Pre-Midterm 2- Standards 10 and 17

Due Date ..... TODO
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Collaborators ..... **None**

Contents

1 Instructions

- The solutions may be typed or handwritten, using proper mathematical notation. If you handwrite your solutions, you must embed them as an image in the template and orient your image so we do not have to rotate our screens to grade it.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
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2 Honor Code (Make Sure to Virtually Sign)

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

(I agree to the above, Alex Ojemann). □

3 Standards 10 and S17- Classify Languages

3.1 Problem 1

Problem 1. Classify each language as regular (REG), nonregular but context-free (CF), or not context-free (NCF). Justify your answer (see below). **Note: if you are attempting S10 but not S17, classify as regular or non-regular only.**

For the justification, when showing a language is regular or context-free you can demonstrate using the usual constructions (DFA,NFA,regexp,CFG,PDA) and do not need to prove that your construction works. You could also use closure properties. To justify the claim that a language is non-regular or non-context-free, you can use e.g. closure properties or the pumping lemma (note: you will not need the CFL pumping lemma, but you may use it).

Recall that you may use any languages in the book, problem sets, or in-class exercises, which we have shown to be regular or non-regular, context-free or non-context-free. For example, you may assume the languages $\{a^n b^n : n \geq 0\}$, $\{a^n b^m : n \geq m\}$, $\{a^n b a^n : n \geq 0\}$ are context-free but nonregular, and the languages $\{a^n b^n a^n : n \geq 0\}$, $\{a^n b^n c^n : n \geq 0\}$, $\{a^n b a^n b a^n : n \geq 0\}$ are not context-free.

(a) $L = \{xy : x, y \in \{a, b, c\}^*, |x| = |y|\}$

Proof. L can be expressed as $((a + b + c)(a + b + c))^*$

This language is regular. □

(b) $L = \{xy : x \in \{a, b\}^*, y \in \{b, c\}^*, |x| = |y|\}$

Proof. Suppose L is regular.

$L \cap a^*c^*$ must also be regular because regular languages are closed under intersection.

$L \cap a^*c^* = a^n c^n$ where $n = |x| = |y|$, which isn't regular.

Therefore, L isn't regular.

However, L is context free because it can be represented by the following context free grammar.

S $->$ aSb | aSc | bSb | bSc | ϵ

Therefore, L is nonregular but context free. □

(c) $L = \{xy : x, y \in \{a, b, c\}^*, x = y\}$

Proof. Assume L is context-free.

k given where k ≥ 0

Let $z = a^k b^k c^k a^k b^k c^k$ where z = uvwxy, $vx \neq \epsilon$, and $|vwx| < k$.

Take i = 2:

If vwx is entirely contained in the first part of the string (the original x from the definition of L), then uv^2wx^2y would have more of at least one of as, bs or cs in the first part of the string than in the second part of the string.

If vwx is entirely contained in the second part of the string (the original y from the definition of L), then uv^2wx^2y would have more of at least one of as, bs or cs in the second part of the string than in the first part of the string.

If vwx contains part of both the first and second parts of the string, then uv^2wx^2y must have more cs in the first part than in the second part and more as in the second part than in the first part because $|vwx| < k$.

Therefore, in any case, $uv^2wx^2y \notin L$.

So, by the CFL pumping lemma, L is not context-free. □

(d) $L = \{xy : x \in \{a, b\}^*, y \in \{b, c\}^*, x = y\}$

Proof. x and y can only contain bs.

L can be expressed as $(bb)^*$

This language is regular. □