S21 retake

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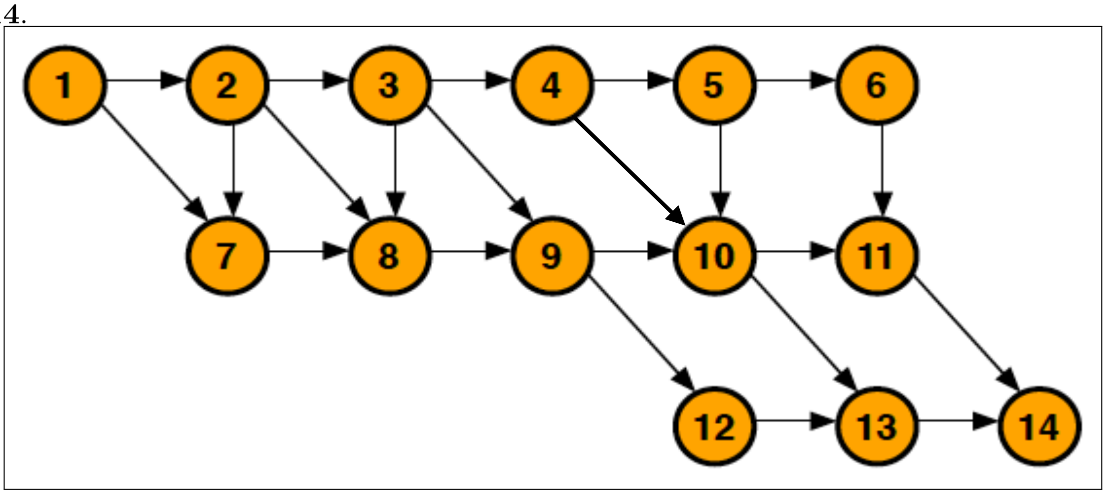
1 Problem (5 in PS7)

Given the following directed acyclic graph. Use dynamic programming to fill in a **one-dimensional** lookup table that counts number of paths from each node j to 14, for $j \geq 1$. Note that a single vertex is considered a path of length 0. **Fill in the lookup table for all vertices 1-14; and in addition, clearly show work for vertices 9-14**.



2 Original Solution

Answer.

Vertex:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of Paths:	1	1	1	1	1	1	2	4	5	7	8	5	12	20

Let P(x) represent the number of paths to vertex x.

Vertex 9: P(9) = P(3) + P(8) = P(2) + P(3) + P(2) + P(7) = 1 + P(2) + 1 + P(2) + 1 = 1 + 1 + 1 + 1 + 1 = 5

Vertex 10: P(10) = P(9) + P(4) + P(5) = P(3) + P(8) + P(3) + P(4) = P(2) + P(3) + P(2) + P(7) + P(2) + P(3) = 1 + P(2) + 1 + P(2) + 1 + 1 + P(2) = 1 + 1 + 1 + 1 + 1 + 1 + 1 = 7

Vertex 11: P(11) = P(10) + P(6) = P(9) + P(4) + P(5) + P(5) = P(3) + P(8) + P(3) + P(4) + P(4) = P(2) + P(3) + P(2) + P(7) + P(2) + P(3) + P(3) = 1 + P(2) + 1 + P(2) + 1 + 1 + P(2) + P(2) = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8

Vertex 12: P(12) = P(9) = P(3) + P(8) = P(2) + P(3) + P(2) + P(7) = 1 + P(2) + 1 + P(2) + 1 = 1 + 1 + 1 + 1 + 1 = 5

Vertex 13: P(13) = P(12) + P(10) = P(9) + P(9) + P(4) + P(5) = P(3) + P(8) + P(3) + P(8) + P(3) + P(4) = P(2) + P(3) + P(2) + P(7) + P(2) + P(3) + P(2) + P(7) + P(2) + P(3) = 1 + P(2) + 1 + P(2) + 1 + 1 + P(2) + 1 + P(2) + 1 + 1 + P(2) = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 12

Vertex 14: P(14) = P(11) + P(13) = P(10) + P(6) + P(12) + P(10) = P(9) + P(4) + P(5) + P(5) + P(9) + P(9) + P(4) + P(5) = P(3) + P(8) + P(3) + P(4) + P(4) + P(3) + P(8) + P(3) + P(8) + P(3) + P(4) = P(2) + P(3) + P(2) + P(7) + P(2) + P(3) + P(3) + P(2) + P(3) + P(2) + P(7) + P(2) + P(3) + P(2) + P(7) + P(2) + P(3) = 1 + P(2) + 1 + P(2) + 1 + 1 + P(2) + P(2) + 1 + P(2) + 1 + P(2) + 1 + 1 + P(2) + 1 + P(2) + 1 + 1 + P(2) = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 20 ∎

3 Revised Solution

Answer.

Vertex:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of Paths:	20	17	11	5	3	1	3	3	3	2	1	1	1	1

Let P(x) represent the number of paths from vertex x to vertex 14.
Vertex 14: P(14) = 1
Vertex 13: P(13) = P(14) = 1
Vertex 12: P(12) = P(13) = P(14) = 1
Vertex 11: P(11) = P(14) = 1
Vertex 10: P(10) = P(11) + P(13) = P(14) + P(14) = 2
Vertex 9: P(9) = P(10) + P(12) = P(11) + P(13) + P(13) = P(14) + P(14) + P(14) = 3

$\square$

4 Reflections

The mistake I made was misreading this problem and believing that we were supposed to calculate all the paths from vertex 1 to each other vertex as opposed to calculating the number of paths from each vertex to standard 14. I believe that my understanding of the recurrences was good when I attempted the original problem and that was confirmed when I reworked the problem in the same manner but finding the paths to vertex 14 instead of from vertex 1.