CIVIL ENGINEERING

Strength of Materials

Transformation of Strain

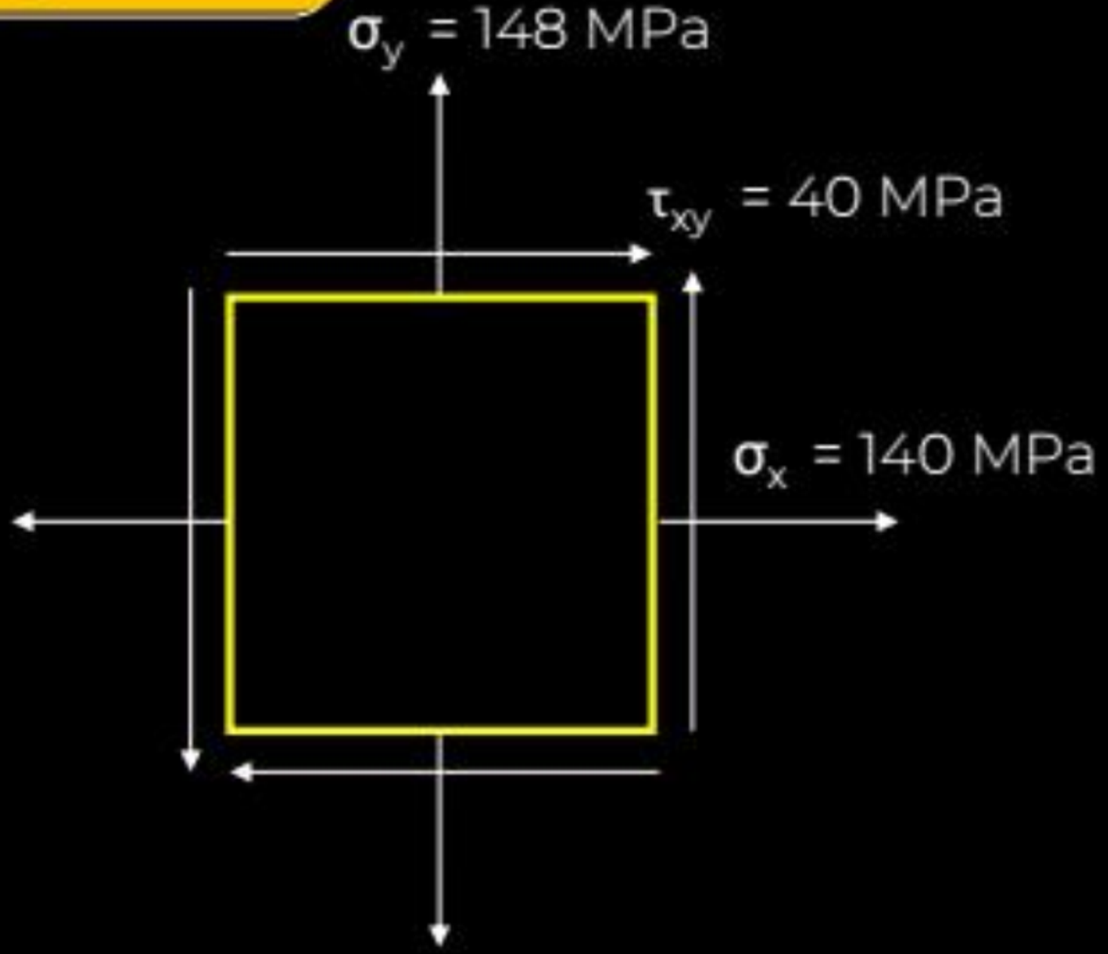
One Shot

Satyajeet Sahu Sir

01 Transformation of Stress

01 Transformation of Strain

#Question



Find the following

a) Centre of Mohr Circle
b) Radius of Mohr Circle
c) Major Principal Stress
d) Minor Principal Stress
e) Maximum Shear Stress
f) Angles of Principal Planes
g) Angles of Planes of Maximum Shear
h) Resultant Stress on Planes of Maximum Shear

$\sigma_x = 140$ MPa, $\sigma_y = 148$ MPa

$\tau_{xy} = 40$ MPa

(a) $C = 144$ MPa

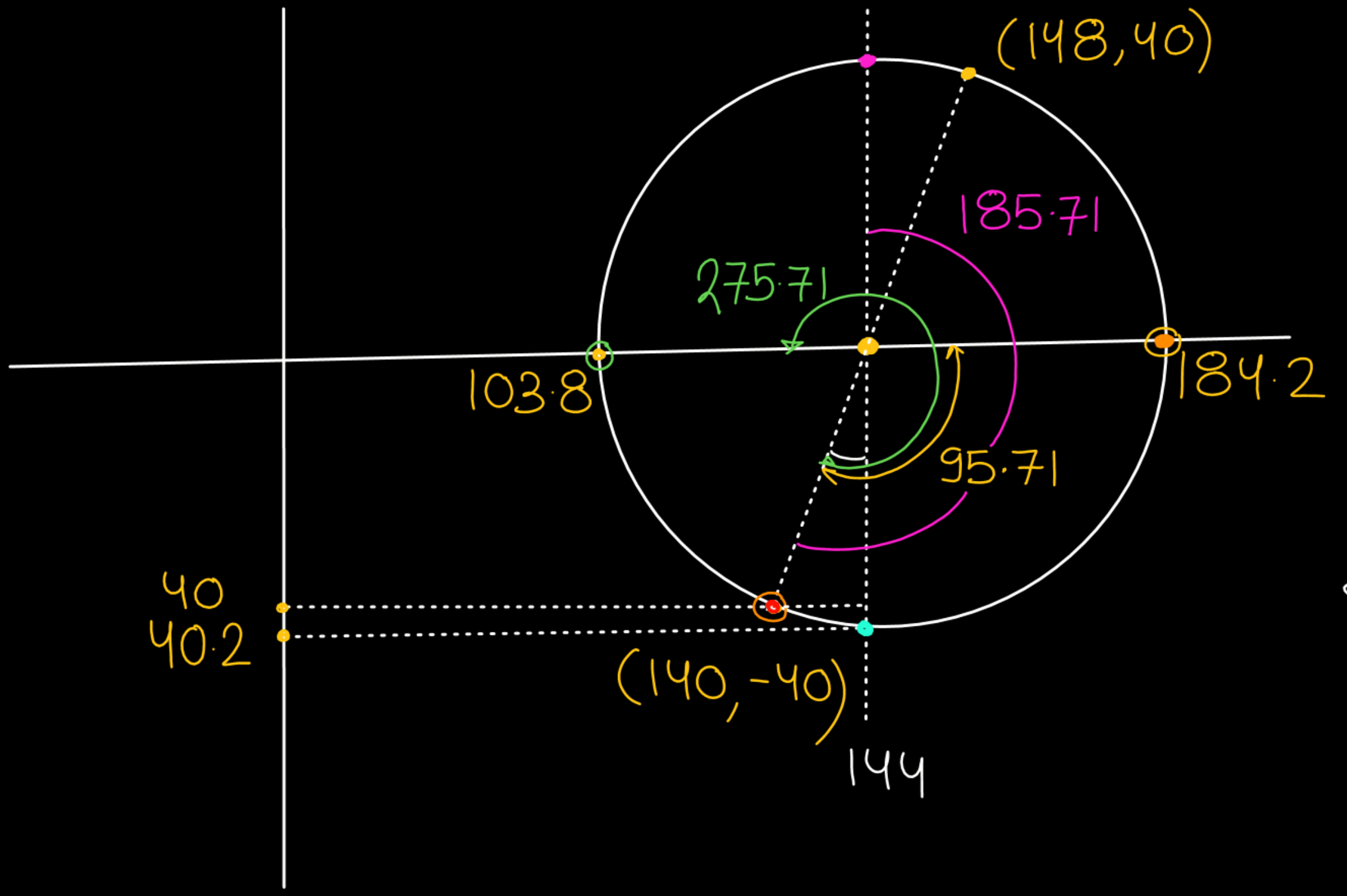
(b) $R = \sqrt{(-4)^2 + 40^2}$

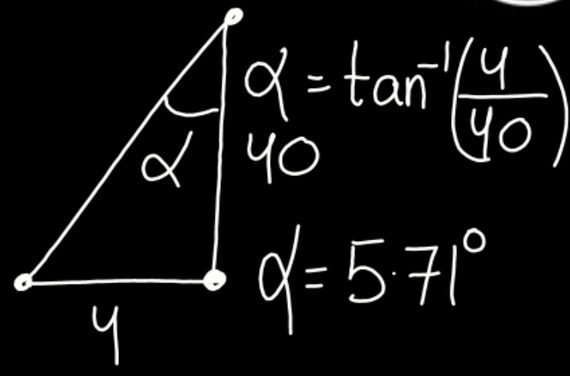
$= 40.2$ MPa

(c) $\sigma_1 = 184.2$ MPa.

(d) $\sigma_2 = 103.8$ MPa

(e) $\tau_{max} = R = 40.2$ MPa



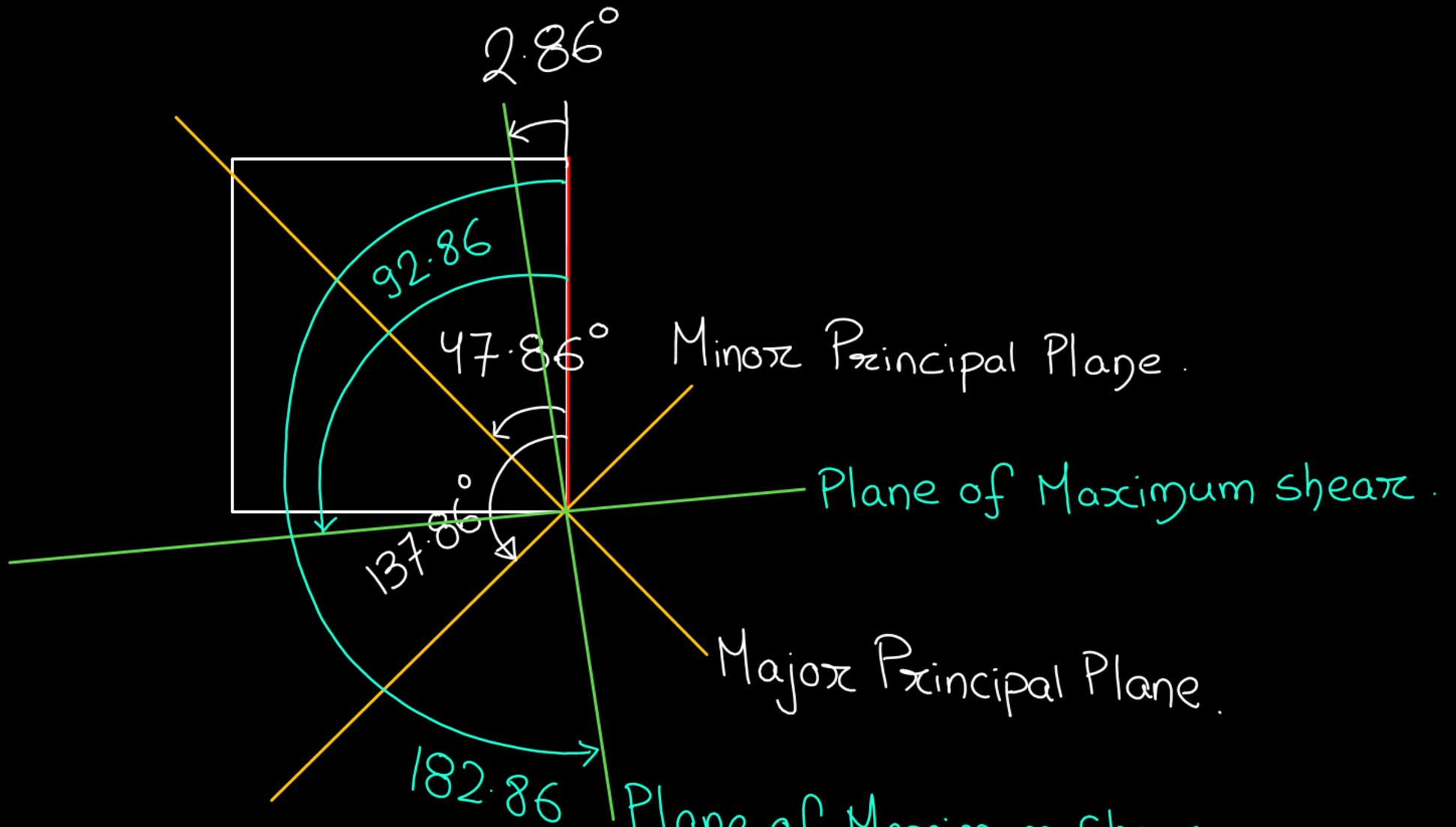


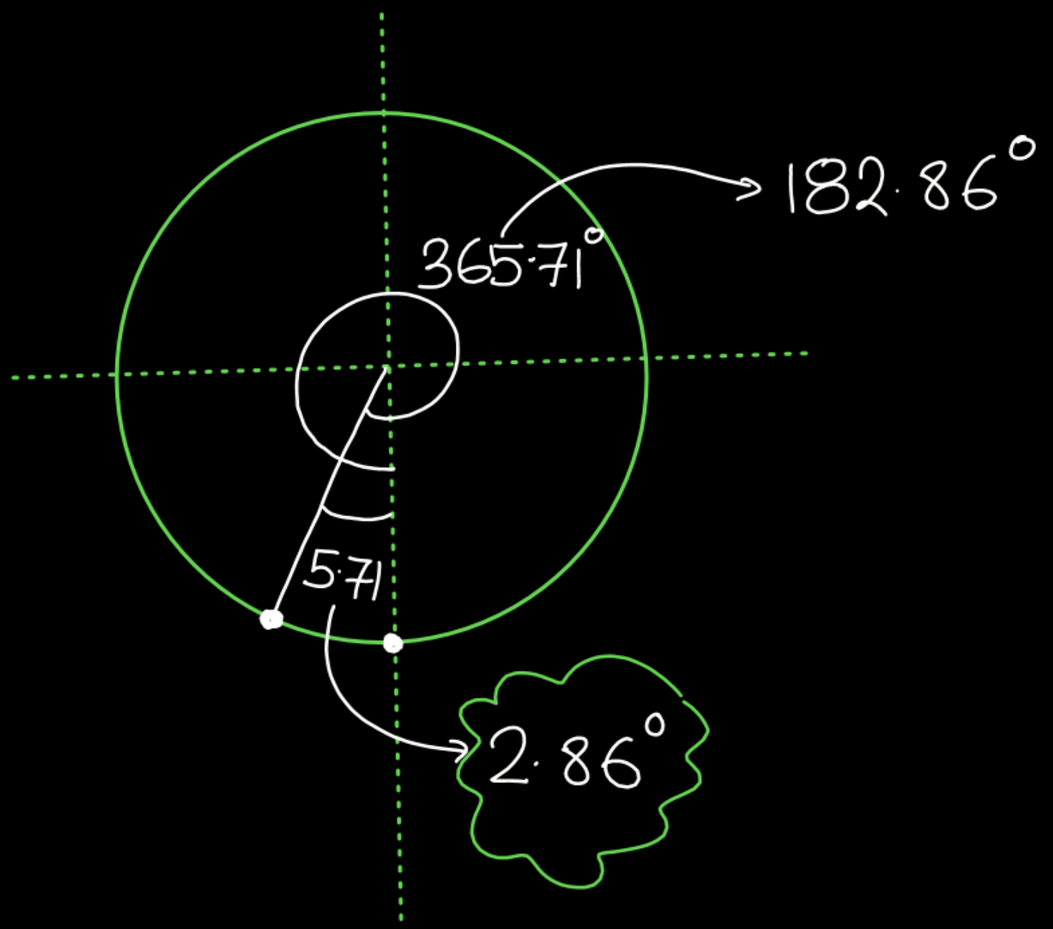
140 , -40

$\sigma_x , -\tau_{xy}$

σ_y , τ_{xy}

→ 148, 40


2·86°
92·86
47·86°
Minor Principal Plane.
Plane of Maximum shear.
137·86°
Major Principal Plane.
182·86
Plane of Maximum Shear



Major Minor.

$\theta_p = 47.86^\circ, 137.86$

$\theta_s = 92.86^\circ, 2.86^\circ$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{80}{-8}$$

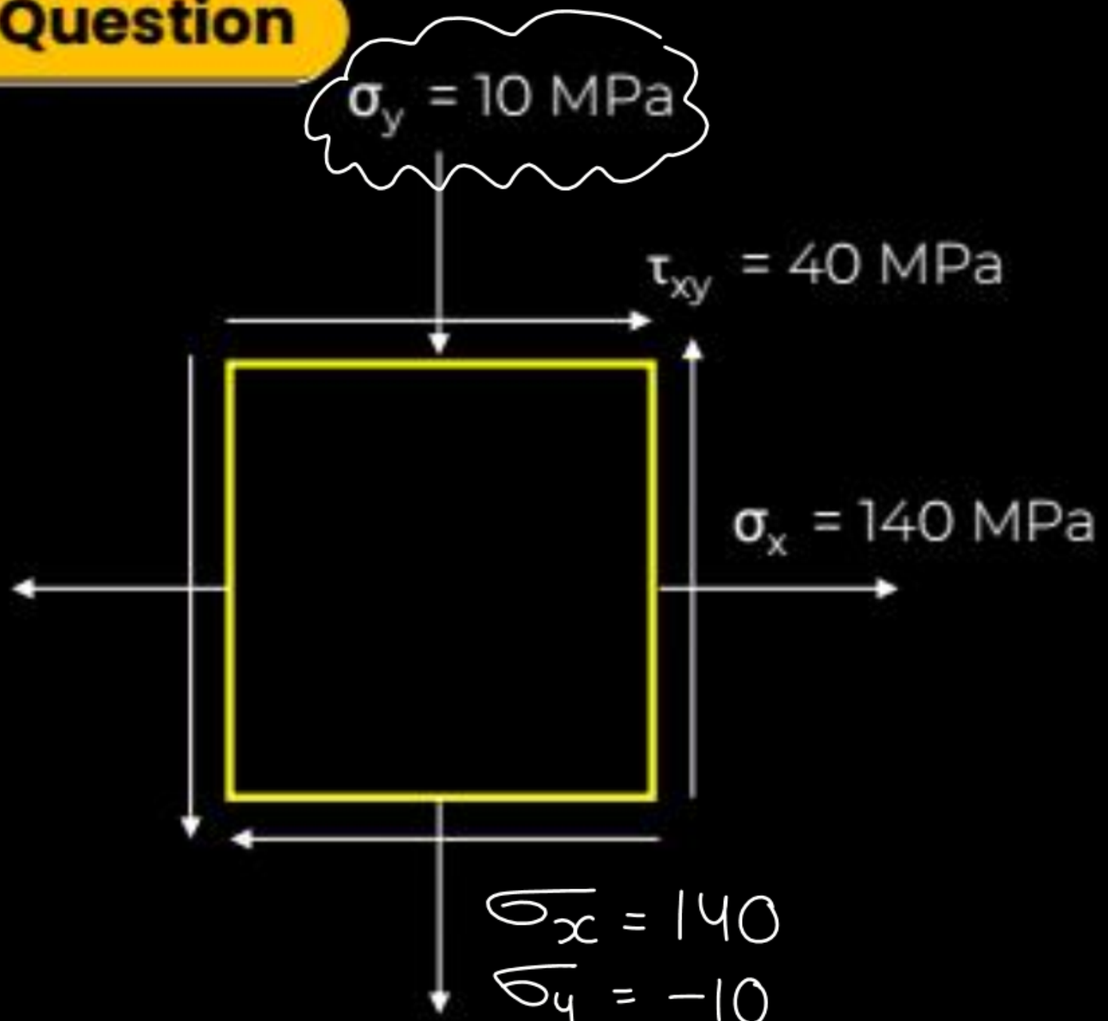
$2\theta_p = \tan^{-1}(-10) = 360 - 84.29$

$= 275.71$

Minor Major

$$\begin{bmatrix} \theta_p = 137.86^\circ, 47.86^\circ \\ \theta_s = 2.86^\circ, 92.86^\circ \end{bmatrix}$$

#Question



$\sigma_x = 140$
$\sigma_y = -10$
$\tau_{xy} = 40$

Find the following

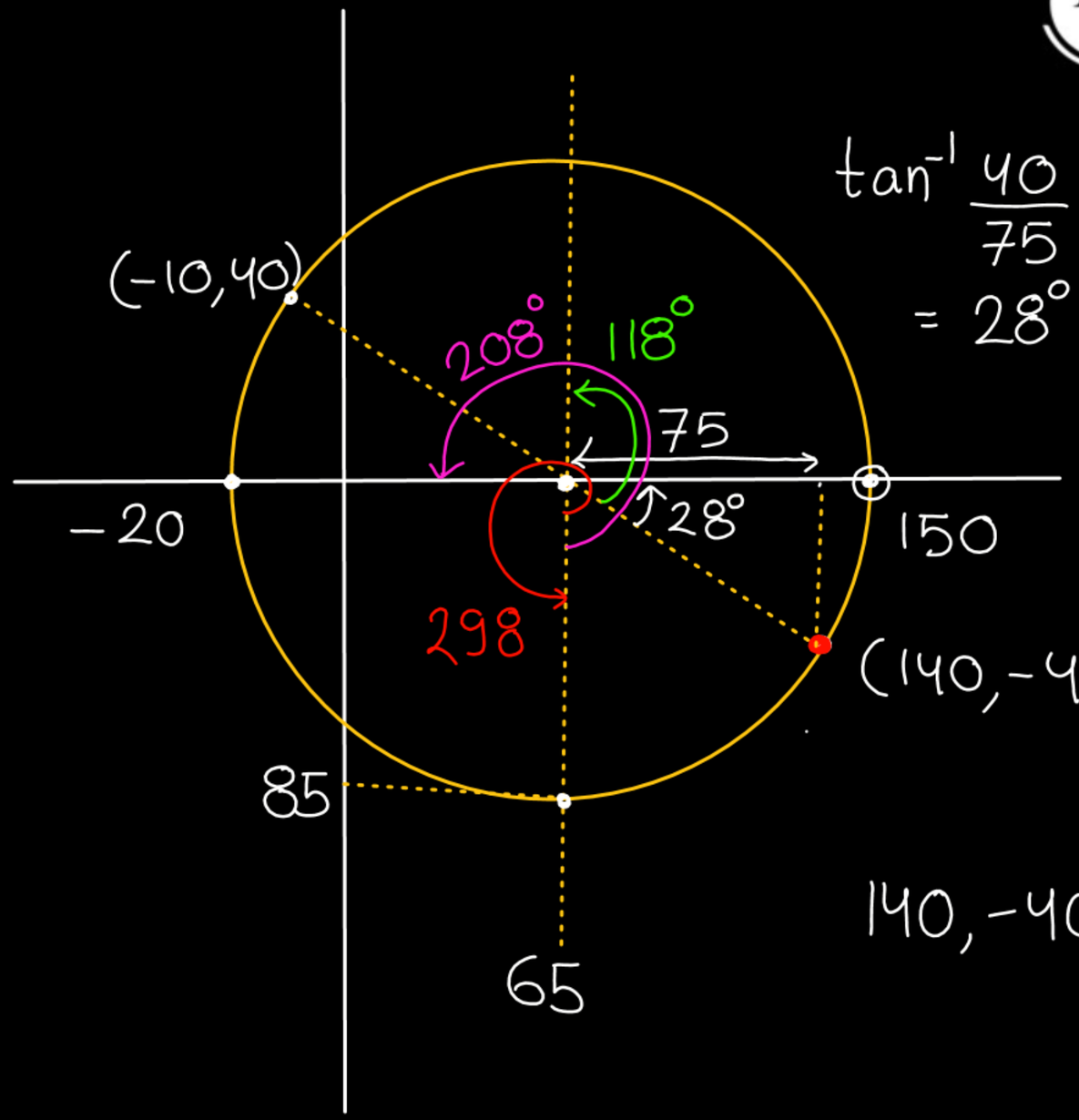
a) Centre of Mohr Circle 65 MPa.
b) Radius of Mohr Circle 85 MPa
c) Major Principal Stress 150 MPa [T]
d) Minor Principal Stress -20 MPa [C]
e) Maximum Shear Stress 85 MPa
f) Angles of Principal Planes
g) Angles of Planes of Maximum Shear
h) Resultant Stress on Planes of Maximum Shear

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$2\theta_p = \tan^{-1}\frac{80}{150}$$

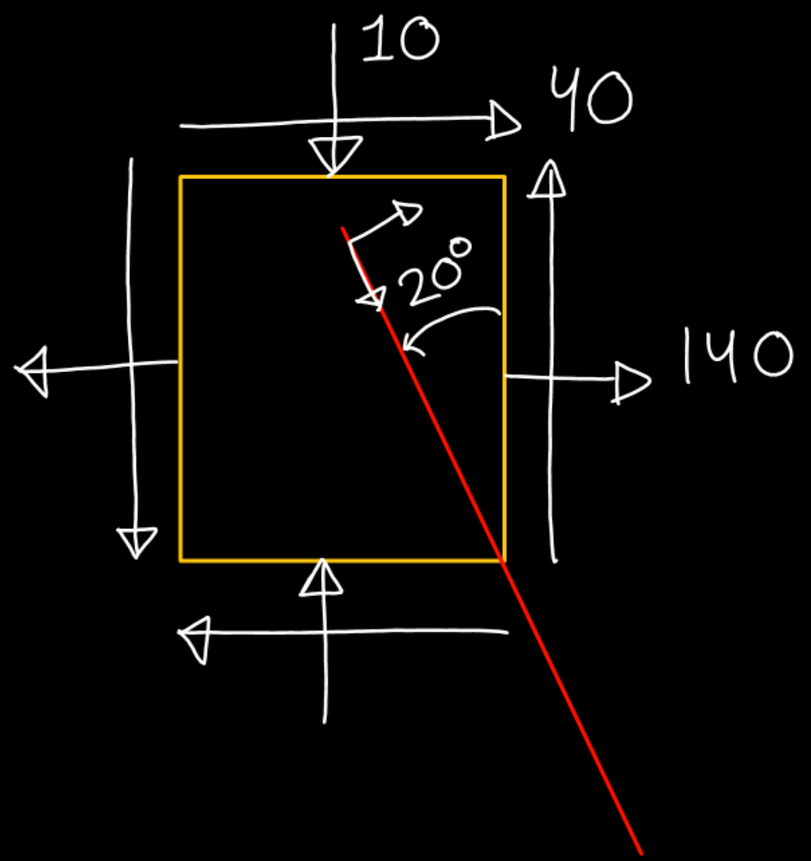
$$\theta_p = 14^\circ, 104^\circ$$

$$\theta_s = 59^\circ, 149^\circ$$



$\tan^{-1}\frac{40}{75} = 28^\circ$

140, -40

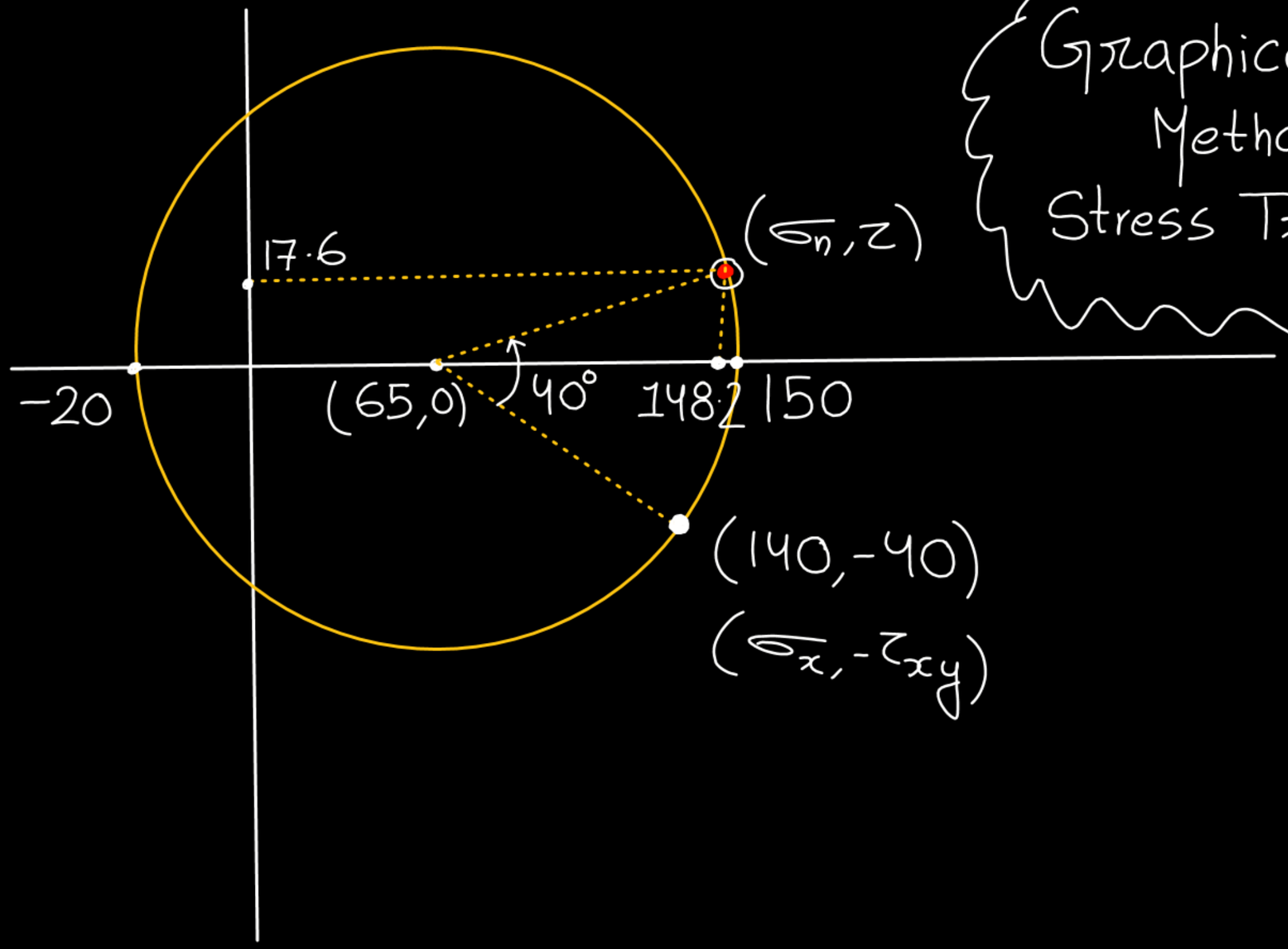


$$\sigma_\eta = \left(\frac{140-10}{2}\right) + \frac{140-(-10)}{2}\cos 40^\circ + 40 \sin 40 = 148.2 \text{ MPa}$$

$$\tau = \frac{140-(-10)}{2}\sin 40 - 40\cos 40 = 17.6 \text{ MPa}.$$

Analytical Method of Stress Transformation

Graphical Method of Stress Transformation



Topic : Transformation of Strain

Plane Strain Condition

$\varepsilon_x = \varepsilon_{xx} \neq 0$ [Normal Strain]

$\varepsilon_y = \varepsilon_{yy} \neq 0$ [Normal Strain]

ϕ_{xy} $\gamma_{xy} \neq 0$ (Shear strain)

$$\sigma_\eta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Normal Stress.

Normal Strain.

$$\varepsilon_\theta = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta$$

Shear stress.

$$\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta$$

Shear strain

$$\frac{\phi_\theta}{2} = \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta - \frac{\phi_{xy}}{2}\cos 2\theta$$

Mohr Circle for Strain

$$C = \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

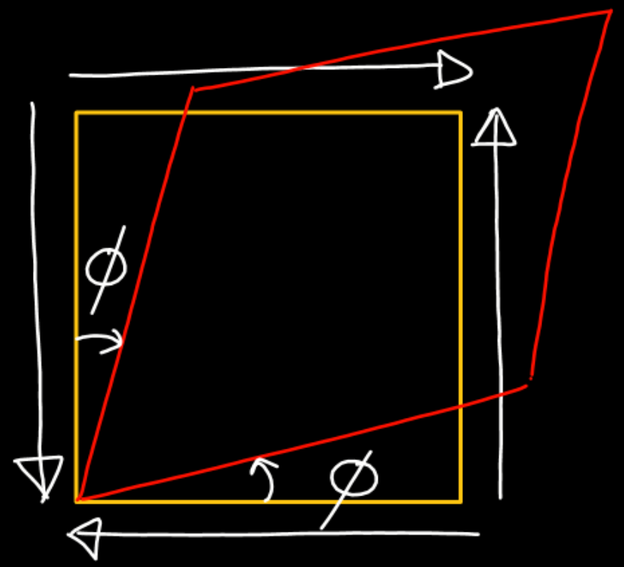
ε_1 = Major Principal Strain = $C + R$

ε_2 = Minor " " = $C - R$

$$\varepsilon_{max} = R$$

$$\frac{\phi_{max}}{2} = R$$

$$\phi_{max} = 2R$$



$$\phi_{xy} = \phi + \phi$$

$$\left(\phi = \frac{\phi_{xy}}{2}\right)$$

Topic : Mohr Circle of Strain

$$C = \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

Topic : Principal Strains

$$\varepsilon_1 = C + R$$

$$\varepsilon_2 = C - R$$

Topic : Maximum Shear Strain

$$\frac{\phi_{max}}{2} = R$$

$$\phi_{max} = 2R$$

Topic : Principal Strains from Principal Stresses

$\varepsilon_1, \varepsilon_2$ ← Principal Strains

σ_1, σ_2 ← Principal Stresses

$$\varepsilon_x = \frac{\sigma_x - \mu(\sigma_y)}{E}$$

$$\varepsilon_1 = \frac{\sigma_1 - \mu\sigma_2}{E}$$

$$\varepsilon_2 = \frac{\sigma_2 - \mu\sigma_1}{E}$$

$$\varepsilon_1 + \mu\varepsilon_2$$

$$\frac{\sigma_1 - \mu\sigma_2 + \mu\sigma_2 - \mu^2\sigma_1}{E}$$

$$\frac{\sigma_1(1-\mu^2)}{E} = \varepsilon_1 + \mu\varepsilon_2$$

$$\sigma_1 = \left(\frac{E}{1-\mu^2}\right)\left(\varepsilon_1 + \mu\varepsilon_2\right)$$

Topic : Principal Stresses from Principal Strains

σ_1, σ_2 ← $\varepsilon_1, \varepsilon_2$

$$\sigma_1 = \frac{E}{1-\mu^2}\left(\varepsilon_1 + \mu\,\varepsilon_2\right)$$

$$\sigma_2 = \frac{E}{1-\mu^2}\left(\varepsilon_2 + \mu\,\varepsilon_1\right)$$

#Question

$\varepsilon_{xx} = 100\ \mu$ → $\times 10^{-6}$

$\varepsilon_{yy} = -50\ \mu$

$\gamma_{xy} = 60\ \mu = \phi_{xy}$

Find Principal Strains and Stresses

$E = 2 \times 10^5$, Poisson ratio $= 0.25 = \nu$

μ (Micron)

$$\sigma_1 = \frac{2\times10^5\times10^{-6}}{1-0.25^2}\left(105.8 + 0.25\times(-55.8)\right)$$

$$= 19.6\ \text{MPa}$$

$$\sigma_2 = \frac{2\times10^5\times10^{-6}}{1-0.25^2}\left(-55.8 + 0.25\times105.8\right)$$

$$= -6.26\ \text{MPa}$$

$$C = \frac{100\mu - 50\mu}{2} = 25\ \mu$$

$$R = \sqrt{\left(\frac{100-(-50)}{2}\right)^2 + \left(\frac{60}{2}\right)^2}\ \mu$$

$$R = 80.8\ \mu$$

$\varepsilon_1 = 105.8\ \mu$, $\phi_{max} = 2R = 161.6\ \mu$

$\varepsilon_2 = -55.8\ \mu$,

$\times 10^{-6}$

#Question – ESE Prelims 2004

$\varepsilon_{xx} = 800\ \mu$

$\varepsilon_{yy} = 100\ \mu$

$\gamma_{xy} = -800\ \mu$

Find Principal Strains

a)981 μ, -81 μ

b)981 μ, -18 μ

c)881 μ , 71 μ

d)839 μ, -81 μ

$C = 450\ \mu$

$R = \sqrt{350^2 + 400^2} = 10\sqrt{2825} = 530\ \mu$

$\varepsilon_1 = 980\ \mu, \ -80\ \mu$

#Question – ESE Prelims 2014

$\varepsilon_{xx} = 800\ \mu$

$\varepsilon_{yy} = 400\ \mu$

$\gamma_{xy} = 300\ \mu$

Find maximum shearing strain

a)150 μ

b)355 μ

c)250 μ

d)500 μ

$$R = \sqrt{200^2 + 150^2} = 10\sqrt{400+225} = 250\ \mu$$

$$\phi_{max} = 2R = 500\ \mu$$

#Question – ESE Prelims 2018

If strains on a piece of metal are $\varepsilon_x = -120\ \mu m/m$, $\varepsilon_y = -30\ \mu m/m$, and $\gamma = 120\ \mu m/m$, what is the maximum principal strain ?

(a) 0

(b) 50 $\mu m/m$

(c) 75 $\mu m/m$

(d) 150 $\mu m/m$

$C = -75\ \mu$

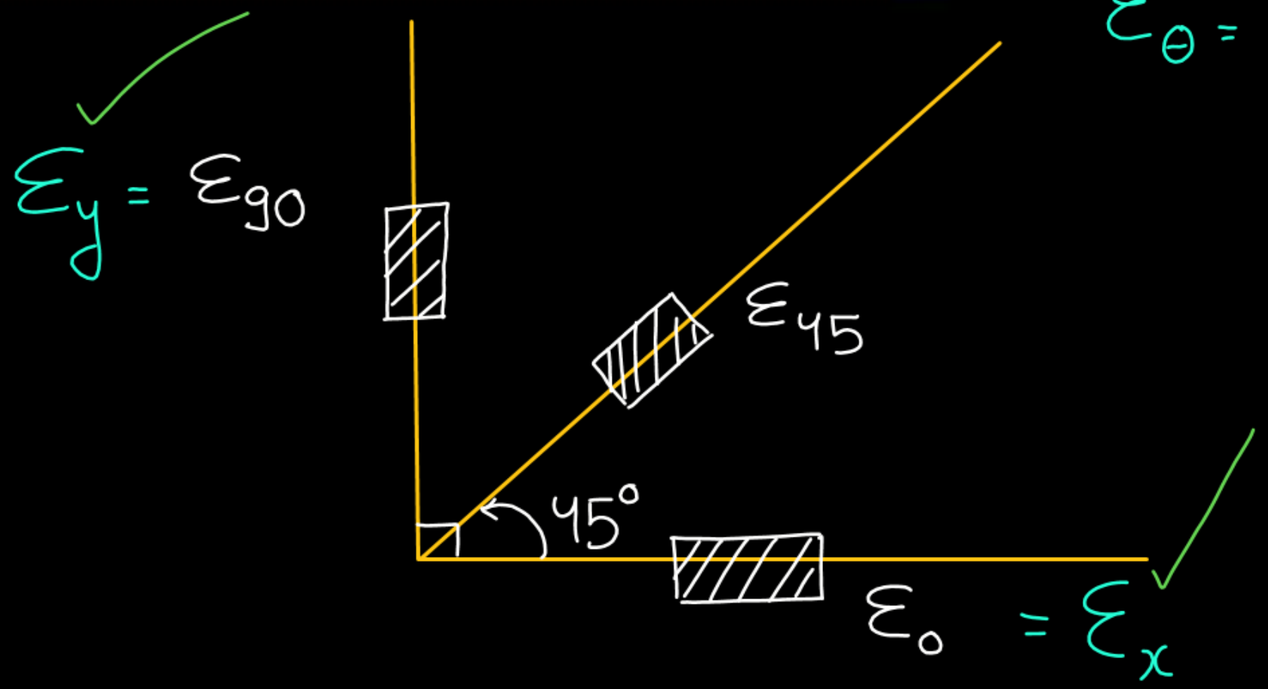
$R = 75\ \mu$

$\varepsilon_1 = 0 = C + R$

$\varepsilon_2 = C - R = -75 - 75$

$= -150\ \mu$

Topic : Strain Rosette



$$\varepsilon_\theta = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta$$

$$\phi_{xy} = (2\varepsilon_{45}) - (\varepsilon_0 + \varepsilon_{90})$$

$$2\varepsilon_{45} = \varepsilon_x + \varepsilon_y + \phi_{xy} = \varepsilon_0 + \varepsilon_{90} + \phi_{xy}$$

$$\varepsilon_x = \varepsilon_0$$

$$\varepsilon_y = \varepsilon_{90}$$

$$\phi_{xy} = 2\varepsilon_{45} - (\varepsilon_0 + \varepsilon_{90})$$

#Question

$\varepsilon_0 = 600\ \mu$ $= \varepsilon_x$

$\varepsilon_{90} = 200\ \mu$ $= \varepsilon_y$

$\varepsilon_{45} = 500\ \mu$

Find Principal Strains

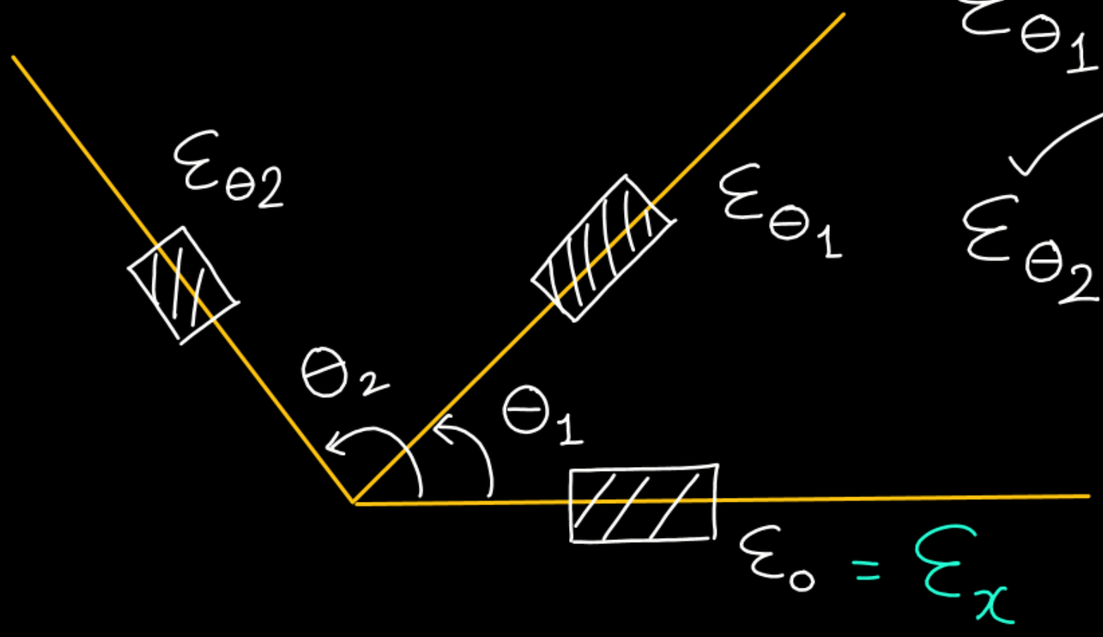
$$\phi_{xy} = 2\times500 - (600+200) = 200\ \mu$$

$$C = 400\ \mu$$

$$R = \sqrt{\left(\frac{600-200}{2}\right)^2 + \left(\frac{200}{2}\right)^2} = \sqrt{200^2+100^2} = 223.6\ \mu$$

$$\varepsilon_1 = 623.6\ \mu$$

$$\varepsilon_2 = 176.4\ \mu$$



$$\varepsilon_{\theta_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta_1 + \frac{\gamma_{xy}}{2}\sin 2\theta_1$$

$$\varepsilon_{\theta_2} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta_2 + \frac{\gamma_{xy}}{2}\sin 2\theta_2$$

Solve and Find

$\varepsilon_y, \gamma_{xy}$

#Question

$\varepsilon_0 = -100\ \mu$ $= \varepsilon_x = -100\ \mu$

$\varepsilon_{60} = 700\ \mu$

$\varepsilon_{120} = -600\ \mu$

Find Principal Strains

$\varepsilon_{120} = -600\ \mu$

$\varepsilon_y = a$, $\gamma_{xy} = b$

$1400 = (-100 + a) + (-100 - a)\cos 120 + b \cdot \sin 120$ ($\cos 120 = -0.5$)

$1500 = a + 50 + 0.5a + 0.866b$

$1.5a + 0.866b = 1450\ \mu$

$$-1200 = -100 + a + (-100 - a)\cos 240 + b \sin 240$$

$\cos 240 = -0.5$

$$-1100 = a + 0.5a + 50 - 0.866b$$

$$1.5a - 0.866b = -1150$$

$$1.5a + 0.866b = 1450$$

$$3a = 300$$

$$a = \varepsilon_y = 100\ \mu$$

$$b = 1501.1\ \mu = \phi_{xy}$$

$$(\gamma_{xy})$$

$\varepsilon_x = -100\ \mu$

$\varepsilon_y = 100\ \mu$

$\gamma_{xy} = 1501.1\ \mu$

$C = 0$

$R = \sqrt{(-100)^2 + (750.55)^2}$

$= 757.2\ \mu$

$\varepsilon_1 = 757.2\ \mu$

$\varepsilon_2 = -757.2\ \mu$

01 Mohr's Circle for Strain

Thank You GW Soldiers