Secant method

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Key idea behind Newton:
iterates are the roots of the tangent line
$\Rightarrow$ we most have access to $f'(x)$ the derivative.

If we don't want to ask for derivative we make an iteration out of secant lines.

2 pts make a line. $(x_0, f(x_0))$ & $(x_1, f(x_1))$

What is the secant line through these pts?

$$y = m(x - x_1) + f(x_1)$$

$$= \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0} \right)(x - x_1) + f(x_1)$$

Solve for the root

$$x = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$[\;\;] = \frac{1}{m}$

Our iteration is given x_0 & x_1

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

Pseudocode: Secant Method

Input: x_0, x_1 - 2pts (initial guesses)
$f(x)$ - function
tol - tolerance
N_{max} - Max # of iterations

Output: p^* - approx. root

$$ier = \begin{cases} 1 & \text{failure} \\ 0 & \text{success} \end{cases}$$

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Steps:

Step 1: if $|f(p_0)| = 0$
    $p^* = p_0$
    $ier = 0$
    return
if $|f(p_1)| = 0$
    $p^* = p_1$; ier; return

Step 2: $fp_1 = f(p_1)$; $fp_0 = f(p_0)$

Step 3: for $j = 1, \cdots N_{max}$
    if $|fp_1 - fp_0| = 0$
        $ier = 1$
        $p^* = p_1$ (display "divide by 0 = BAD")
        return

$$P_2 = P_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)}$$

if $|p_2 - p_1| < tol$ $\left(\text{or } \frac{|p_2 - p_1|}{|p_2|} < tol\right)$

$p^* = p_2$; ier = 0 ; return

$$\left.\begin{array}{ll} p_0 = p_1 ; & f_{p_0} = f_{p_1} \\ p_1 = p_2 ; & f_{p_2} = f(p_2) \end{array}\right\} \text{update to march forward}$$

Step 4: $p^* = p_2$
ier = 1
return