Homework 14

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8.5: 1.d. $(1-\lambda)(1-\lambda)(1-\lambda) - 9(1-\lambda) - 4(4(1-\lambda)) = 0$

$(1-\lambda)(\lambda^2-2\lambda-24)=0$

$(1-\lambda)(\lambda-6)(\lambda+4)$

$\boxed{\lambda = 1, 6, -4}$

$\lambda=6$: $\begin{bmatrix} -5 & 0 & 4 \\ 0 & -5 & 3 \\ 4 & 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $x = \frac{4z}{5}$ $y = \frac{3z}{5}$ $v_1 = \begin{bmatrix} 4/5 \\ 3/5 \\ 1 \end{bmatrix} = \begin{bmatrix} 4/5\sqrt{2} \\ 3/5\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

$\lambda=1$: $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 3 \\ 4 & 3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $4z=0$ $4x+3y=0$ $v_2 = \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} -3/5 \\ 4/5 \\ 0 \end{bmatrix}$

$\lambda=-4$: $\begin{bmatrix} 5 & 0 & 4 \\ 0 & 5 & 3 \\ 4 & 3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $5x=-4z$ $y=-3z/5$ $\begin{bmatrix} -4/5 \\ -3/5 \\ 1 \end{bmatrix} = \begin{bmatrix} -4/5\sqrt{2} \\ -3/5\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

5.a. Given matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\lambda^2-(a+d)\lambda+(ad-bc)=0$

For real values of λ: $(a+d)^2-4(ad-bc) \geq 0$

$a^2+d^2+2ad-4ad+4bc \geq 0$

$(a-d)^2+4bc \geq 0$ this ensures that

b. Symmetric matrix: $b=c$

$(a-d)^2+4b^2 \geq 0$

$(a-d)^2$ and $4b^2$ are always ≥ 0, so their sum is also always ≥ 0

c. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

15. b. $D = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1/2 & 3/2 \\ 3/2 & -1/2 \end{bmatrix}$

8.5 15.d. $D=\begin{bmatrix}2&0\\0&2\end{bmatrix}$ $P=\begin{bmatrix}2&1\\1&2\end{bmatrix}$

$A=\begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}2&0\\0&2\end{bmatrix}\begin{bmatrix}2&1\\1&2\end{bmatrix}^{-1}=\begin{bmatrix}2&0\\0&2\end{bmatrix}$

22. $(2-\lambda)(4-5x+\lambda^2)=0$

$(2-\lambda)(\lambda-4)(\lambda-1)=0$

$\lambda=1,2,4$

$\lambda=2$: $\begin{bmatrix}0&1&0\\1&1&1\\0&1&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$ $v_1=\begin{bmatrix}1\\0\\-1\end{bmatrix}$

$\lambda=4$: $\begin{bmatrix}-2&1&0\\1&-1&1\\0&1&-2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$ $v_2=\begin{bmatrix}1\\2\\1\end{bmatrix}$

$\lambda=1$: $\begin{bmatrix}1&1&0\\1&2&1\\0&1&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$ $v_3=\begin{bmatrix}-1\\0\\-1\end{bmatrix}$

8.7 1. d. $AA^T=\begin{bmatrix}2&0&0\\0&3&0\end{bmatrix}\begin{bmatrix}2&0\\0&3\\0&0\end{bmatrix}=\begin{bmatrix}4&0\\0&9\end{bmatrix}$

$(4-\lambda)(9-\lambda)=0$ $\lambda=4,9$

Singular eigenvalues $=\sqrt{4},\sqrt{9}=2,3$

e. $AA^T=\begin{bmatrix}2&1&0&-1\\0&-1&1&1\end{bmatrix}\begin{bmatrix}2&0\\1&-1\\0&1\\-1&1\end{bmatrix}=\begin{bmatrix}6&-2\\-2&3\end{bmatrix}$

$\lambda^2-9\lambda+14=0$ $(\lambda-7)(\lambda-2)=0$ $\lambda=2,7$

Singular eigenvalues $=\sqrt{2},\sqrt{7}$

8.7.12. $Ax = \lambda x$ where λ is the singular value and x is the eigenvector

$A^{-1}Ax = A^{-1}\lambda x$

$x = \lambda A^{-1}x$

$A^{-1}x = \frac{1}{\lambda}x$

thus the singular value of A is $1/\lambda$.

The condition number of A is $\|A\|\,\|A^{-1}\|$

Thus the condition number of $A^{-1} = \|A^{-1}\|\,\|(A^{-1})^{-1}\| = \|A\|\,\|A^{-1}\|$

They're the same.

18. True

34.b. $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

$A^TAx = A^Tb$

$\begin{bmatrix} 5 & -1 & 3 \\ -1 & 2 & 0 \\ 5 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 7 \end{bmatrix} \quad -R1$

$\left[\begin{array}{ccc|c} 5 & -1 & 3 & 9 \\ -1 & 2 & 0 & 3 \\ 0 & 0 & 0 & -2 \end{array}\right] \quad +1/5 R1$

$\left[\begin{array}{ccc|c} 5 & -1 & 3 & 9 \\ 0 & 9/5 & 3/5 & 24/5 \\ 0 & 0 & 0 & -2 \end{array}\right]$

$x_1 = 9/5 - 3/5\,x_3 + 1/5\,x_2$

$x_2 = 8/3 - 1/3\,x_3$

x_3 is free

8.7.36. $Ax = b$

$$A^{-1}Ax = A^{-1}b$$

$$x = A^{-1}b$$

$$x = A^{+}b, \text{ because } A^{+} = A^{-1}$$

If x_1^* and x_2^* are two solutions of $Ax = b$,

$$x_1^* = A^{+}b \quad x_2^* = A^{+}b$$

$$x_1^* = x_2^*$$

Unique solution.