Alex Ojemann
James Curry
Section 002

1. a. False
   b. True
   c. False

$$\vec{x}\vec{y}^T = \begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}-1 & 2\end{bmatrix} = \begin{bmatrix}-1 & 2\\-2 & 4\end{bmatrix} \quad -4-(-4)=0$$

   d. True
   e. False
   f. False

$$A = \begin{bmatrix}1 & 2\\0 & 0\end{bmatrix} \quad b = \begin{bmatrix}3\\0\end{bmatrix} \quad x = \begin{bmatrix}1\\1\end{bmatrix}$$ (and other possible solutions)

   g. True
   h. True
   i. False
   j. True

2. a.

$$\begin{bmatrix}0 & 2 & 2\\1 & 0 & 3\\0 & 1 & -1\end{bmatrix} \leftrightarrow \begin{bmatrix}1 & 0 & 3\\0 & 2 & 2\\0 & 1 & -1\end{bmatrix} -\tfrac{1}{2}R2 \quad \begin{bmatrix}1 & 0 & 3\\0 & 2 & 2\\0 & 0 & -2\end{bmatrix}$$

$$PA = \begin{bmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 2 & 2\\1 & 0 & 3\\0 & 1 & -1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & -\tfrac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 3\\0 & 2 & 2\\0 & 0 & -2\end{bmatrix} = LU$$

b. $-2x_3 = 2 \quad x_3 = -1$

$2x_2 - 2 = 0 \quad x_2 = 1$

$x_1 - 3 = -2 \quad x_1 = 1$

$$\vec{x} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$$

c. $\det(A) = \dfrac{\det(L)\det(U)}{\det(P)}$

$$= \frac{1 \cdot 4}{-1} = \boxed{4}$$

3. $x^2+2xy+y^2=(x+y)^2$

✓ Zero vector: $\begin{bmatrix}0\\0\\0\end{bmatrix}$ is in W because $(0+0)^2=0$

✓ Closed under addition: If $\begin{bmatrix}a\\b\\c\end{bmatrix}\in W$ and $\begin{bmatrix}d\\e\\f\end{bmatrix}\in W$,

$(a+b)^2=0$ and $(d+e)^2=0$, so $a+b=0$ and $d+e=0$,

so $a+b+d+e=0$, so $\left(\begin{bmatrix}a\\b\\c\end{bmatrix}+\begin{bmatrix}d\\e\\f\end{bmatrix}\right)\in W$.

Closed under scalar multiplication:

If $\begin{bmatrix}g\\h\\i\end{bmatrix}\in W$, $(g+h)^2=0$, so $g+h=0$,

so $cg+ch=0$, so $c\cdot\begin{bmatrix}g\\h\\i\end{bmatrix}\in W$.

Since W is a subset of V and it's closed under addition and scalar multiplication, V is a subspace of W.

4.
$$c_1\begin{bmatrix}0\\0\\0\\1\end{bmatrix}+c_2\begin{bmatrix}0\\0\\-1\\1\end{bmatrix}+c_3\begin{bmatrix}0\\-1\\2\\1\end{bmatrix}+c_4\begin{bmatrix}1\\0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\x\\y\\z\end{bmatrix}$$

$$\left[\begin{array}{cccc|c}0&0&0&c_4&0\\0&0&-c_3&0&x\\0&-c_2&2c_3&0&y\\c_1&c_2&c_3&0&z\end{array}\right]\begin{array}{l}\\ \\ +2R2\\ \\ \end{array}$$

$$\left[\begin{array}{cccc|c}0&0&0&c_4&0\\0&0&-c_3&0&x\\0&-c_2&0&0&y+2x\\c_1&c_2&c_3&0&z\end{array}\right]\begin{array}{l}\\ \\ \\ +R2+R3\end{array}$$

$$\left[\begin{array}{cccc|c}0&0&0&c_4&0\\0&0&c_3&0&x\\0&c_2&0&0&y+2x\\c_1&0&0&0&z+y+x\end{array}\right]$$

This system is consistent for all values of x, y, and z, so this set of polynomials spans the set of all polynomials with degree ≤ 2.

rank = 2

5. a. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \right\}$

b. $\text{Img}(A^T) = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \right\}$

c. $x_1 = -3x_3$ Solution $= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$

$x_2 = -2x_3$

$\left\{ \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} \right\}$

d. A^T: $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -3 \\ 3 & -1 & 0 \end{bmatrix} \begin{matrix} \\ \\ -3R1 \end{matrix}$

$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -3 \\ 0 & 2 & -6 \end{bmatrix} \begin{matrix} \\ \\ -2R2 \end{matrix}$ Solution: $x_3 \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$ $x_2 = 3x_3$

$x_1 = (3x_3) - 2x_3 = x_3$

$\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \right\}$