Homework 12

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14.c.

t_i	-1	0	1
y_i	1	2	-1

$$y(t) = \frac{(t-t_1)(t-t_2)}{(t_0-t_1)(t_0-t_2)} y_0 + \frac{(t-t_0)(t-t_2)}{(t_1-t_0)(t_1-t_2)} y_1 + \frac{(t-t_0)(t-t_1)}{(t_2-t_0)(t_2-t_1)} y_2$$

$$y(t) = \frac{(t-1)t}{2} - 2(t-1)(t+1) - \frac{(t+1)t}{2}$$

$$= \frac{t^2 - t - 4t^2 + 4 - t^2 - t}{2} = \boxed{-2t^2 - t + 2}$$

34.a. Let $x_{-1} = x_0 - h$, $x_0 = x_0$, $x_1 = x_0 + h$

x_{-1}	x_0	x_1
y_{-1}	y_0	y_1

$$P_2(x) = L_0(x) y_{-1} + L_1(x) y_0 + L_2(x) y_1$$

$$= \frac{(x-x_0)(x-x_1)}{(x_{-1}-x_0)(x_{-1}-x_1)} y_{-1} + \frac{(x-x_{-1})(x-x_1)}{(x_0-x_{-1})(x_0-x_1)} y_0 + \frac{(x-x_{-1})(x-x_0)}{(x_1-x_{-1})(x_1-x_0)} y_1$$

$$= \frac{(x-x_0)(x-x_0-h)}{2h^2} y_{-1} + \frac{(x-x_0+h)(x-x_0-h)}{-h^2} y_0 + \frac{(x-x_0+h)(x-x_0)}{2h^2} y_1$$

$$f'(x) = P_2'(x) = \boxed{\frac{2x-2x_0-h}{2h^2} y_{-1} - \frac{2x-2x_0}{h^2} y_0 + \frac{2x-2x_0+h}{2h^2} y_1}$$

b. $$f''(x) = P_2''(x) = \boxed{\frac{1}{h^2} y_{-1} - \frac{2}{h^2} y_0 + \frac{1}{h^2} y_1}$$

41. $1 = a\cos(0) + b\sin(0)$

$.5 = a\cos(\pi/2) + b\sin(\pi/2)$

$.25 = a\cos(\pi$

$$\begin{bmatrix} 1 \\ .5 \\ .25 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$b = .5$

$a = \frac{(1-.25)}{2} = .375$ $\boxed{\begin{bmatrix} .375 \\ .5 \end{bmatrix}}$

5.5. 47. a. $y = a + bt$

$E = \int_0^1 (t^{1/3} - a - bt)^2\,dt$

$\frac{dE}{da} = 0 \quad \int_0^1 a + bt\,dt = \int_0^1 t^{1/3}\,dt$

$a + b/2 = 3/4$

$\frac{dE}{db} = 0 \quad \int_0^1 (a + bt)t\,dt = \int_0^1 t^{4/3}\,dt$

$a/2 + b/3 = 3/7$

$-\frac{1}{2}(a + b/2 = 3/4)$

$b/12 = 3/56 \quad b = 12/56 = 9/14$

$a + 9/28 = 3/4 \quad a = 12/28 = 3/7$

$\boxed{3/7 + 9t/14}$

6. 2. $e^{k\pi i} = \cos k\pi + i \sin k\pi$

$= \cos k\pi + 0$

if k is odd $\cos(k\pi) = -1$

if k is even $\cos(k\pi) = 1$

thus $e^{k\pi i} = (-1)^k$

16. a. $e^{2i\theta} = \cos 2\theta + i \sin 2\theta$

$(e^{i\theta})^2 = (\cos\theta + i\sin\theta)^2 = \cos^2\theta - \sin^2\theta + 2i\cos\theta\sin\theta$

$\cos 2\theta$ = real part of $\cos^2\theta - \sin^2\theta + 2i\cos\theta\sin\theta = \cos^2\theta - \sin^2\theta$

$\sin 2\theta$ = imaginary part of $\cos^2\theta - \sin^2\theta + 2i\cos\theta\sin\theta\,/\,i = 2\cos\theta\sin\theta$

26. d. $\begin{bmatrix} 2+i & 4-3i & 2i \\ -i & 1 & 1-5i \end{bmatrix} \begin{matrix} (-2-i) \\ (i) \end{matrix}$

$\begin{bmatrix} 5 & -11+2i & -4+2 \\ -1 & i & i-5 \end{bmatrix} +\frac{1}{5}R1$

$\begin{bmatrix} 5 & -11+2i & -4+2 \\ 0 & -\frac{11}{5}+\frac{7}{5} & \frac{4}{5}-\frac{23}{5} \end{bmatrix} \begin{matrix} (-11-2i) \\ (-\frac{11}{5}-\frac{7}{5}) \end{matrix}$

$\begin{bmatrix} -51+10i & 125 & 30+40i \\ 0 & 170/5 & \frac{38}{5}-8i \end{bmatrix}$ can't be linearly dependent because second vector has 1st component of 0 and first doesn't

(linearly independent)

26 27. $a\begin{bmatrix} z \\ \bar{z} \end{bmatrix} = a\begin{bmatrix} a+ib \\ a-ib \end{bmatrix} \in S$ $z = (a+ib)$ $a = c+id$

$(c+id)\begin{bmatrix} a+ib \\ a-ib \end{bmatrix}$

$\begin{bmatrix} ca+cib+aid-db \\ ca-cib+aid+db \end{bmatrix} \not\in S$

this is not a subspace of $\mathbb{C}^2$

False

31. $\begin{bmatrix} x+iy \\ x-iy \end{bmatrix} + R1$

$\begin{bmatrix} x+iy \\ 2x \end{bmatrix} - \tfrac{1}{2}R2$

$\begin{bmatrix} iy \\ x \end{bmatrix} \cdot -i$

$\begin{bmatrix} y \\ x \end{bmatrix}$ thus x and y must be linearly independent for v and $\bar{v}$ to be linearly independent,

36. a. bilinear $\langle cu+dv, w\rangle = c(u_1w_1+v_1w_1)+2c(u_2w_2+v_2w_2) - c(u_1w_1, v_1w_1)$

Symmetric $\langle v,w\rangle = v_1w_1+2v_2w_2 = w_1v_1+2w_2v_2 = \langle w,v\rangle$

Positive: $v_1w_1+2v_2w_2 \geq 0$

b. bilinear $\langle cu+dv, w\rangle = c(u_1w_1+v_1w_1)+c(iu_1w_2+iv_1w_2)+c(iu_2w_1+iv_2w_1) + 2c(u_2w_2+v_2w_2) = c(\langle u,w\rangle + \langle v,w\rangle)$

Symmetric: $\langle v,w\rangle = v_1w_1+iv_1w_2-iv_2w_1+2v_2w_2 = \langle w,v\rangle$

Positivity: $v_1w_1+iv_1w_2-iv_2w_1+2v_2w_2 \geq 0$

3.6 #4.c. $A=\begin{bmatrix} a+bi & c+di \\ e+fi & g+hi \end{bmatrix}$ $B=\begin{bmatrix} j+ki & l+mi \\ n+oi & p+qi \end{bmatrix}$

$(AB)^{*}=\begin{bmatrix} (aj+cn-bk-do)+(ak+bj+co+dn)i & (ej+gn-fk-ho)+(ek+fj+go+hn)i \\ (al+cp-bm-dq)+(am+bl+cq+dp)i & (el+gp-fm-hq)+(em+fl+gq+hp)i \end{bmatrix}$

$=\begin{bmatrix} j-ki & n-oi \\ l-mi & p-qi \end{bmatrix}\begin{bmatrix} a-bi & e-fi \\ c-di & g-hi \end{bmatrix}=B^{*}A^{*}$