Pre-Final 1- Standard 22

Due Date ....

Name .... **Alex Ojemann**

Student ID **109722375**

Collaborators ....

Contents

1 Instructions

- The solutions may be typed or handwritten, using proper mathematical notation. If you handwrite your solutions, you must embed them as an image in the template and orient your image so we do not have to rotate our screens to grade it.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

(I agree to the above, Alex Ojemann). □

3 Standard 22: Computability Classifications

Problem 1. Classify the following languages as below, and justify your answer:

- (RE) recursive enumerable (r.e.),
- (CO-RE) co-r.e. (i.e. the complement is r.e.),
- (BOTH) both r.e. and co-r.e., or
- (NEITHER) neither r.e. nor co-r.e.

Note this means showing two things each: L is r.e. or not, and $\sim L$ is r.e. or not. If you provide a reduction, to L_1 or L_2, you do not have to prove that your reduction is correct.

(a) $L_1 = \{M \mid M \text{ accepts } aaa \text{ but not } bbb\}$

Proof. RE: We may design a TM M' to simulate M(aaa) and M(bbb). Now M' accepts $M\#aaa\#bbb$ if and only if M(aaa) $= 1$ and M(bbb) $= 0$. So L_1 is RE.

not co-RE: To see that L_1 isn't co-RE, we reduce from the membership problem. Let $M\#x \in$ MP. We design a TM M' such that on input bbb, M' rejects. Otherwise, M' simulates M. So we map $M\#x-> M'\#x\#bbb$. As MP is undecidable, so is L. As $L \in$ RE and decidable languages are closed under complementation, $L \notin$ co-RE. □

(b) $L_2 = \{M \mid M \text{ takes at least } |M| \text{ steps (length of the description of } M) \text{ on all inputs}\}$

Proof. not RE: To see that L_2 isn't RE, we reduce from $\sim HP$. Let $M\#x$ be an instance of $\sim HP$. We construct a TM M' such that M' simulates $M\#x$ on any input string. If $M\#x$ is 1, M' takes at least $|M|$ steps on all strings. We map $M\#x-> M'\#x \in L_2$. As $\sim HP$ isn't RE, neither is L.

co-RE: We may design a TM M' to simulate M(There exists an input where M halts in less than $|M|$ steps). Now M' accepts $M\#x$ if and only if there exists an input x where M halts in less than $|M|$ steps. So $\sim L_2$ is RE, thus L_2 is co-RE. □