Homework 1
UMN STAT 5511 (Fall 2024)
Charles R. Doss
Assigned: Mon, September 23
Due: Mon, September 30

The usual formatting rules:

- Your homework (HW) should be formatted to be easily readable by the grader.
- You may use knitr or Sweave in general to produce the code portions of the HW. However, the output from knitr/Sweave that you include should *be only what is necessary to answer the question*, rather than just any automatic output that R produces. (You may thus need to avoid using default R functions if they output too much unnecessary material, and/or should make use of `invisible()` or `capture.output()`.)
  - For example: for output from regression, the main things we would want to see are the estimates for each coefficient (with appropriate labels of course) together with the computed OLS/linear regression standard errors and p-values. If other output is not needed to answer the question, it should be suppressed!
- Code snippets that directly answer the questions can be included in your main homework document; ideally these should be preceded by comments or text at least explaining what question they are answering. Extra code can be placed in an appendix.
- All plots produced in R should have appropriate labels on the axes as well as titles. Any plot should have explanation of what is being plotted given clearly in the accompanying text.
- Plots and figures should be *appropriately sized,* meaning they should not be too large, so that the page length is not too long. (The arguments fig.height and fig.width to knitr chunks can achieve this.)
- **Directions for "by-hand" problems:** In general, credit is given for (correct) shown work, not for final answers; so show **all** work for each problem and explain your answer fully.

Further instructions:

- You may see `acf` and/or `astsa::acf1`.

Questions:

1. (MA process and ACF) Shumway and Stoffer, Question 1.7. (Page 39.) In addition to plotting the true ACF, you will plot a sample ACF: Generate $n = 100$ observations for $\{W_t\} \sim WN(0, 1)$ (white noise with mean 0 and variance 1). Compute and plot the sample ACF for $X_t = W_{t+1} + 2W_t + W_{t-1}$, and (as the book asks) also plot the true ACF. Compute the sample ACF "by hand".[1] Either make two plots, one above the other (you can use `par(mfrow=)` for instance), or put the two functions on the same plot (visually distinguished in some way).

   [Note: you can use the "type" argument with the plot function to change the way the plot is drawn.]

2. Shumway & Stoffer, Question 2.1 (page 70, 4th ed.) You can skip the part of the question in (c) that asks "And, by what percentage does it increase or decrease."

3. (Stationarity) For each of the following, state if it is a stationary process. If so, give the mean and autocovariance functions. Here, $\{W_t\}$ is i.i.d. $N(0, 1)$.

   (a) $X_t = W_t - W_{t-3}$.
   (b) $X_t = W_3$.
   (c) $X_t = t + W_3$.
   (d) $X_t = W_t^2$.
   (e) $X_t = W_t W_{t-2}$.

4. Pick *one* of the following two questions to do:

   - (White noise is not necessarily i.i.d.). Suppose that $\{W_t\}$ and $\{Z_t\}$ are independent and identically distributed (i.i.d.) sequences, also independent of each other, with $P(W_t = 0) = P(W_t = 1) = 1/2$ and $P(Z_t = -1) = P(Z_t = 1) = 1/2$. Define the time series X_t by $X_t = W_t(1 - W_{t-1})Z_t$. Show that $\{X_t\}$ is white but not i.i.d.

[1] Here "by hand" means: do not use acf(), but rather write your own code. (It does not mean that you should actually use a calculator.) You could use acf() to check your results for your own edification, but this should not be included in the output.

- Shumway and Stoffer, Question 1.25 (page 42).

5. ("Plot your data!" [This question is due to Andy Poppick.]) Find the file "mystery.csv" which contains a data series of mysterious origin.

   (a) Plot the sample ACF of the data series. Based on the sample ACF, suggest a model for these data.

   (b) Plot your data: make a time series plot (a plot of the series over time) and a "lag-1" plot (meaning plot X_t against X_{t-1}). Do you still believe your answer to the previous part?