Assignment - 6 Parameter Estimation

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Q1) Let $(X_1, X_2, \ldots)$ be a random sample of size n taken from a Normal Population with parameters : mean = θ_1 & variance = θ_2. Find the maximum likelihood estimator of these two parameters.

⇒ Soln:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Let $x_1, x_2, x_3, \ldots, x_n$ be sample of size n

let,

$$L(x_1, x_2, x_3, \ldots x_n) = f(x_1) \cdot f(x_2) \cdot f(x_3) \cdots f(x_n)$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) \cdots$$

Taking ln on both sides:

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma^2}\right) \quad \text{—— (1)}$$

Taking derivative w.r.t μ :

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} \frac{-2(x_i-\mu)}{2\sigma^2} = 0$$

$$\Rightarrow \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$\Rightarrow n\bar{x} - n\mu = 0$$

$$\Rightarrow \boxed{\bar{x} = \mu}$$

Hence $\theta_1 = \mu$ is the sample mean.

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Taking derivative w.r.t σ^2 of eq (1)

$$\frac{\partial(\ln(L))}{\partial\sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} \frac{-(x_i - \mu)^2}{2(\sigma^2)^2} = 0$$

$$\Rightarrow -n + \sum_{i=1}^{n} - \frac{(x_i - \mu)}{\sigma^2} = 0$$

$$\Rightarrow n = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

Hence, $\theta_2 = \sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n} x_i - \mu\right)^2$ is the sample variance.

Proved..

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Q) let $(x_1, x_2, \dots x_n)$ be a random sample from $B(m, \theta)$ distribution, where $\theta = \Theta = (0,1)$ is unknown & m is a known positive integer. Compute value of θ using M.L.E.

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$\Rightarrow$ Soln:

Binomial distribution : $^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$ is given as

Let, $L = \prod_{i=1}^{n} {^{n}C_{x_i}}\, \theta^{x_i} (1-\theta)^{n-x_i}$

Taking log on both sides,

$$\log(L) = \sum_{i=1}^{n} \left\{ \log(^{n}C_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right\}$$

Differentiate w.r.t θ

$$\frac{d \log(L)}{d\theta} = 0$$

$$\frac{1}{\theta} \Sigma x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \Sigma x_i = 0$$

$$\frac{1}{\theta(1-\theta)} \cdot \Sigma x_i = \frac{n^2}{1-\theta}$$

$$\frac{\Sigma x_i}{\theta} = n^2$$

$\therefore$ $\boxed{\theta = \frac{\Sigma x_i}{n^2}}$ is the value of θ.