Homework 4

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1 Problem 1

1.1 Part a

$\delta_k(x) = \pi_k(x) f_k(x) = \pi_k(x) p_k^x (1-p_k)^{1-x}$
$\delta_1(x) = \pi_1(x) f_1(x) = \pi_1(x) p_1^x (1-p_1)^{1-x}$
$\delta_2(x) = \pi_2(x) f_2(x) = \pi_2(x) p_2^x (1-p_2)^{1-x}$

1.2 Part b

$\delta_1(x) = \delta_2(x)$
$\pi_1(x) p_1^x (1-p_1)^{1-x} = \pi_2(x) p_2^x (1-p_2)^{1-x}$
$0.5 * p_1^x (1-p_1)^{1-x} = 0.5 * p_2^x (1-p_2)^{1-x}$
$p_1^x (1-p_1)^{1-x} = p_2^x (1-p_2)^{1-x}$
$ln(p_1^x (1-p_1)^{1-x}) = ln(p_2^x (1-p_2)^{1-x})$
$ln(p_1^x) + ln((1-p_1)^{1-x}) = ln(p_2^x) + ln((1-p_2)^{1-x})$
$x * ln(p_1) + (1-x) * ln(1-p_1) = x * ln(p_2) + (1-x) * ln(1-p_2)$
$x * ln(p_1) + ln(1-p_1) - x * ln(1-p_1) = x * ln(p_2) + ln(1-p_2) - x * ln(1-p_2)$
$x * ln(p_1) + x * ln(1-p_2) - x * ln(1-p_1) - x * ln(p_2) = ln(1-p_2) - ln(1-p_1)$
$x(ln(p_1) + ln(1-p_2) - ln(1-p_1) - ln(p_2)) = ln(1-p_2) - ln(1-p_1)$
$x = \frac{ln(1-p_2) - ln(1-p_1)}{ln(p_1) + ln(1-p_2) - ln(1-p_1) - ln(p_2)}$

1.3 Part c

This implies that the distribution X given Y=1 will have more 1s than 0s and the distribution X given Y=2 will have more 0s than 1s.

1.4 Part d

$x = \frac{ln(1-p_2) - ln(1-p_1)}{ln(p_1) + ln(1-p_2) - ln(1-p_1) - ln(p_2)}$

When p_1 is greater than 0.5 and p_2 equal to $1 - p_1$ in this equation, the numerator $ln(1-p_2) - ln(1-p_1)$ is equal to $ln(p_1) - ln(p_2)$ and the denominator $ln(p_1) + ln(1-p_2) - ln(1-p_1) - ln(p_2)$ is equal to $ln(p_1) + ln(p_1) - ln(p_2) - ln(p_2)$ or $2 * ln(p_1) - 2 * ln(p_2)$, resulting in $x = 1/2$. This makes sense because p_1 and p_1 are equidistant from 1/2 and the probability of being in each class π_k is the same for both classes, so the discriminant function should split the two classes down the middle at 1/2.