Homework 6

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APPM 3310

Grading Grid

3.1: 2. d. no; $\langle av, w\rangle = a^2 v_1^2 w_1^2 + a^2 v_2^2 w_2^2 \neq a(v_1^2 w_1^2 + v_2^2 w_2^2) = a\langle v, w\rangle$

e. No; $\langle u+v, w\rangle \neq \langle u, w\rangle + \langle v, w\rangle$

3. Not positive definite; if $v = \langle 1, -1\rangle$ then $\langle v, v\rangle = 0$

7. $\|cv\| = \sqrt{\langle cv, cv\rangle} = \sqrt{c^2\langle v, v\rangle} = |c|\|v\|$

12. a. $\frac{1}{4}(\|u+v\|^2 - \|u-v\|^2) = \frac{1}{4}(2\|u\|^2 + 2\|v\|^2) = \langle u, v\rangle$

b. $\langle u, v\rangle = \frac{1}{4}(\|u+v\|^2 - \|u-v\|^2) = \dfrac{\sqrt{v_1^2 + 3v_1 w_2 + 5w_2^2} - \sqrt{v_1^2 - 3v_1 w_2 + 5w_2^2}}{2}$

15. $v \cdot (Aw) = v^T A w = (A^T v)^T w = (A^T v) \cdot w$

25. d. Yes; Bilinearity and symmetry hold and it's always positive

3.2: 5. b. $|\langle v, w\rangle| = 11 \leq 11.747 = \|v\|\|w\|$

7. Let $v = (a_1, a_2, \ldots, a_n)$ $w = (1, 1, \ldots, 1)$
$|v \cdot w| = |a_1 + a_2 + \cdots + a_n| \leq \sqrt{n}\sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} = \|v\|\|w\|$
So Cauchy-Schwarz inequality is true.

15. a. $\langle 2, a, -3\rangle \cdot \langle -1, 3, -2\rangle = -2 + 3a + 6 = 0$ $\boxed{a = -4/3}$

b. $\langle 2, a, -3\rangle \times \langle -1, 3, -2\rangle = \langle -2a - 9, 7, 6 + a\rangle$
No value of a makes that 0 so no.

22. $\langle x, y, z\rangle \cdot \langle x, y, z\rangle = 0$
$x^2 + y^2 + z^2 = 0$ This is only possible when all components are 0

3.2. 24. $\langle v+w, v-w \rangle = \|v\|^2 - \|w\|^2 = 0$ if $\|v\| = \|w\|$
$v+w$ is a unit vector if $\langle v, w \rangle = -1/2$, $v-w$ is a unit vector if $\langle v, w \rangle = -1/2$

31. a $\langle 1,2,3 \rangle \cdot \langle 1,-1,2 \rangle = 5$

$5 = \sqrt{14} \cdot \sqrt{6} \cos\theta$

$\theta = \cos^{-1}(5/\sqrt{84}) = .994$ radians

b. $v \cdot w = 5 < 9.165 = \sqrt{84} = \|v\|\|w\|$

c. $\langle 1,2,3 \rangle \times \langle 1,-1,2 \rangle = \langle 7, 1, -3 \rangle$

$\langle 1,-1,2 \rangle \times \langle 1,2,3 \rangle = \langle -7, -1, 3 \rangle$

$\boxed{\langle -7/3t, -t/3, t \rangle}$