Problem Set 6

Due Date ..... October 20, 2022
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Collaborators ..... **None**

Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. (See this short intro to LaTeX plus other resources on Canvas.)
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

(I agree to the above, Alex Ojemann). □

3 Standard 17: Classification of Context-Free Languages

Problem 1. Classify each language as regular (REG), nonregular but context-free (CF), or not context-free (NCF). Justify your answer. (Note: you can show NCF using closure properties—for real this time—and use of the CFL pumping lemma is optional.)

(a) $\{xcy \mid x, y \in \{a, b, c\}^*,\ \#a(x) = \#b(x) = \#b(y)\}$

Proof. The language $a^*b^*cb^*$ is regular.

Assume the given language to be context-free, the language $\{xcy \mid x, y \in \{a, b, c\}^*,\ \#a(x) = \#b(x) = \#b(y)\} \cup a^*b^*cb^* = a^nb^ncb^n$ must also be context-free.

Assume $a^nb^ncb^n$ is context-free.

k given where k ≥ 0

Let $z = a^kb^kcb^k$ where z = uvwxy, $vx \neq \epsilon$, and $|vwx| < k$.

Take i = 2:

If either v or x contains any cs, then uv^2wx^2y would contain more than one c and therefore wouldn't follow the required form of abcb.

If v and/or x contains both as and bs, then uv^2wx^2y wouldn't follow the required form of abcb.

If v and/or x contains only as, then uv^2wx^2y would have the group of as be larger than either group of bs.

If v and/or x contains only bs, then uv^2wx^2y would have one or both group of bs be larger than the group of as.

Therefore, in any case, $uv^2wx^2y \notin a^nb^ncb^n$.

So, by the CFL pumping lemma, $a^nb^ncb^n$ is not context-free and $\{xcy \mid x, y \in \{a, b, c\}^*,\ \#a(x) = \#b(x) = \#b(y)\}$ is not context-free by closure properties. □

(b) $\{xyz \mid x, y, z \in \{a, b, c\}^*,\ \#a(x) = \#a(y) = \#a(z)\}$

Proof. regex: $((b|c)^*a(b|c)^*a(b|c)^*a(b|c)^*)^*$ □

4 Novel Construction

Problem 2. Let x be a string. Define MoreA(x) to be the set of strings where each instance of a is replaced with one or more a's. So MoreA(bac) = $\{bac, baac, baaac, \ldots\}$. Let L be a language. Define:

$$\text{MoreA}(L) := \bigcup_{x \in L} \text{MoreA}(x).$$

4.1 Standard 11: Novel Construction

(a) Suppose that L is regular. Give a construction (e.g., regexp, DFA/NFA) to justify that MoreA(L) is regular. You need not prove that your construction works to get credit for Standard 11. [**Hint:** Can you adapt the regexp solution for S11 from PS3?]

Answer. Suppose L is regular. So there exists a DFA $M = (Q_M, \Sigma, \delta_M, s_M, F_M)$ such that L(M) = A. We construct an NFA $N = (Q_N, \Sigma, \delta_N, S_N, F_N)$ accepting MoreA(L), as follows.

We let $Q_N := Q_M$.

The alphabet for N is the same alphabet as for M.

$s_M = s_N$

We define δ_N as follows. δ_N contains every transition in δ_M. In addition, for every transition in M δ_M(p,a) = q where the input symbol is a, there is a transition in N δ_N(q,a) = q.

$F_N = F_M$ □

4.2 Standards 2/3: Proofs

(b) Carefully prove that your construction from part (a) works. That is, again suppose L is regular, and let K be the language accepted by your construction. Carefully prove that $K = \text{MoreA}(L)$.

Proof. □