

Homework 9

Grading

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Ex. 27. d. $y_1 = x_1 = \langle 0, -1, -1 \rangle$

$$y_2 = x_2 - \text{proj}_{y_1} x_2 = \langle 1, 1, 1 \rangle - \frac{\langle 1, 1, 1 \rangle \cdot \langle 0, -1, -1 \rangle}{\sqrt{2}} \langle 0, -1, -1 \rangle = \langle 1, 0, 0 \rangle$$

$$y_3 = x_3 - \text{proj}_{y_1} x_3 - \text{proj}_{y_2} x_3 = \langle 2, 1, 3 \rangle - \frac{\langle 2, 1, 3 \rangle \cdot \langle 0, -1, -1 \rangle}{\sqrt{2}} \langle 0, -1, -1 \rangle - \frac{\langle 2, 1, 3 \rangle \cdot \langle 1, 0, 0 \rangle}{1} \langle 1, 0, 0 \rangle = \langle 0, -1, 1 \rangle$$

$$Q = \begin{bmatrix} 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$R = Q^T A = \begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} \\ 1 & 0 & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -\sqrt{2} & -\sqrt{2} & -2\sqrt{2} \\ 0 & 1 & 2 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

$$A = QR = \begin{bmatrix} 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -\sqrt{2} & -\sqrt{2} & -2\sqrt{2} \\ 0 & 1 & 2 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

28. iii. $y_1 = x_1 = \langle 1, -1, 0 \rangle$

$$y_2 = \langle 1, 0, -1 \rangle - \frac{\langle 1, 0, -1 \rangle \cdot \langle 1, -1, 0 \rangle}{\sqrt{2}} \langle 1, -1, 0 \rangle = \langle 1/2, 1/2, -1 \rangle$$

$$y_3 = \langle 0, 1, 1 \rangle - \frac{\langle 0, 1, 1 \rangle \cdot \langle 1, -1, 0 \rangle}{\sqrt{2}} \langle 1, -1, 0 \rangle - \frac{\langle 0, 1, 1 \rangle \cdot \langle 1/2, 1/2, -1 \rangle}{\sqrt{3/2}} \langle 1/2, 1/2, -1 \rangle = \langle 2/3, 2/3, 2/3 \rangle$$

$$Q = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{3} \\ 0 & -1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$$

$$R = Q^T A = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{3} \\ 0 & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 2/\sqrt{3} \end{bmatrix}$$

$$A = QR = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{3} \\ 0 & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} \sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 2/\sqrt{3} \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 2/\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 1/\sqrt{3} \end{bmatrix}$$

$$2/\sqrt{3} z = 1/\sqrt{3} \quad z = 1/2$$

$$1/\sqrt{2} y - 1/\sqrt{2} = 1/\sqrt{2} \quad y = 1/2$$

$$\sqrt{2} x + 1/2\sqrt{2} - 1/2\sqrt{2} = -1/\sqrt{2} \quad x = -1/2$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$$

32. d.i. $y_1 = x_1 = \langle 1, 2, 0 \rangle$

$y_2 = \langle -1, 3, 2 \rangle \Rightarrow \frac{\langle 1, 2, 0 \rangle \cdot \langle -1, 3, 2 \rangle}{\sqrt{5}} = \frac{-1+6+0}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$
 $\Rightarrow \langle -1, 3, 2 \rangle - \sqrt{5} \langle 1, 2, 0 \rangle = \langle -2, 1, 2 \rangle$

$Q = \begin{bmatrix} 1/\sqrt{5} & -2/3 \\ 2/\sqrt{5} & 1/3 \\ 0 & 2/3 \end{bmatrix} \quad R = Q^T A = \begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} & 0 \\ -2/3 & 1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} \sqrt{5} & \sqrt{5} \\ 0 & 3 \end{bmatrix} \quad A = QR = \begin{bmatrix} 1/\sqrt{5} & -2/3 \\ 2/\sqrt{5} & 1/3 \\ 0 & 2/3 \end{bmatrix} \begin{bmatrix} \sqrt{5} & \sqrt{5} \\ 0 & 3 \end{bmatrix}$

Check: $\begin{bmatrix} 1/\sqrt{5} & -2/3 \\ 2/\sqrt{5} & 1/3 \\ 0 & 2/3 \end{bmatrix} \begin{bmatrix} \sqrt{5} & \sqrt{5} \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 0 & 2 \end{bmatrix} \quad \checkmark$

e. If $\text{rank } A < n$ we can't do a QR decomposition because there are too many column vectors.

35 (27.d.). $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 1 \\ -1 & 1 & 3 \end{bmatrix} \quad |a_1| = \sqrt{2} \quad |a_2| = \sqrt{3} \quad |a_3| = \sqrt{14}$

$v_1 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} + \sqrt{2} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ -1 \\ -1 \end{bmatrix} \quad H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} -1 & \sqrt{2} & -1 \end{bmatrix} \begin{bmatrix} \sqrt{2} \\ -1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & 1/2 & -1/2 \\ \sqrt{2}/2 & -1/2 & 1/2 \end{bmatrix} \quad H_1 A_1 = \begin{bmatrix} 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & 1/2 & -1/2 \\ \sqrt{2}/2 & -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -\sqrt{2} & \sqrt{2} & 2\sqrt{2} \\ 0 & \sqrt{2}/2 & \sqrt{2}-1 \\ 0 & \sqrt{2}/2 & \sqrt{2}-1 \end{bmatrix}$

$A_2 = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}-1 \\ \sqrt{2}/2 & \sqrt{2}-1 \end{bmatrix} \quad v_2 = \begin{bmatrix} \sqrt{2}/2 \\ -\sqrt{2}/2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 + 1 \\ \sqrt{2}/2 \end{bmatrix} \quad H_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{2}{2+\sqrt{2}} \begin{bmatrix} 1+\sqrt{2} & \sqrt{2} \end{bmatrix}$

$\begin{bmatrix} 1+\sqrt{2} & \sqrt{2} \end{bmatrix} = \begin{bmatrix} -\sqrt{2}/2 & -\sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \quad H_2 A_2 = \begin{bmatrix} -1 & -2 \\ 0 & \sqrt{2} \end{bmatrix}$

$A_3 = \begin{bmatrix} \sqrt{2} \end{bmatrix} \quad v_3 = \begin{bmatrix} 2\sqrt{2} \end{bmatrix} \quad H_3 = 1 - \frac{2}{2\sqrt{2}} \begin{bmatrix} 2\sqrt{2} \end{bmatrix} = -1 \quad H_3 A_3 = -1 \cdot \begin{bmatrix} \sqrt{2} \end{bmatrix} = \begin{bmatrix} -\sqrt{2} \end{bmatrix}$

$R = H_3 H_2 H_1 A = \begin{bmatrix} \sqrt{2} & -\sqrt{2} & 2\sqrt{2} \\ 0 & 1 & 2 \\ 0 & 0 & \sqrt{2} \end{bmatrix} \quad Q = H_1 H_2 H_3 = \begin{bmatrix} 0 & 1 & 0 \\ -1/2 & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \quad A = QR = \begin{bmatrix} 0 & 1 & 0 \\ -1/2 & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{2} & -\sqrt{2} & 2\sqrt{2} \\ 0 & 1 & 2 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$