Problem Set 1

Due Date ..... September 6, 2022
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Collaborators ..... **None**

Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. (See this short intro to LaTeX plus other resources on Canvas.)
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

(I agree to the above, Alex Ojemann). □

3 Standard 5: DFAs/NFAs

Problem 1. Complete the relevant problems on Automata Tutor. You need not submit anything on the PDF.

4 Standard 6: Product Constructions

Problem 2. Consider the following two deterministic finite automata.

	a	b
$\rightarrow 1$	1	2
2F	2	1

	a	b
$\rightarrow 1$	2	3
2	3	1
3F	1	2

Do the following. [**Note:** You may either draw a diagram or a table. If you draw a diagram, you may embed it as an image, provided we do not have to rotate our screens to grade your work.]

4.1 Part (a)

(a) Use the product construction to produce deterministic finite automata accepting the intersection of the two languages accepted by these automata.

Answer.

	a	b
$\rightarrow \{1,1\}$	{1,2}	{2,3}
{2,1}	{2,2}	{1,3}
{1,2}	{1,3}	{2,1}
{2,2}	{2,3}	{1,1}
{1,3}	{1,1}	{2,2}
{2,3}F	{2,1}	{1,2}

□

4.2 Part (b)

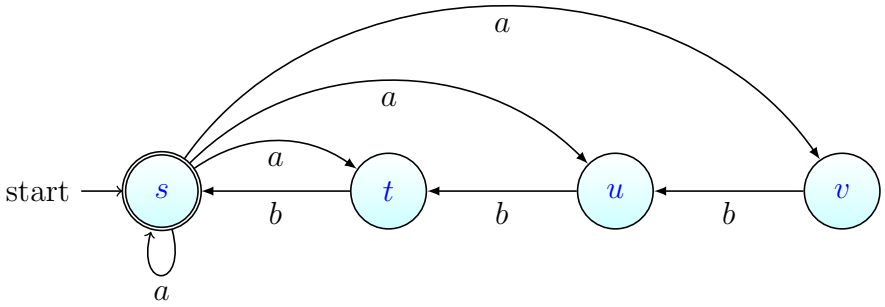
(b) Use the product construction to produce deterministic finite automata accepting the union of the two languages accepted by these automata.

Answer.

	a	b
$\rightarrow \{1,1\}$	{1,2}	{2,3}
{2,1}F	{2,2}	{1,3}
{1,2}	{1,3}	{2,1}
{2,2}F	{2,3}	{1,1}
{1,3}F	{1,1}	{2,2}
{2,3}F	{2,1}	{1,2}

□

Problem 3. Consider the following NFA.



Do the following.

(a) Give a string beginning with a that is *not* accepted.

Answer. abbbb is not accepted because it transitions from s to v to u to t to s then gets another b at state s which doesn't have an output state. □

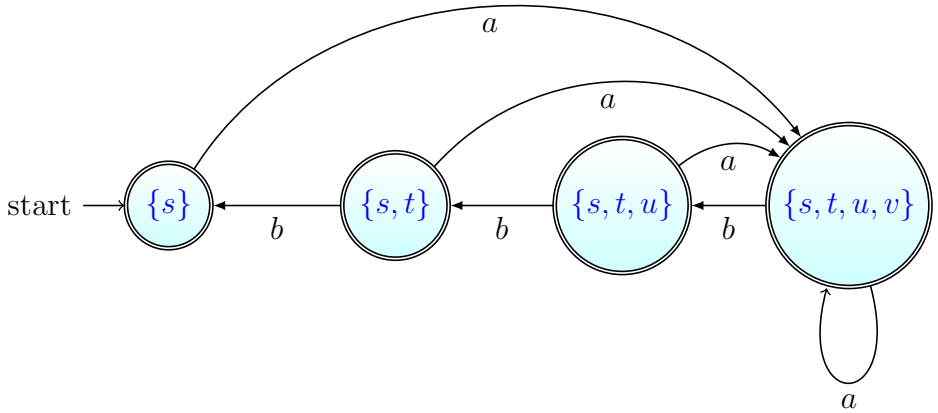
(b) Construct an equivalent deterministic automaton using the subset construction. Clearly indicate which subset of $\{s, t, u, v\}$ corresponds with each state of the deterministic automaton. Omit inaccessible states.

Answer. NFA:

	a	b
$\rightarrow sF$	s,t,u,v	None
t	None	s
u	None	t
v	None	u

DFA:

	a	b
$\rightarrow \{s\}F$	{s,t,u,v}	None
{s,t,u,v}F	{s,t,u,v}	{s,t,u}
{s,t,u}F	{s,t,u,v}	{s,t}
{s,t}F	{s,t,u,v}	s



□

5 Standards 11 and 2 & 3

Problem 4. Let Σ be an alphabet. The *reverse* of a string x, denoted $\mathbf{rev}\, x$, is x written backwards. Formally,

$$\mathbf{rev}\, \varepsilon := \varepsilon \quad \text{and } \mathbf{rev}\, xa := a\mathbf{rev}\, x.$$

For example, $\mathbf{rev}\, abbaaab = baaabba$. For a language $A \subseteq \Sigma^*$, define

$$\mathbf{rev}\, A := \{\mathbf{rev}\, x \mid x \in A\}.$$

For example, $\mathbf{rev}\, \{a, ab, aab, aaab\} = \{a, ba, baa, baaa\}$. Do the following.

5.1 Standard 11: Novel Constructions

(a) Suppose that A is regular. Carefully explain how to either construct (i) a finite state automaton or (ii) a regular expression to accept $\mathbf{rev}\, A$.
[**Hint:** You can start with a finite automaton / regular expression for A (why?), and then describe how to modify it in some way to obtain one for $\mathbf{rev}\, A$.]
[**Note:** To earn full credit for Standard 11, your construction should be correct. However, you do not have to prove that your construction is correct to earn credit for Standard 11.]

Answer. Steps to reverse a finite automaton:

Step 1: Make the final state of the original FA the initial state and the initial state of the original FA the final state.

Step 2: Reverse the direction of the transitions. That is, if $\delta(x, y) = z$, where $x \in Q$, $y \in \Sigma$ and $z \in Q$, then define the new transition function $\delta'(z, y) = x$. Repeat this for every input of the original transition function δ.

Step 3: Remove any inaccessible states. □

5.2 Standards 2 & 3: Proofs

(b) Provide a proof of correctness for your construction in part (a). That is, let L be the language accepted by your construction in part (a). Prove that $L = \mathbf{rev}\, A$.

Proof. Given that the initial language A is regular we know that L must also be regular.

Inductive Hypothesis: Let x be a string in a. Let y be the last character in that string and x' equal x without the last letter. Suppose that $\mathbf{rev}\,(\mathbf{rev}\,(x)) = x$.

Base Case: $x = \epsilon$. $rev\epsilon = \epsilon$ so $\mathbf{rev}\,\mathbf{rev}\,\epsilon = \epsilon$.

Inductive Step: $\mathbf{rev}\,(\mathbf{rev}\,(x)) = \mathbf{rev}\,(y\mathbf{rev}\,(x')) = \mathbf{rev}\,(\mathbf{rev}\,(y)\mathbf{rev}\,(x')) = x'y = x$ □