APPM 4600 — HOMEWORK # 12

For all homeworks, if coding is required, you should use Python. **Do not use** symbolic software such as Maple or Mathematica.

1. Consider the linear system

$$\begin{aligned} 6x + 2y + 2z &= -2 \\ 2x + 2/3y + 1/3z &= 1 \\ x + 2y - z &= 0 \end{aligned}$$

   (a) Verify that $(x, y, z) = (2.6, -3.8, -5)$ is the exact solution.

   (b) Using 4 digit floating point arithmetic with rounding, solve the system via Gaussian elimination without pivoting.

   (c) Repeat part (a) with partial pivoting.

   (d) Which method is more accurate? i.e. stable.

   (Remember to do the rounding to 4 significant digits as the machine would.)

   Soln:

   (a) Plug in the answer into the left-hand side equation and verify that you get the vector on the right-hand side.

   (b) After the first round of Gaussian elimination we get

$$\left[\begin{array}{ccc|c} 6 & 2 & 2 & -2 \\ 0 & 1 \times 10^{-4} & -0.3334 & 1.667 \\ 0 & 1.333 & -1.333 & 0.3334 \end{array}\right]$$

   Now we eliminate the (3,2) entry and get

$$\left[\begin{array}{ccc|c} 6 & 2 & 2 & -2 \\ 0 & 1 \times 10^{-4} & -0.3334 & 1.667 \\ 0 & 0 & 4443 & 0.3334 \end{array}\right]$$

   Doing the back solve, we find $x = -2 \times 10^5$, $y = 6664$ and $z = 7.503 \times 10^{-5}$.
   This is very wrong.

   (c) Lets try with row (partial) pivoting. The first reduced system is the same.

$$\left[\begin{array}{ccc|c} 6 & 2 & 2 & -2 \\ 0 & 1 \times 10^{-4} & -0.3334 & 1.667 \\ 0 & 1.333 & -1.333 & 0.3334 \end{array}\right]$$

   Now we will swap the second and third row.

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$$\left[\begin{array}{ccc|c} 6 & 2 & 2 & -2 \\ 0 & 1.333 & -1.333 & 0.3334 \\ 0 & 1\times 10^{-4} & -0.3334 & 1.667 \end{array}\right]$$

We get the following after reduction

$$\left[\begin{array}{ccc|c} 6 & 2 & 2 & -2 \\ 0 & 1.333 & -1.333 & 0.3334 \\ 0 & 0 & -0.3332 & 1.667 \end{array}\right]$$

Performing the back solve we get $z = -5.003$, $y = -3.801$ and $x = 2.601$. While these are still off they are much closer.

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