Homework 2

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1.9: 11.a. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ d. $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

14. a. True b. False c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} = I$ Fals

16. a. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ b. True c. True

21. a. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$ $x_2 = 3$
$2x_1 - 3 = 2$ $x_1 = 5/2$

b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 7 \\ 0 & 0 & -4 \end{bmatrix}$ $-4x_3 = 1$ $x_3 = -1/4$
$x_2 + 7(-1/4) = -1$ $x_2 = 3/4$
$x_1 + 2(3/4) + 3(-1/4) = 2$ $x_1 = 5/4$

c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 9 \end{bmatrix}$ $9x_3 = 0$ $x_3 = 0$
$x_2 - 3(0) = 1$ $x_2 = 1$
$x_1 + 2(0) = -1$ $x_1 = -1$

23. First: Permutations: $R2 \Rightarrow R1$, $R3 \Rightarrow R2$, $R1 \Rightarrow R3$ Row ops: $R3 = R3 + R1 - R2$ valid ✓

Second: Permutations: $R3 \Rightarrow R1$, $R1 \Rightarrow R3$ Row ops: $R2 = R2 + R1$, $R3 = R3 + R2$ Valid ✓

Third: Permutations: $R3 \Rightarrow R1$, $R1 \Rightarrow R2$, $R2 \Rightarrow R3$ Row ops: $R3 = R3 + R1 + R2$ Valid ✓

1.5: 1. a. $\begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & -3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. a. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ -b & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ -b & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ a & 1 & 0 & 0 & 1 & 0 \\ b & c & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R1 \\ -cR2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ a & 1 & 0 & 0 & 1 & 0 \\ b-ca & 0 & 1 & 0 & -c & 1 \end{array}\right] \begin{array}{l} \\ -aR1 \\ -(b-ca)R1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -a & 1 & 0 \\ 0 & 0 & 1 & ca-b & -c & 1 \end{array}\right] \qquad M^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ca-b & -c & 1 \end{bmatrix}$$

5. This is because there isn't a matrix you can multiply it by that will result in a 1 on the diagonal in the row where the original matrix had all zeroes.

12. $(A^2 - 3A + I = 0)A^{-1}$

$A(AA^{-1}) - 3(AA^{-1}) + IA^{-1} = 0$

$A - 3I + A^{-1} = 0$

$\boxed{A^{-1} = 3I - A}$

1.5: 19. a. $S = I$

$I^{-1}AI = IAI = A$

b. $(A = S^{-1}BS)S^{-1}$

$S(AS^{-1} = S^{-1}BSS^{-1})$

$SAS^{-1} = SS^{-1}BSS^{-1} = B$

25. a. $\left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 3 & -3 & 0 & 1 \end{array}\right] -3R1$

$\left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 1 \end{array}\right] +\frac{2}{3}R2$

$\left[\begin{array}{cc|cc} 1 & 0 & -1 & \frac{2}{3} \\ 0 & 1 & -3 & 1 \end{array}\right]$

b. $\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right] -3R1$

$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -8 & -3 & 1 \end{array}\right] +\frac{3}{8}R2$

$\left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{8} & \frac{3}{8} \\ 0 & -8 & -3 & 1 \end{array}\right] /-8$

$\left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{8} & \frac{3}{8} \\ 0 & 1 & \frac{3}{8} & -\frac{1}{8} \end{array}\right]$

c. $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 9 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -2R1 \\ -3R1 \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 0 & -2 & 1 & 0 \\ 4 & 2 & 0 & -3 & 0 & 1 \end{array}\right] -2R2$

$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{array}\right]$

Not Linvertible

1.6: 7. $\begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$\begin{bmatrix} a^2+c^2 & ab+cd \\ ab+cd & b^2+d^2 \end{bmatrix} = \begin{bmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{bmatrix}$

All 2x2 matrices of the form $\begin{bmatrix} a & b \\ -b & d \end{bmatrix}$

1.6: 10. $v = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $w = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$ $vw^T = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} x_3 & x_4 \end{bmatrix} = \begin{bmatrix} x_1x_3 & x_1x_4 \\ x_2x_3 & x_2x_4 \end{bmatrix}$

$wv^T = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} x_1x_3 & x_2x_3 \\ x_1x_4 & x_2x_4 \end{bmatrix}$ False

12. a. ~~Because~~ a $m \times n$ times ~~a~~ $1 \times n$ results in an $m \times 1$ and since the only entry is a 1 in the jth column that's the only part of the result that isn't 0.

b. Because the size ends up being 1×1 and a_{ij} is the only element present in A, e_i, and e_j

4. a. Because flipping the entries about the diagonal results in the opposite row operation which is the inverse (ex. R2→R1, R3→R2, R1→R3 becomes R1→R2, R2→R3, R3→R1).

17 b. $a = -1$ $b = 2$ $c = 3$

25. d. $\begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$