APPM 4600 Project Proposal for Regularized Least Squares

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1 Introduction

Regularized least squares are a set of modifications to the least squares regression method that aim to constrain the basic least square regression and account for noise and other effects. We plan to compare Ridge Regression and Tikhonov regularization's effects on the least squares problem. To further our understanding of regularized least squares, we will be exploring the LASSO and elastic net constraints for the least squares problem.

2 Project Content

For our project, we plan on starting with the derivation of the ridge and Tikhonov regressions, before going into comparisons of their effects on the least squares regression. Specifically, we will first be testing how well the ridge regression model fits after sampling with Gaussian noise along a linear and a trigonometric equation. We will analyze errors in this method and explore how varying a constant to penalize the l_2-norm of x affects performance. Finally, we will show how Ridge Regression can improve a polynomial fit of an exponential function.

Then, we will be testing how well the Tikhonov regression model fits to a sine curve and comparing this to the ridge regression model. We will also be using the forward finite differences formula to penalize the Tikhonov model and comparing the results. Finally, we will apply LASSO regression and elastic net constraints to the least squares problem and comparing this method to the previous results.

We intend to derive the LASSO estimator and compare the feature coefficients when using ridge, Tikhonov and LASSO regression for each of the numerical examples in the project document. These examples include a linear function, a fifth order polynomial function and a sinusoidal function all with Gaussian noise included when sampling, for which we will fit a regression model using each of these techniques and compare the performance and sparsity of features.

After comparing the performance for relatively simple known functions with noise, we will apply these methods to a more complex least squares problem and comparing the behaviors of each regularization method. The problem we are considering is expanded on in Section 2.1.

2.1 Example

Suppose we have a data set with four features and one response variable that we want to fit a regression model to. Three of the features, x_1, x_2 and x_3 are truly correlated with the response and x_4, is not, but noise is added to all features. When we fit an ordinary least squares regression model and a regression model with two of the techniques, ridge and LASSO, to this data, here are the results. Both the ridge and LASSO models have an alpha value of 0.1.

Linear Regression Formula: $y = 6.09x_1 + 1.71x_2 + 0.91x_3 + -0.24x_4 - 0.19$

Ridge Regression Formula: $y = 5.34x_1 + 1.88x_2 + 1.05x_3 - 0.22x_4 - 0.00$

LASSO Regression Formula: $y = 4.17x_1 + 2.47x_2 + 0.00x_3 - 0.00x_4 + 0.41$

As shown, both the ridge and LASSO models have smaller feature coefficients than the ordinary least squares model and the LASSO model is more likely to drop a coefficient all the way to 0 as it did for x_3 and x_4. For the project report we can then compare these results to the actual behavior of the modeled function.

3 Timeline

We expect to be done with the derivation of the Ridge estimator and Tikhonov regressions the week the rough draft is due on Nov. 18, as well as some preliminary exploration applying these regressions to a theoretical problem. We expect to complete the theoretical derivation of the LASSO regularization before the rough draft is due, and include it in the rough draft.

We expect to complete the benchmark comparisons of approximating a sampled function with added noise of all 3 methods by Dec. 1, and to fully complete our analysis of the larger problem by Dec. 8, and use the remaining week before the final draft is due to finalize our report and practice for presentations.

Completion Date	Work Done
Nov. 10	Tikhonov and Ridge derivation rough draft
Nov. 17	Finish rough draft
Dec. 1	Finish analysis of simple functions
Dec. 8	Finish all analysis
Dec. 15	Finish final report and presentation

Table 1: Expected project schedule

4 Division of Work

Since this project will be exploring 3 major regularized least squares methods, we will be roughly dividing the work by the method examined. Alex Ojemann will provide the initial derivation and preliminary analysis of the LASSO regression, Luke Stuckenbruck will provide the analysis of the Ridge method, and Steven Liu will provide analysis of Tikhonov's Regularization. Each member is responsible for the code written for numerical analysis of their method. Since the LASSO

regularization will likely take more work to complete, we will likely work together on analysis for LASSO as the Tikhonov and Ridge methods are finalized.

The written reports will be a collaborative effort between all group members, with each member focusing on writing the sections corresponding to their assigned method of regularization.