1.8: 1.b. $\left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 1 & 4 & -2 & -3 \end{array}\right] -2R2$ $\boxed{\vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix} x_3}$

$\left[\begin{array}{ccc|c} 0 & -7 & 7 & 7 \\ 1 & 4 & -2 & -3 \end{array}\right]$

$x_3 = x_2 - 1$
$x_1 = 2x_3 - 4(x_3 - 1) - 3$

2.f. $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 9 & 8 \\ 0 & 1 & 2 & 8 & 7 \\ -3 & 0 & 1 & -7 & 9 \end{array}\right] -R2$ $\boxed{\vec{x} = \begin{bmatrix} 1 \\ 19 \\ -6 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} x_4}$

compatible

$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 8 & 7 \\ -3 & 0 & 1 & -7 & 9 \end{array}\right] +3R1$

$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 2 & 8 & 7 \\ 0 & 0 & -2 & -4 & 12 \end{array}\right] +R3$

$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 4 & 19 \\ 0 & 0 & -2 & -4 & 12 \end{array}\right] -\frac{1}{2}R3$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 1 \\ 0 & 1 & 0 & 4 & 19 \\ 0 & 0 & -2 & -4 & 12 \end{array}\right]$

4. i. $\det(A) \neq 0$ $b(2a-2b) \neq 0$ all values where $b \neq 0$ and $a \neq b$
ii. all values where $a = b$ other than $a = b = 0$
iii. $a = b = 0$

7.h. S

1.8: 12. The determinant is 0.

22, a. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t$

23. $\left[\begin{array}{ccc|c} 0 & 2 & -1 & 0 \\ -2 & 0 & 3 & 0 \\ 1 & 3 & 0 & 0 \end{array}\right] +2R3$

$\left[\begin{array}{ccc|c} 0 & 2 & -1 & 0 \\ 0 & 6 & 3 & 0 \\ 1 & 3 & 0 & 0 \end{array}\right] -3R1$

$\left[\begin{array}{ccc|c} 0 & 2 & -1 & 0 \\ 0 & 0 & 6 & 0 \\ 1 & 3 & 0 & 0 \end{array}\right]$

$6x_3 = 0 \quad x_3 = 0$

$2x_2 - (0) = 0 \quad x_2 = 0$

$x_1 + 3(0) = 0 \quad x_1 = 0$

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

1.9: 1.a. $\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} +2R1$

$\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$ Det = 2

1.c. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 10 \end{bmatrix} -2R1$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 8 & 10 \end{bmatrix} -3R1-2R2$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{bmatrix}$

det = -3

1.9: 4, a. True because inverse only doesn't exist when det = 0

4, g. True because the RREF of a nxn matrix must have a row of 0s if det = 0.

5. $\det(B) = \det(S^{-1}AS)$

$= \det(S^{-1})\det(A)\det(S)$

$= \det(A)\det(S^{-1})\det(S)$

$= \det(A)\det(S^{-1}S)$

$= \det(A)\det(I)$

$\det(B) = \det(A)$

6. $\det(cA) = \det(cI)\det(A)$

$\det(cI) = c \times c \times c \times \dots c = c^n$

(n times because cs on diagonal of nxn matrix)

$\det(cA) = c^n \det(A)$

2.1: 6, a. $a\begin{bmatrix} 0 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$a = -4 \quad b = 3$

b. $a\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}^2 + b\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + c = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$

$a = -3 \quad b = 1 \quad c = 1$

2.1: 10. Addition: Adding any infinite real sequence to another infinite real sequence results in an infinite real sequence.

Scalar Mult: Multiplying a scalar by an infinite real sequence results in an infinite real sequence

12. Assume 0 and $0'$ are both zeroes of a vector space

$v + 0 = v + 0'$

Add v's additive inverse to both sides

$v + 0 + (-v) = v + 0' + (-v) = 0 = 0'$

$0 = 0'$

So all zero vectors are the same, and since all vector spaces must have a zero vector, they all have a unique zero vector

13. a. Zero: (0 from V, 0 from W) $= \vec{0}$.

Addition: $(v_i, w_i) + (v_j, w_j) = (v_i + v_j, w_i + w_j) \in V \times W$

($v_i + v_j \in V$ and $w_i + w_j \in W$ because V and W are vector spaces)

Scalar mult: $c(v_i, w_i) = (cv_i, cw_i) \in V \times W$

b. Because $\mathbb{R} \times \mathbb{R}$ is the set of all pairs of two real numbers which is the definition of $\mathbb{R}^2$.

2.2: 2.d. Yes because zero is included, if you add them the first component is still 0, and any scalar times 0 is still 0.

2.e. No because if you add them the last ~~compo~~ component becomes 2.

2.f. Yes because $(0\ 0\ 0)^T$ satisfies $x \geq y \geq z$, and if you add them together or multiply by a scalar $x \geq y \geq z$ remains true.

2.2: 6.a. Yes because $v_i + v_j$ could not be a member of S

6.b Yes because cv could not be a member of S

10. b. No (addition fails) d. Yes e. No (addition fails) g. Yes

15. a. Yes e. No (scalar mult fails) g. Yes

17. Zero: $u'' = 0$ ✓

Addition: $u'' = xu_1 + xu_2 \in$ space ✓

Scalar mult: $u'' = cxu \in$ space ✓

19. a. Zero: $([0,0],(0,0))$ ✓

add: $([a,b],(c,d)) + ([e,f],(g,h)) \in$ space ✓

scalar: $c([a,b],(c,d)) \in$ space ✓

b. Yes