Homework 4

CSCI 2824 Section 1
SID: 109722375
Alex Osmann

1\. a. $8 = 2^3$
$9 = 3^2$
8 and 9 are consecutive
QED
Disproof by counterexample

b. Let $a = 4x+1$ and $b = 4y+1$
$a \cdot b = (4x+1)(4y+1) = 16xy + 4x + 4y + 1$
let $z = 4xy + x + y$
$a \cdot b = 4z + 1$
QED
Direct Proof

c. Contrapositive: If $ax + by = e$ and $cx + dy = f$ can't be solved for x and y (real numbers), then a, b, c, d, e, and f aren't real numbers such that $ad - bc \neq 0$.
Let $x = mi$ and $y = ni$ (since they're nonreal numbers)
$ami + bni \neq e$ where a, b, e, m, and i are real constants because the addition of two complex numbers can't result in a real number
QED
Proof by contraposition

2\. $W(x)$ = "x is a wizard" $P(x)$ = "x likes to play"
$Q(x)$ = "x plays Quidditch" $R(x)$ = "x likes to relax" $S(x)$ = "x likes to sleep"
Given $\exists x (W(x) \wedge (R(x) \Rightarrow \neg Q(x)))$
$\neg Q(x) = \neg P(x)$ by modus tollens
$\exists x (W(x) \wedge (R(x) \Rightarrow \neg P(x)))$
$\neg P(x) \Rightarrow S(x)$ given $R(x)$ by disjunctive syllogism
$\exists x (W(x) \wedge (R(x) \wedge S(x)))$
QED
Direct Proof

~~3. a. Contrapositive~~

3\. a. $n = 2k$

$7n+4 = 14k+4 = 2(7k+2)$

$7n+4 = 2k$

$n = \frac{2k-4}{7} = 2\left(\frac{k-2}{7}\right)$ ← always even if n is an integer

QED

Direct proof

b. $n = 2k+1$

$5n+6 = 10k+11 = 2(5k+5)+1$

$5n+6 = 2k+1$

$n = \frac{2k-5}{5} = 2\left(\frac{k-5}{5}\right)+1$ ← always odd if n is an integer

QED

Direct proof

4\. ~~Contr~~ Contradiction: r is odd if p and q are even

$p = 2k \quad q = 2k$

$r = 4k^2 + 2k + 2k$

$= 2k(2k+2)$

an even number times an even number can't be odd

~~QED~~ contradiction: p and q are odd if r is even

$r = (2k+1)^2 + 2k+1 + 2k+1 = 4k^2+8k+3 =$

$= 2(2k^2+4k+1)+1$

2 times any integer plus 1 can't be even

QED

Proof by contradiction

5. $1^4=1$ $2^4=16$ $3^4=81$ $4^4=256$ $5^4=625$ $6^4=1296$
$7^4=2401$

We only need to check combinations of 1^4 to 6^4.

$1+1=1 \neq 2401$
$1+16=17 \neq 2401$
$1+81=82 \neq 2401$
$1+256=257 \neq 2401$
$1+625=626 \neq 2401$
$1+1296=1297 \neq 2401$
$16+81=97 \neq 2401$
$16+256=282 \neq 2401$
$16+625=641 \neq 2401$
$81+256=337 \neq 2401$
$81+625=706 \neq 2401$
$81+1296=1377 \neq 2401$
$256+625=881 \neq 2401$
$256+1296=1552 \neq 2401$
$625+1296=1921 \neq 2401$

QED

Proof by Exhaustion