CSCI 3104 Spring 2022
Instructors: Profs. Chen and Layer

Quiz 12 - Nested Independent Loops

Due Date ..... March 4
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1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 12 - Nested Independent Loops

Problem 1. Analyze the runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) \in \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops. [**Note:** $A[n, n]$ denotes an n-by-n matrix with entries $a(i, j),\ 1 \leq i \leq n,\ 1 \leq j \leq n$.]

```
1: column_power_sum(A[n, n])
2:   sum = 0
3:   for i = 1, i <= n
4:       i = i + 1
5:       for j = 1, j <= n
6:           j = 2 * j
7:           sum = sum + a(i, j)
8:   return sum
```

Answer. Initializing the variable sum in line 2 takes one unit of time.

Initializing the loop in line 3 takes one unit of time.

For each iteration of the first loop, the comparison statement $i <= n$ in line 3 takes one unit of time, the computation $i + 1$ in line 4 takes one unit of time, the assignment of that value to i in line 4 takes one unit of time, and the initialization of the second loop on line 5 takes one unit of time resulting in four units of time per iteration. The loop runs n times for a total of 4n units of time from this loop.

For each iteration of the second loop, the comparison statement $j <= n$ in line 5 takes one unit of time, the computation $2 * j$ in line 6 takes one unit of time, the assignment of that value to j in line 6 takes one unit of time, the computation of $sum + a(i, j)$ on line 7 takes one unit of time, and the assignment of this value to sum on line 7 takes one unit of time for a total of five units of time per iteration. The loop runs $n * (\lfloor \log_2(n) \rfloor + 1)$ times for a total of $5n * (\lfloor \log_2(n) \rfloor + 1)$ units of time from this loop.

The return statement on line 8 takes one unit of time.

So we have that $T(n) = 3 + 4n + 5n * (\lfloor \log_2(n) \rfloor + 1)$.

Thus, $T(n) \in \Theta(n \log_2(n))$. □