S21 retake

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1 Problem (Quiz 13)

Analyze the runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) \in \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops.

```
1: procedure foo(integer n):
2:   for i = 1, i <= 2*n
3:       i = i + 1
4:       for j = 1, j <= i
6:           j = j + 1
7:           print ‘foo’
```

2 Original Solution

Answer. The first loop takes one step to initialize.

For each iteration of the first loop we have two steps for the calculation and comparison on line 2, two steps for the calculation and assignment on line 3, and one step for the initialization of the second loop on line 4, so five steps per iteration. Since it iterates 2n times the total steps from the loop is 10n.

For each iteration of the second loop we have one step for the comparison on line 4, two steps for the calculation and assignment on line 6 and one step for the print statement on line 7, so four steps per iteration. Since it iterates $2n * i$ times and the average value of i during the first loop is $n + 0.5$ the total steps from the loop is $8n^2 + 4n$.

So we have that $T(n) = 1 + 14n + 8n^2$

Thus, $T(n) \in \Theta(n^2)$. □

3 Revised Solution

Answer.

Vertex:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of Paths:	20	17	11	5	3	1	3	3	3	2	1	1	1	1

Let P(x) represent the number of paths from vertex x to vertex 14.

Vertex 14: P(14) = 1

Vertex 13: P(13) = P(14) = 1

Vertex 12: P(12) = P(13) = P(14) = 1

Vertex 11: P(11) = P(14) = 1

Vertex 10: P(10) = P(11) + P(13) = P(14) + P(14) = 2

Vertex 9: P(9) = P(10) + P(12) = P(11) + P(13) + P(13) = P(14) + P(14) + P(14) = 3

□

4 Reflections

The mistake I made was misreading this problem and believing that we were supposed to calculate all the paths from vertex 1 to each other vertex as opposed to calculating the number of paths from each vertex to standard 14.

I believe that my understanding of the recurrences was good when I attempted the original problem and that was confirmed when I reworked the problem in the same manner but finding the paths to vertex 14 instead of from vertex 1.