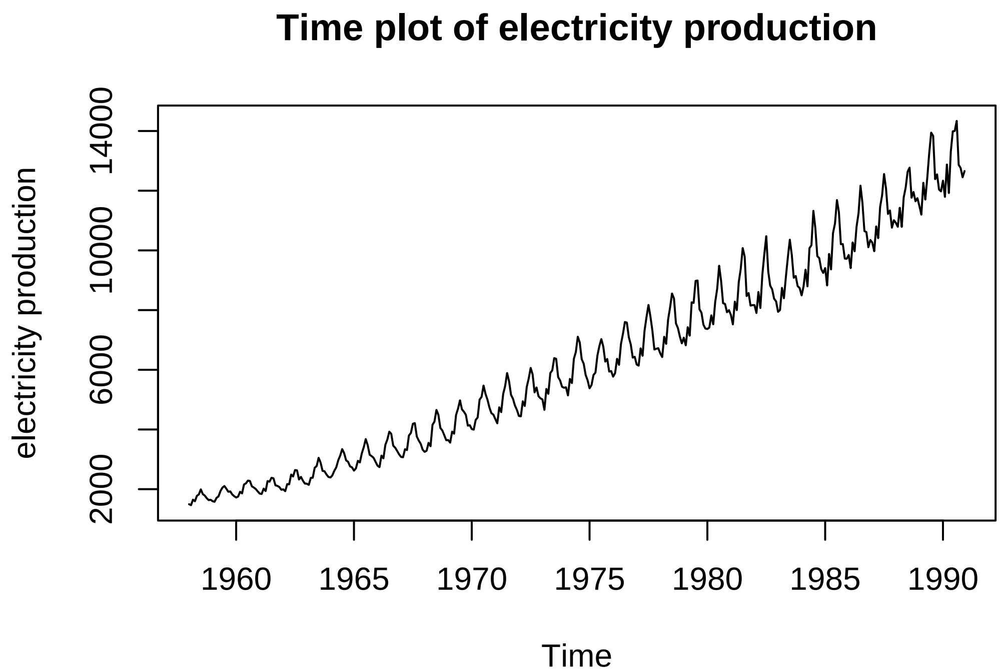
Problem 1

(a)

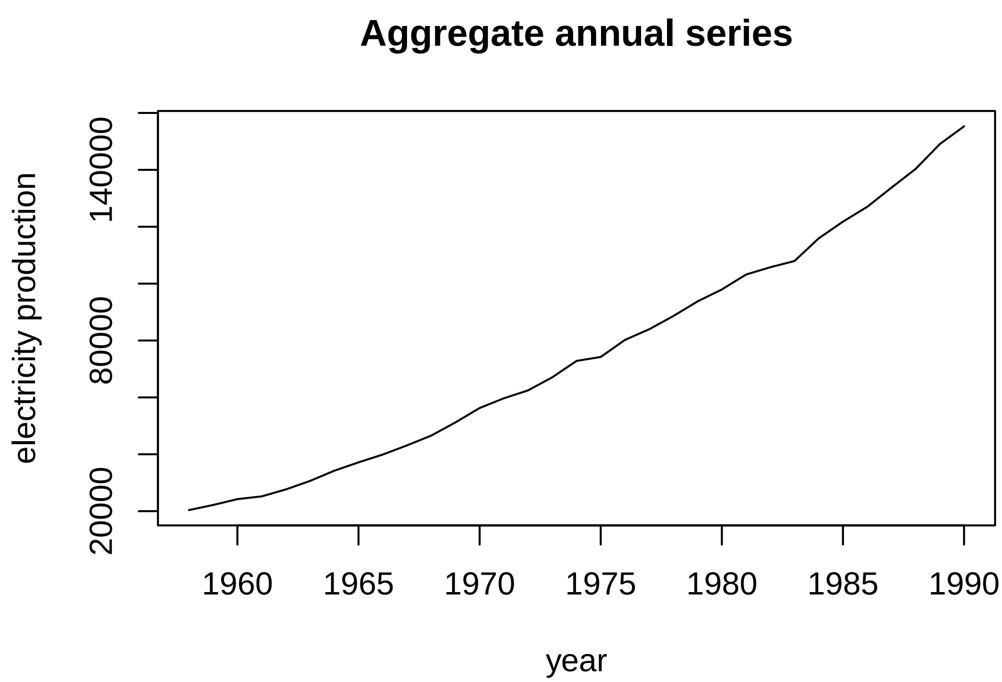


They have the similiar fluctuation in the phase P and phase S.

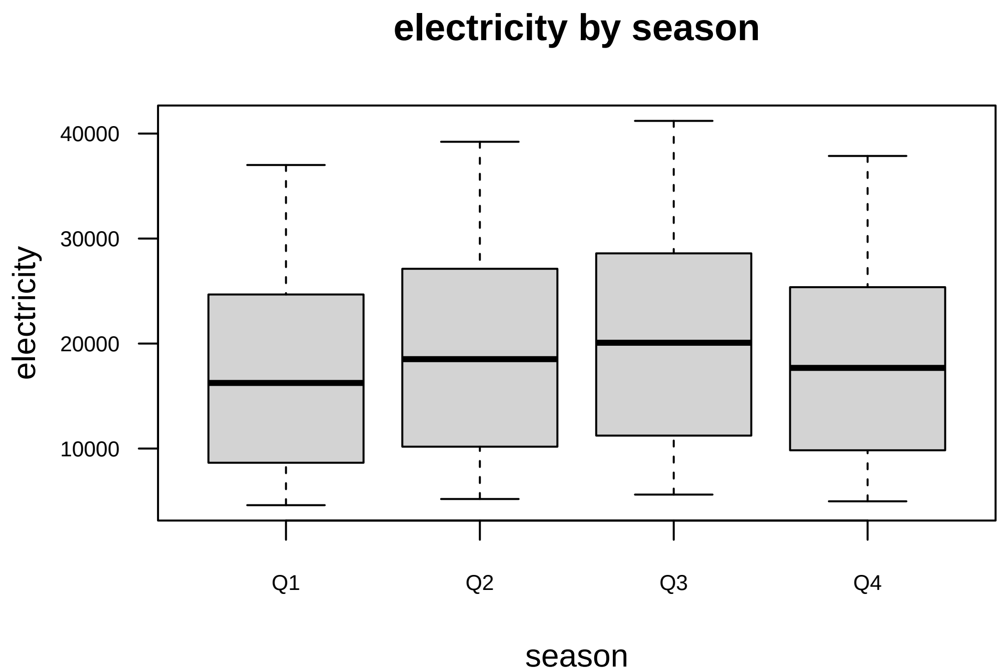
(b)



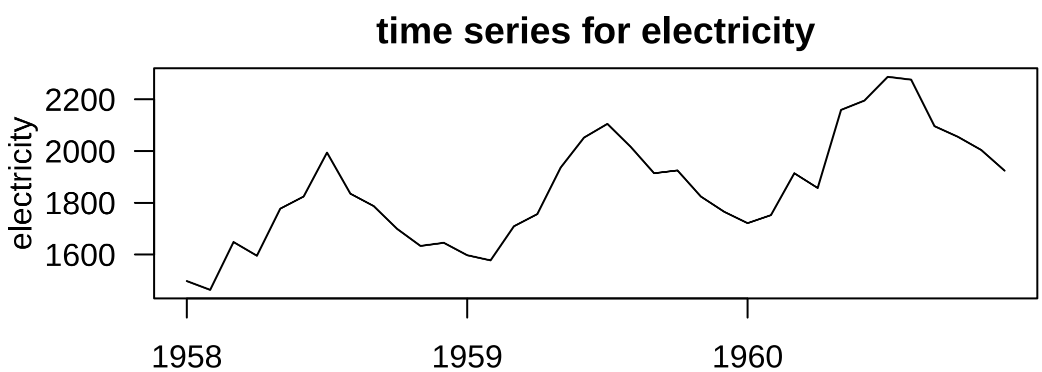
It has an increasing trend with a variation in each year.

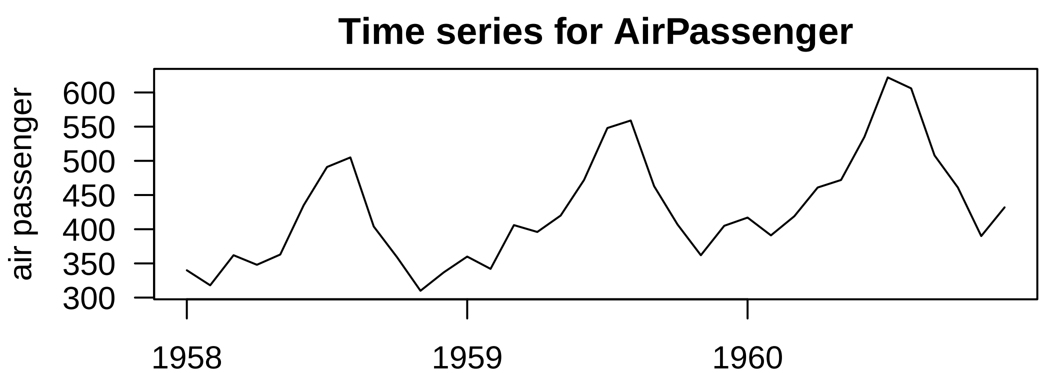


It has an increasing trend.



The mean of first three season increases and it drops at the fourth one.





They have very similar periodicity.

Problem 2

(a)

The density f is, for $0 \le x, y \le 1$,

$$\begin{aligned} f(x,y) &= \frac{\partial^2}{\partial x \partial y} F(x,y) \\ &= \frac{\partial}{\partial x}\{x[1+\theta(1-x)(1-2y)]\} \\ &= [1+\theta(1-x)(1-2y)] - \theta x(1-2y) \\ &= 1+\theta(1-2x)(1-2y) \end{aligned}$$

(b)

One can integrate the density to find the marginal density. A simpler way is to take $y = 1$ in the joint CDF which gives $F_X(x) = F_{X,Y}(x,1) = x$ so the marginal density is:

$$f_X(x) = 1 \qquad 0 \le x \le 1$$

is the pdf of a uniform $[0,1]$ distribution

(c)

By integration or by facts about the uniform distribution, we have

$$E[X] = E[Y] = \frac{1}{2} \qquad \mathrm{Var}(X) = \mathrm{Var}(Y) = \frac{1}{12}$$

(d)

Compute

$$\begin{aligned} E[XY] &= \int_0^1 \int_0^1 xyf(x,y)\,dxdy \\ &= \int_0^1 \int_0^1 xy[1+\theta(1-2x)(1-2y)]\,dxdy \\ &= \left(\int_0^1 x\,dx\right)^2 + \theta\left(\int_0^1 x(1-2x)\,dx\right)^2 \\ &= \left(\frac{1}{2}\right)^2 + \theta\left(-\frac{1}{6}\right)^2 \\ &= \frac{1}{4} + \frac{\theta}{36} \end{aligned}$$

Thus,

$$\mathrm{Cov}(X,Y) = E[XY] - E[X]E[Y] = \frac{\theta}{36}$$

and

$$\mathrm{Cor}(X,Y) = \frac{\text{Cov(X, Y)}}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}} = \frac{\theta/36}{1/12} = \frac{\theta}{3}$$

(e)

By the independence of X_i and X_j for $i \neq j$, we have

$$\mathrm{Cov}(\bar{X}_n, \bar{X}_n^2) = \frac{1}{n^2}\sum_{i=1}^n \sum_{j=1}^n \mathrm{Cov}(X_i, X_j^2) = \frac{1}{n^2}\sum_{i=1}^n \mathrm{Cov}(X_i, X_i^2) = \frac{1}{n}\mathrm{Cov}(X, X^2) = \frac{1}{n}\big(E[X^3] - E[X]E[X^2]\big)$$

and

$$E[X^3] = \int_0^1 x^3 \cdot f(x)\,dx = \int_0^1 x^3 \cdot 1\,dx = \frac{1}{4}x^4\Big|_0^1 = \frac{1}{4}$$

so that

$$\text{Cov}(\bar{X}_n, \bar{X}_n^2) = \frac{1}{n}\left(\frac{1}{4} - \frac{1}{6}\right) = \frac{1}{12n}$$

(f)

By the Law of Large Numbers (LLN), as $n \to \infty$,

$$\bar{X}_n \xrightarrow{\text{P}} E[X] = \frac{1}{2} \qquad \bar{X}_n^2 \xrightarrow{\text{P}} E[X^2] = \frac{1}{3}$$

(g)

By the Central Limit Theorem (CLT), for large n, we have

$$\sqrt{n}\left(\bar{X}_n - \frac{1}{2}\right) \xrightarrow{\text{d}} N\left(0, \frac{1}{12}\right)$$

Problem 3

(a)

- i. $1 + 2i$
- ii. $-2 - 6i$
- iii. $11 - 2i$
- iv. $\frac{11}{25} - \frac{2}{25}i$
- v. -6

(b)

Since $|z| = \frac{1}{8} < 1$, we have

$$\sum_{j=0}^{\infty} z^j = \frac{1}{1-z} = \frac{1}{1 - 1/4 - i/4} = \frac{1}{3/4 - i/4} = \frac{3/4 + i/4}{10/16} = \frac{6}{5} + \frac{2}{5}i$$

(c)

By Euler's formula, we have:

$$|e^{i\pi t}| = |\cos(\pi t) + i\sin(\pi t)| = \sqrt{\cos^2(\pi t) + \sin^2(\pi t)} = 1$$

(d)

By the fundamental theorem of algebra, this equation has 10 roots.

(e)

Apply the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, which yields

$$\frac{-1 \pm \sqrt{1 - 4 \times 3 \times 2}}{4} = -\frac{1}{4} \pm \frac{i\sqrt{23}}{4}$$

Appendix

Problem 1a

```
library(astsa)
data("EQ5")
data("EXP6")
ts.plot(EQ5,EXP6,col=c(1,2),lty=c(1,2),
main="Comparing the Earthquake and Explosion signals")
legend("bottomleft",legend=c("earthquake","explosion"),
inset=0.04,col=c(1,2),lty=c(1,2),cex=0.8)
```

Problem 1b

```
cbe<-read.csv("cbe.txt",header=T,sep='')
times<-ts(cbe$elec,frequency=12,start=c(1958,1))
ts.plot(times,main="Time plot of electricity production",
ylab="electricity production")

timesb<-aggregate(times,nfrequency=1)
ts.plot(timesb,main="Aggregate annual series",
ylab="electricity production",xlab="year")

times_quarter=aggregate(times,nfrequency=4)
boxplot(times_quarter~cycle(times_quarter),main="electricity by season",
ylab="electricity",xlab="season",las=1,names=paste("Q",1:4,sep=""),cex.axis=0.7)

data("AirPassengers")
timesel<-window(times,start=1958,end=c(1960,12))
timesair<-window(AirPassengers,start=1958)
par(mfrow=c(2,1),
    mar=c(2 ,4, 2, 1))
times_air=time(timesair,frequency=12,start=c(1958,1))
times_el=time(timesel,frequency=12,start=c(1958,1))
axis_time_el <- seq(times_el[1], times_el[length(times_el)], by = 1)
axis_time_air <- seq(times_air[1], times_air[length(times_air)], by = 1)
plot(timesel,main="time series for electricity",ylab="electricity",xaxt="n",las=1)
axis(1, at = axis_time_el, labels = TRUE)
plot(timesair,main="Time series for AirPassenger",ylab="air passenger",xaxt="n",las=1)
axis(1, at = axis_time_air, labels = TRUE)
```