Induction Quiz 1

Due Date .... September 10, 2022
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Collaborators .... **None**

Contents

1 Instructions

- The solutions may be typed or handwritten, using proper mathematical notation. If you handwrite your solutions, you must embed them as an image in the template and orient your image so we do not have to rotate our screens to grade it.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

(I agree to the above, Alex Ojemann). $\square$

3 Standard 1: Induction

Problem 1. Let $M = (Q, \Sigma, \delta, s, F)$ be an arbitrary DFA. Prove by induction on y that for any strings $x, y \in \Sigma^*$ and any $q \in Q$, that:

$$\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q, x), y).$$

Recall that $\hat{\delta} : Q \times \Sigma^* \to Q$ is the extended version of δ defined on all strings described in lecture 3. For convenience, we recall $\hat{\delta}$, which is defined inductively on the length of x.

- **Basis:** $\hat{\delta}(q, \varepsilon) := q$,
- **Inductive Step:** For any $a \in \Sigma$, $\hat{\delta}(q, xa) = \delta(\hat{\delta}(q, x), a)$.

Proof. Base Case:

$$\begin{aligned} \hat{\delta}(q, x\epsilon) &= \hat{\delta}(q, x) && (1) \\ &= \hat{\delta}(\hat{\delta}(q, x), \epsilon) && (2) \end{aligned}$$

Line 2 by the definition that $\hat{\delta}(a, \epsilon) = a$
Inductive Case:

$$\begin{aligned} \hat{\delta}(q, xya) &= \bigcup_{p \in \hat{\delta}(q, xy)} (\delta(p, a)) && (3) \\ &= \bigcup_{p \in \hat{\delta}(\hat{\delta}(q,x), y)} (\delta(p, a)) && (4) \\ &= \hat{\delta}(\hat{\delta}(q, xy), a) && (5) \end{aligned}$$

Line 3 by the definition that $\hat{\delta}(q, xa) = \bigcup_{p \in \hat{\delta}(q,x)} (\delta(p, a))$
Line 4 by the inductive hypothesis
Line 5 by the definition that $\hat{\delta}(q, xa) = \bigcup_{p \in \hat{\delta}(q,x)} (\delta(p, a))$ $\square$