

Quiz 1 - Induction

Due Date February 4
Name Alex Ojemann
Student ID 109722375

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1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to \LaTeX .
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this \LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

Problem 1. • My submission is in my own words and reflects my understanding of the material.

- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (Alex Ojemann). I agree to the above, Alex Ojemann.

□

3 Standard 1- Proof by Induction

3.1 Problem 2

Problem 2. Consider the sequence defined by:

$$T_n = \begin{cases} 1 & : n = 1, \\ 3T_{n-1} + 1 & : n > 1. \end{cases}$$

Prove by induction that for every $n \geq 1$, we have:

$$\sum_{i=1}^n T_i = \frac{T_{n+1} - n - 1}{2}.$$

Proof. The proof is by induction on $n \geq 1$.

- **Base Case:** When $n = 1$, we have that:

$$\begin{aligned} \sum_{i=1}^n T_i &= 1 \\ &= \frac{4 - 1 - 1}{2} \\ &= \frac{T_{n+1} - n - 1}{2} \end{aligned}$$

- **Inductive Hypothesis:** Now suppose that for some $k \geq 1$ we have that $\sum_{i=1}^k T_i = \frac{T_{k+1} - k - 1}{2}$.
- **Inductive Step:** We now consider the $k + 1$ case. We will show that $\sum_{i=1}^{k+1} T_i = \frac{T_{k+2} - (k+1) - 1}{2}$.

$$\begin{aligned} \sum_{i=1}^{k+1} T_i &= \sum_{i=1}^k T_i + T_{k+1} \\ &= \frac{T_{k+1} - k - 1}{2} + T_{k+1} \\ &= \frac{3T_{k+1} - k - 1}{2} \\ &= \frac{(T_{k+2} - 1) - k - 1}{2} \\ &= \frac{T_{k+2} - k - 2}{2} \\ &= \frac{T_{k+2} - (k+1) - 1}{2} \end{aligned}$$

In the second line we replaced $\sum_{i=1}^k T_i$ with $\frac{T_{k+1} - k - 1}{2}$ because they are assumed to be equal in our inductive hypothesis. In the fourth line we replaced $3T_{k+1}$ with $(T_{k+2} - 1)$ based on the given function T_n .

The result follows by induction.

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