Homework 10

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section 002

4.4: 1. b. $\langle 1,1,0\rangle \cdot \langle 1,-1,1\rangle = 0$, $\langle 1,1,0\rangle \cdot \langle 2,1,1\rangle = 3 \neq 0$

$\langle -2,2,2\rangle \cdot \langle 1,-1,1\rangle = -2 \neq 0$

$\langle 2,-1,-3\rangle \cdot \langle 1,-1,1\rangle = 0$ $\langle 2,-1,-3\rangle \langle 2,1,1\rangle = 0$ ✓

$\langle -1,3,4\rangle \langle 1,-1,1\rangle = 0$ $\langle -1,3,4\rangle \cdot \langle 2,1,1\rangle = 5 \neq 0$

$\boxed{v_3 \text{ only}}$

d. $\langle 1,1,0\rangle \cdot \langle 1,-1,-1\rangle = 0$ $\langle 1,1,0\rangle \langle 3,-2,-4\rangle = 1 \neq 0$

$\langle -2,2,2\rangle \cdot \langle 1,-1,-1\rangle = -6 \neq 0$

$\langle 2,-1,3\rangle \cdot \langle 1,-1,-1\rangle = 6 \neq 0$

$\langle -1,3,4\rangle \cdot \langle 1,-1,-1\rangle = -8 \neq 0$

$\boxed{\text{None}}$

e. $\langle 1,1,0\rangle \cdot \langle -3,3,-1\rangle = 0$ $\langle 1,1,0\rangle \cdot \langle 1,-1,0\rangle = 0$ ✓

$\langle -2,2,2\rangle \cdot \langle -3,3,-1\rangle = 2 \neq 0$

$\langle 2,-1,3\rangle \cdot \langle -3,3,-1\rangle = -6 \neq 0$

$\langle -1,3,4\rangle \cdot \langle -3,3,-1\rangle = 8 \neq 0$

$\boxed{v_1}$

3. a. $y_1 = v_1 = \langle 1,-1,2,1\rangle$

$y_2 = \langle 2,1,0,-1\rangle - \frac{\langle 1,-1,2,1\rangle \cdot \langle 2,1,0,-1\rangle}{(\sqrt{7})^2} \langle [illegible],-1,2,1\rangle = \langle 2,1,0,-1\rangle$

Projection $= \frac{\langle 1,-1,2,1\rangle \cdot \langle 1,2,-1,2\rangle}{(\sqrt{7})^2} \langle 1,-1,2,1\rangle + \frac{\langle 2,1,0,-1\rangle \langle 1,2,-1,2\rangle}{(\sqrt{6})^2} \langle 2,1,0,-[illegible]$

$= \boxed{\langle 11/21, 10/21, -2/7, -10/21\rangle}$

d. $u \cdot a = 0$ $u = \langle a,b,c,d\rangle$

$\langle a,b,c,d\rangle \cdot \langle 1,-1,0,1\rangle = a-b+d = 0$

$u = \langle 1,1,1,0\rangle$ (can be anywhere $a+d-b=0$)

Projection vector $= \frac{\langle 1,1,1,0\rangle \cdot \langle 1,2,-1,2\rangle}{(\sqrt{3})^2} \langle 1,1,1,0\rangle = \boxed{\langle 2/3, 2/3, 2/3, 0\rangle}$

4.4: 7.2a. i. $2v_1w_1 + 2v_2w_2 + v_3w_3$

$$\frac{2(1)(-1/\sqrt{3}) + 2(1)(1/\sqrt{3}) + 1(1/\sqrt{3})}{()^2} \langle -1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3} \rangle = \boxed{\langle -1/3, 1/3, 1/3 \rangle}$$

ii. $$[v_1\ v_2\ v_3]\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$[v_1\ v_2\ v_3]\begin{bmatrix} 2w_1 - w_2 \\ -w_1 + 2w_2 - w_3 \\ -w_2 + 2w_3 \end{bmatrix}$$

$2v_1w_1 - w_2v_1 - v_2w_1 + 2w_2v_2 - w_3v_2 - w_2v_3 + 2w_3v_3$

$$\frac{2(1)(-1/\sqrt{3}) - (1)(1/\sqrt{3}) - (1)(-1/\sqrt{3}) + 2(1)(1/\sqrt{3}) - (1)(1/\sqrt{3}) - (1)(1/\sqrt{3}) + 2(1)(1/\sqrt{3})}{(1)^2}$$

$\boxed{\langle 0,0,0 \rangle}$

7.2c. i. $2v_1w_1 + 2v_2w_2 + v_3w_3$

$$\frac{2(1)(1) + 2(1)(1) + (1)(0)}{(\sqrt{2})^2}\langle 1,1,0 \rangle + \frac{2(1)(-2) + 2(1)(2) + (1)(1)}{(\sqrt{9})^2}\langle -2,2,1 \rangle$$

$\langle 2 - 2/9, 2 + 2/9, 1/9 \rangle = \boxed{\langle 16/9, 20/9, 1/9 \rangle}$

ii. $2v_1w_1 - w_2v_1 - v_2w_1 + 2w_2v_2 - w_3v_2 - w_2v_3 + 2w_3v_3$

$2(1)(1) - (1)(1)$

4.8 12. a. Let $u = \langle 3, -1, 1 \rangle$ $W = \text{span}\{u\}$

$W^{T} = \text{span}\{v : v \in \mathbb{R}^3 \text{ and } \langle u, v \rangle = 0\}$

If $v = \langle x, y, z \rangle$, then we have to find $x, y,$ and z so that $\langle u, v \rangle = 0$

$y = 3x + z$

case 1: $x = 0$ $z = 1$

$y = 1$

$v_1 = \langle 0, 1, 1 \rangle$

case 2: $x = 1, z = 0$

$y = 3$

$v_2 = \langle 1, 3, 0 \rangle$

$v_1, v_2 \in W^{T}$ hence v_1 and v_2 span W^{T}

$3 = \dim W + \dim W^{T}$

$\dim W = 1$ so $\dim W^{T} = 2$

c. $u = \langle 1, 2, 3 \rangle$ $v = \langle 2, 4, 6 \rangle$ $v = 2u$ so $\dim = 1$

$x = -2y - 3z$

case 1: $y = 1$ $z = 0$

$w_1 = \langle -2, 1, 0 \rangle$

case 2: $z = 1$ $y = 0$

$w_2 = \langle -3, 1, 0 \rangle$

$w_1, w_2 \in W^{T}$ w_1 and w_2 span W^{T}

$3 = \dim W + \dim W^{T}$ $\dim W = 1$ so $\dim W^{T} = 2$

15. c. $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$x = -z$ $y = z$

$W^{T} = \{\langle 1, 1, 0 \rangle \langle 1, 0, 1 \rangle, \langle -1, 1, 1 \rangle\}$

$\langle 1, 0, 0 \rangle = 1/3 \langle 1, 1, 0 \rangle + 1/3 \langle 1, 0, 1 \rangle - 1/3 \langle -1, 1, 1 \rangle$

4.8 29. d. i. Basis for range = $(\langle 1,-1,0\rangle \langle 2,1,3\rangle)$

corange = $(\langle 1,2,0,1\rangle \langle 0,3,3,2\rangle)$

Kernel = $(\langle 2,-1,1,0\rangle, \langle 1/3,-2/3,0,1\rangle)$

Cokernel = $(\langle -1,-1,1\rangle)$

ii. $\langle 1,-1,0\rangle \cdot \langle -1,-1,1\rangle = 0$

$\langle 2,1,3\rangle \langle -1,-1,1\rangle = 0$ ✓

iii. $\langle 1,2,0,1\rangle \cdot \langle 2,-1,1,0\rangle = 0$

$\langle 1,2,0,1\rangle \cdot \langle 1/3,-2/3,0,1\rangle = 0$

$\langle 0,3,3,2\rangle \cdot \langle 2,-1,1,0\rangle = 0$

$\langle 0,3,3,2\rangle \cdot \langle 1/3,-2/3,0,1\rangle = 0$ ✓

34. b. System: $2x+3y=-1$

$3x+7y=1$

$-3x+2y=8$

$$A=\begin{bmatrix}2 & 3 & -1\\ 3 & 7 & 1\\ -3 & 2 & 8\end{bmatrix}$$

rank A = 2

system is compatible

$3x+7y=1$

$+\ -3x+2y=8$

$9y=9$ $\quad \boxed{y=1\ x=-2}$

5.2: 3.c. $f(x,y) = x^2 + 5xy + 3y^2 + 2x - y$

$$f_x = 2x + 5y + 2 = 0$$

$$f_y = 6y + 5x - 1 = 0$$

$$10x + 25y + 10 = 0$$

$$-10x + 12y - 2 = 0$$

$$13y = -12$$

$$\boxed{\begin{array}{l} y = -12/13 \\ x = 38/26 \end{array}}$$

4.a $\det \begin{pmatrix} 1-x & 6 \\ 6 & 4-x \end{pmatrix} = 0$

$$4 - 5x + x^2 - 6^2 = 0$$

$$\boxed{\text{thd} < 4}$$