An Exploration of the Riemann Zeta Function

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Abstract:

This report is an exploration of the Riemann Zeta Function. It will start with Euler's Product and consider Riemann's extension into the Complex plane via complex arguments. It will be shown that when the real component of the argument is greater than one, the infinite sum can be represented as an integral. Furthermore the Analytic Continuation of the function beyond its singular points as well as the implications that the function's zeros have on the distribution of prime numbers will be shown as well as discussed. Visualizations not limited to the functions singular points and zeros will be included.

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