Friday, September 22, 2023 2:06 PM

Warm-up

Given a function $f(x)$ & the goal of finding a fixed pt. What requirement do we have in gaurantee convergence to the unique fixed pt?

Soln: We need $|f'(x)|<1$ in a neighborhood of the fixed pt & our intial guess should be in that neighborhood.

Non-linear systems of equations: Fixed pt

$f(x,y)$ & $g(x,y)$ what is a fixed pt of the non-linear set of functions

$$\begin{bmatrix} f(x,y) \\ g(x,y) \end{bmatrix} \quad ?$$

Soln: (α, β) is a fixed pt if

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} f(\alpha,\beta) \\ g(\alpha,\beta) \end{bmatrix}$$

The fixed pt iteration $x_{n+1} = f(x_n)$

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} f(x_n, y_n) \\ g(x_n, y_n) \end{bmatrix} \quad \text{for} \quad n = 0, 1, 2, \ldots$$

How do we know when this will converge?

Answer: follow 1D initition:
Assume f & g are analytic (Nice enough!)

let $\begin{bmatrix} x_n \\ y_n \end{bmatrix} = \begin{bmatrix} \alpha + \Delta x_n \\ \beta + \Delta y_n \end{bmatrix}$