CSCI 3104 Spring 2022
Instructor: Profs. Chen and Layer

Midterm 2 Standard 13 - Analyzing Code II: Dependent nested loops

Due Date ..... May 3rd
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Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 13 - Analyzing Code II: Dependent nested loops

2.1 Problem 1

Problem 1. Analyze the runtime of the following algorithm. Clearly derive the runtime complexity function $T(n)$ for this algorithm, and then find a tight asymptotic bound for $T(n)$ (that is, find a function $f(n)$ such that $T(n) \in \Theta(f(n))$). Avoid heuristic arguments from 2270/2824 such as multiplying the complexities of nested loops.

```
for i = 1, i <= n/2
    i = i + 1
    for j = i, j <= n/2
        j = j + 1
        print(i,j)
```

Answer. The first loop takes one step to initialize.

For each iteration of the first loop we have two steps for the calculation and comparison on line 1, two steps for the calculation and assignment on line 2 and one step for the initialization of the second loop on line 3, so five steps per iteration. Since it iterates n/2 times the total steps from the loop is 5n/2.

For each iteration of the second loop we have two steps for the calculation and comparison on line 3, two steps for the calculation and assignment on line 4 and one step for the print statement on line 5, so five steps per iteration. Since it iterates (n/2)(n/2 - i + 1) times and the average value of i during the first loop is (n + 2)/4 the total steps from the loop is 5(n/2)((n/2) - ((n + 2)/4) + 1) which equals 5(n/2)((n + 2)/4).

So we have that $T(n) = 1 + 5n/2 + 5(n/2)((n+2)/4)$.

Thus, $T(n) \in \Theta(n^2)$. □