CSCI 3104 Spring 2022
Instructor: Profs. Chen and Layer

Problem Set 3

Due Date ..... **February 8, 2022**
Name ..... **Alex Ojemann**
Student ID **109722375**
Collaborators .....

Contents

1 Instructions

- The solutions **must be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

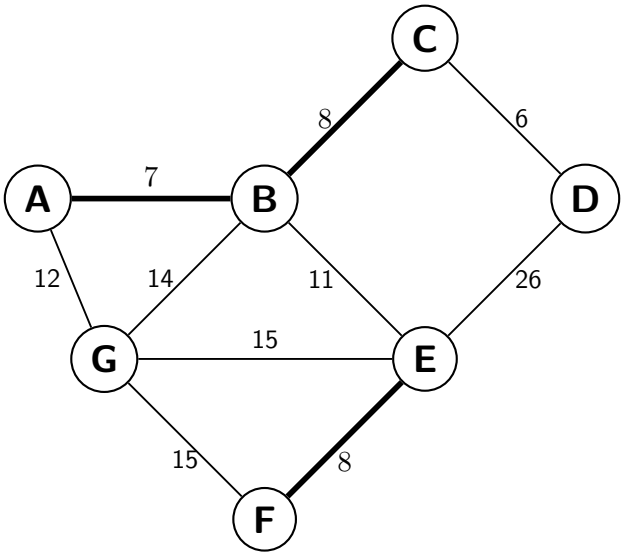
Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (Alex Ojemann). I agree to the above, Alex Ojemann □

3 Standard 7 - MST: safe and useless edges

Problem 2. Consider the weighted graph $G(V, E, w)$ below. Let $\mathcal{F} = \{\{A, B\}, \{B, C\}, \{E, F\}\}$ be an intermediate spanning forest (indicated by the thick edges below). Label each edge that is **not** in $\mathcal{F}$ as safe, useless, or undecided. Provide a 1-2 sentence explanation for each such edge.



Answer. The individual trees in this forest are {G}, {A, B, C}, {F, E} and {D}.

(A,G) is incident to one vertex from {G} and one vertex from {A, B, C}. It's the minimum weight edge incident to {G} so it's a safe edge.

(B,G) is incident to one vertex from {G} and one from {A, B, C}. It's not the minimum-weight edge incident to either of them, so it's undecided.

(E,G) is incident to one vertex from {G} and one from {E, F}. It's not the minimum weight edge incident to either of them so it's undecided.

(F,G) is incident to one vertex from {G} and one from {E, F}. It's not the minimum weight edge incident to either of them so it's undecided.

(C,D) is incident to one vertex from {D} and one from {A, B, C}. It's the minimum-weight edge incident to {D} and {A, B, C} so it's safe.

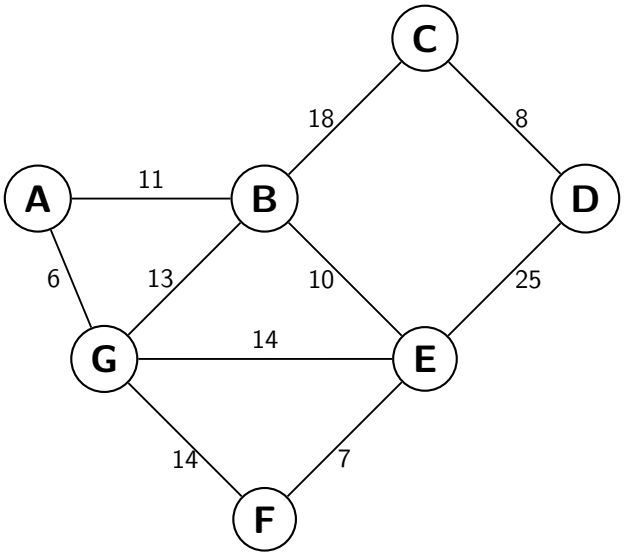
(D,E) is incident to one vertex from {D} and one from {E, F}. It's not the minimum-weight edge incident to either of them, so it's undecided.

(B,E) is incident to one vertex from {E, F} and one from {A, B, C}. It's the minimum-weight edge incident to {E, F} so it's safe.

□

4 Standard 8- Kruskal's Algorithm

Problem 3. Consider the weighted graph $G(V, E, w)$ below. Clearly list the order in which Kruskal's algorithm adds edges to a minimum-weight spanning tree for G. Additionally, clearly articulate the steps that Kruskal's algorithm takes as it selects the first **three** edges.

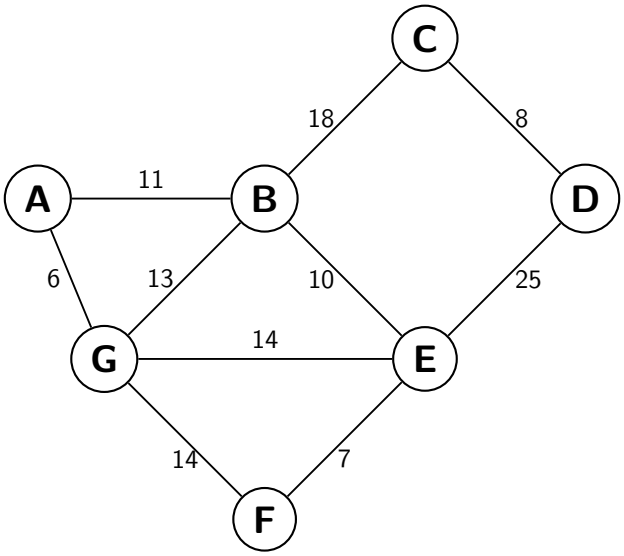


Answer. First three edges:

Start: Q = [((A,G),6),((E,F),7),((C,D),8),((B,E),10),((A,B),11),((B,G),13),((E,G),14),((F,G),14),((B,C),18),((D,E),25
Current individual trees: {A}, {B}, {C}, {D}, {E}, {F}, {G}
(A,G) is dequeued and selected because no other selected edges connect {A} and {G}.
Q = [((E,F),7),((C,D),8),((B,E),10),((A,B),11),((B,G),13),((E,G),14),((F,G),14),((B,C),18),((D,E),25)]
Current individual trees: {A,G}, {B}, {C}, {D}, {E}, {F}
(E,F) is dequeued and selected because no other selected edges connect {E} and {F}.
Q = [((C,D),8),((B,E),10),((A,B),11),((B,G),13),((E,G),14),((F,G),14),((B,C),18),((D,E),25)]
Current individual trees: {A,G}, {B}, {C}, {D}, {E,F}
(C,D) is dequeued and selected because no other selected edges connect {C} and {D}.
Q = [((C,D),8),((B,E),10),((A,B),11),((B,G),13),((E,G),14),((F,G),14),((B,C),18),((D,E),25)]
Current individual trees: {A,G}, {B}, {C,D}, {E,F}
...
Final order of added edges: (A,G), (E,F), (C,D), (B,E), (A,B), (B,C) □

5 Standard 9- Prim's Algorithm

Problem 4. Consider the weighted graph $G(V, E, w)$ below. Clearly list the order in which Prim's algorithm, **using the source vertex** A, adds edges to a minimum-weight spanning tree for G. Additionally, clearly articulate the steps that Prim's algorithm takes as it selects the first **three** edges.



Answer. First three edges:

Add vertex A's edges to the priority queue.

Q = [((A,G),6),((A,B),11)]

Current component containing A: {A}

Dequeue (A,G) and add it to the component containing A because it has only one vertex connected to the component containing A.

Add vertex G's unexplored edges to the priority queue.

Q = [((A,B),11),((B,G),13),((E,G),14),((F,G),14)]

Current component containing A: {A,G}

Dequeue (A,B) and add it to the component containing A because it has only one vertex connected to the component containing A.

Add vertex B's unexplored edges to the priority queue.

Q = [((B,E),10),((B,G),13),((E,G),14),((F,G),14),((B,C),18)]

Current component containing A: {A,G,B}

Dequeue (B,E) and add it to the component containing A because it has only one vertex connected to the component containing A.

Add vertex E's unexplored edges to the priority queue.

Q = [((E,F),7),((B,G),13),((E,G),14),((F,G),14),((B,C),18),((E,D),25)]

Current component containing A: {A,G,B,E}

...

Final order of added edges: (A,G), (A,B), (B,E), (E,F), (B,C), (C,D) □