UBS Investment Bank

Commodity-Linked Products: Best of n/Worst of n

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USD Exotic Fixed Income Derivatives

Product of Sales and Trading, Not Research

For Professional Investors Only

See Disclosures

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Basic Investment Rationale

Structured products are synthetic instruments that consist of the following**:**

Investment view + Payoff Structure + Wrapper

Investment view: offering clients targeted risk exposure

- Underlying asset(s)/indices: bullish, bearish, flat, range bound, high vol; relative value & cross-asset, curve trades
- Volatility: realized vs. implied, skew
- Correlation- convergence/divergence, stability

Payoff structure:

- Capital preservation: principal protection vs. principal at risk (PARs)
- Embedded optionality- callable and autocallable structures, extendible collars, coupon catch-up features
- Leverage (gearing)

Wrapper:

- Bond, note, insurance contract, ERISA-compliant

Rainbow Options (Best of n/Worst of n)

Rainbows are essentially puts and calls written on the best performing (or alternatively, worst performing) asset in a basket of n underlying assets.

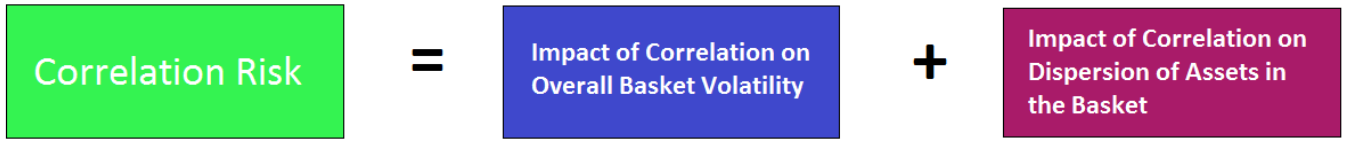
- **The literature on rainbows focuses on equities:** they have the widest participation base, are exchange-traded and the data is easiest to obtain. Much of the general approach for equities can be adapted to FX and commodities. No closed form solution exists for rainbow options of >3 assets. Pricing is determined by Monte Carlo simulation.
- **A rainbow option is intuitively more valuable the lower the correlations between underlying assets** because of the greater variation in combined asset price paths. If constituent pair-wise correlations are negative and approach -1, the option should be in the money for at least one of the assets.
- Indeed**, the pricing of these products is driven by the correlation structure.** Correlation risk is time-dependent and difficult to manage: it is not directly observable in the market, cannot be hedged precisely, and relying on historical data is misleading- from one period to the next, pair-wise correlations may vary substantially. Applying multivariate GARCH models, researchers have found the number of required parameters increases rapidly for even moderate dimensionality.
- Practitioners generally use fixed historical correlation estimates and update them periodically.

Some Variations

- **Best of assets or cash:** $\max(S_1, S_2, \ldots, S_n, K)$
- **Call on max:** $\max(\max(S_1, S_2, \ldots, S_n) - K, 0)$
- **Call on min:** $\max(\min(S_1, S_2, \ldots, S_n) - K, 0)$
- **Put on max:** $\max(K - \max(S_1, S_2, \ldots, S_n), 0)$
- **Put on min:** $\max(K - \min(S_1, S_2, \ldots, S_n), 0)$
- **Spread:** $\max(S_1, S_2, \ldots, S_n) - \min(S_1, S_2, \ldots, S_n)$
- **Out-performance:** $\max(S_1, S_2, \ldots, S_n) - \mathrm{avg}(S_1, S_2, \ldots, S_n)$
- **Best m of n/Worst m of n**

On Correlation

Correlation risk for best/worst and averaging baskets can be decomposed into two constituents:



Calls on a basket (say, 25% Aluminum, 25% Copper, 25% Nickel, 25% Zinc) are redeemed at the total value earned by the portfolio of underlying assets. As a result, **higher correlations increase basket volatility and hence, the option value.** Investors are long correlation.

Bought Best of n: while higher correlations among assets increase the basket's volatility, lower correlations increase the dispersion of asset returns. **The dispersion effect dominates so investors are net short correlation.**

Sold Worst of n: similarly, the low dispersion effect will result in all assets performing quite well or at least several assets performing poorly. Since the sold worst-of option would reference the underperforming asset, **option writers are long correlation,** similar to the equity tranche holder in a CDO.

Case of 'front runner' assets:

At issue date it is unknown which asset will be the best performing in a basket. Yet, **as the basket option nears maturity**, there is often a 'front runner', that is, a candidate likely to be the final winner. As one might guess, **the best-of option is far more exposed to changes in correlation between the best two assets than the worst two, which are less important or even irrelevant**. As a winner emerges, one can argue that correlation is less of an important factor.

Risks for the Dealer on Commodities-Linked Products (Non FX)

- **Second order risks: Gamma and cross gamma**

  Cross gamma is very difficult to measure. Complicating matters, for n assets, there are n(n-1)/2 cross gamma terms.

- **Skew/Smile**

- **Parameter Estimates for Correlation and Volatility: Historical vs. Implied (risk-neutral)**

Two assets close in price: gamma & cross-gamma become unstable

Explanation:

As an aside, for two assets that are close to being the best-of winner, interesting hedging considerations develop for the dealer. It is assumed the dealer hedged most of the client risk on the trade date and subsequently manages residual risk on a book of commodity trades outstanding. If two assets are sufficiently close in price, then the gamma and cross gamma become very unstable (if asset 1>asset 2 near expiry, the dealer is substantially long on asset 1. If the two assets switch places the next day, the dealer must sell a large quantity of asset 1 and buy asset 2).

Typical Trade: 1.5Y USD Laggard Breakfast Note (95% Principal Protected)

Basket Components	Exchange	Bloomberg Code	Price as of Launch Date
Sugar	NYBOT	SB1	13.6 cents per pound
Soybean	CME	S 1	1417 cents per bushel
Corn	CME	C 1	568.5 cents per bushel

- **Redemption Amount: Subject to Early Redemption, in respect of each Note, an amount in USD determined in accordance with the following:**

  (i) If LCR (FVD) < 0%, Specified Denomination x Max [95%, 100% + LCR(FVD)]

  (ii) If LCR (FVD) > 21%, Specified Denomination x [1 + LCR (FVD)];

  (iii) If 0% <= LCR (FVD) <= 21%, Specified Denomination x 121%,

  Where LCR(FVD), means the Laggard Commodity Return as of the Final Valuation Date

- **Early Redemption:If LCR(i) is greater than or equal to 0 in respect of a Calculation Date i, each Note will be early redeemed at the relevant Early Redemption Amount (specified below) on the day that is five (5) Business Days following such Calculation Date i (such date being the "Early Redemption Date") .**
- **Early Redemption Amount: Calculation Date 1:Specified Denomination x 107%**
- **Calculation Date 2: Specified Denomination x 114%**
- **Calculation Dates i: i = 1:September 18, 2008, i = 2:March 18, 2009**

Business Model for Structured Notes/FI Exotics

Client Price

\- **Cost of Replicating Hedge Portfolio (mid market)**

\- **Execution Costs (bid/offer)**

\- **Valuation reserve to cover model risk and illiquidity**

\- **Credit reserve**

= **Revenue from the transaction for UBS**

UBS Competitive Advantages:

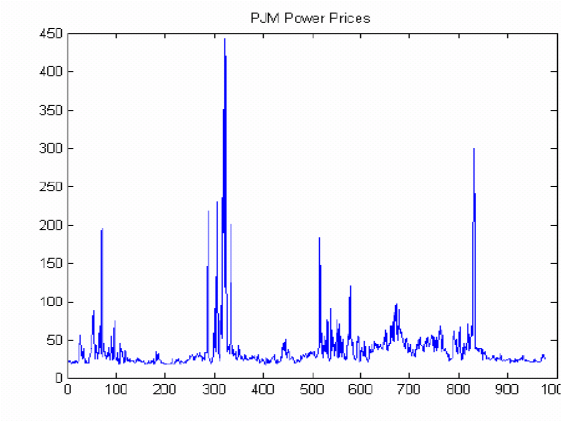
- **Access to the Wealth Management distribution channel and UBS Private Bank, Switzerland.**
- **The market for structured financial products in Switzerland is considered the largest in the world.**
- **Product design: Structuring teams in London, Stamford and Zurich and Quantitative Research groups in London and Stamford.**
- **For vanilla hedges, the FI Exotics desk executes swaps with UBS metals, energy and agricultural commodities desks in London and Stamford.**
- **Model-based risk aggregation for efficient hedging, portfolio effects**

Modeling Considerations for Commodity Prices

- **Commodity prices typically exhibit mean-reversion and elements of seasonality, which are generally not observed in the equity markets. For natural gas, the return distribution has kurtosis (fat tails) and skewness (elongated tails). For electricity, there are sharp and asymmetric spikes that require advanced models (jump-diffusion and Levy processes).**

	Annual Volatility	Skewness	Kurtosis
Nord Pool	182%	1.468	26.34
NP 6 pm	238%	2.079	76.82
DAX	23%	0.004	3.33

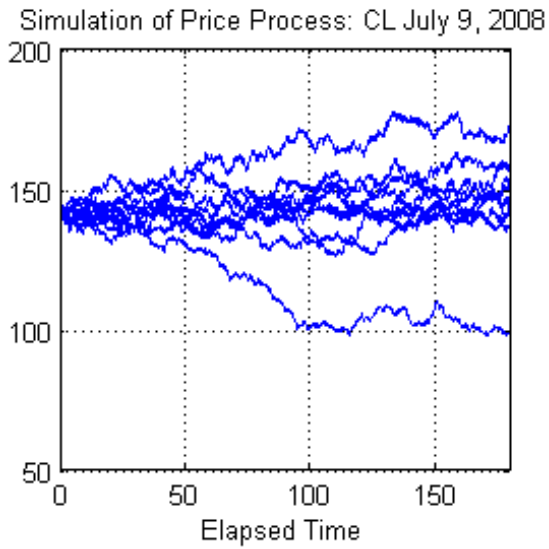
A. Werner, Risk Management for the Electricity Market, 2003

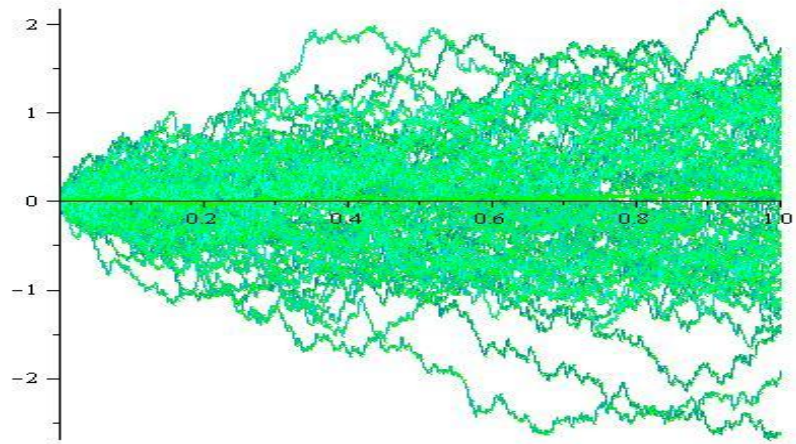


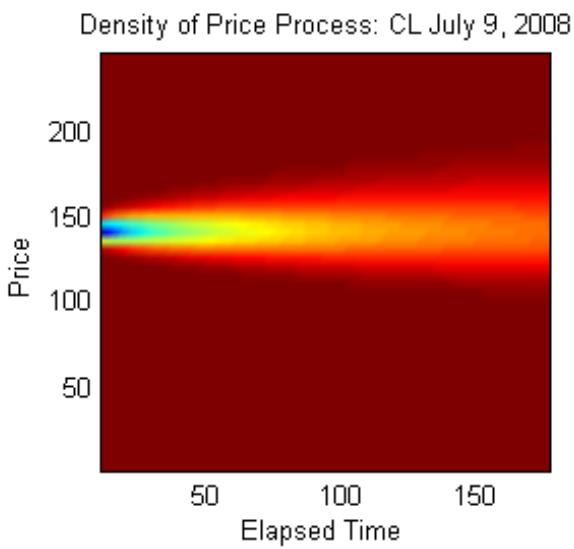
A. Eydeland, Energy Derivatives, 2006

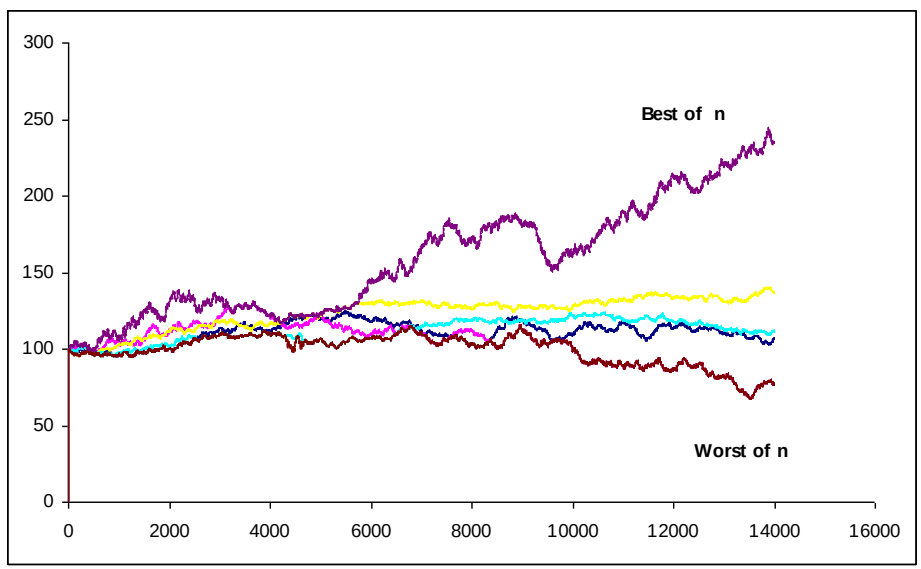
- **There does not appear to be an industry-standard model in terms of dynamics and number of parameters.**
- **Multivariate lognormal diffusion models are often used for multifactor commodity curves.**
- **The Heath-Jarrow-Morton framework for term structure of interest rates is used for oil futures curves as well. The volatility and correlation structure is estimated through Principal Components Analysis (PCA).**
- **There are complex attributes between the spot physical commodity and its forward contracts and other derivatives, e.g. stochastic convenience yield for the owner of a commodity held in storage.**

Monte Carlo Methods: Computational Efficiency vs. Oversimplification









Commonly Used Models for Commodity Price Dynamics

- **Parameters are estimated from calibrating to vanilla instruments- calls, puts, swaps, caps, etc. However, local volatility functions calibrated in this manner generally perform poorly in modeling the shape of the implied volatility surface. That is, deterministic local volatility functions are adequate for static hedges but do not account for observed stochastic volatility (jumps, regime switches).**
- **Since liquid strikes are not present for many contract specifications, synthetic replication often breaks down. However, replication arguments do hold up well if volatility of volatility is not too high.**

Ornstein-Uhlenbeck

$$dX_t = (AX_t + b_t)dt + \sigma_t dW_t$$

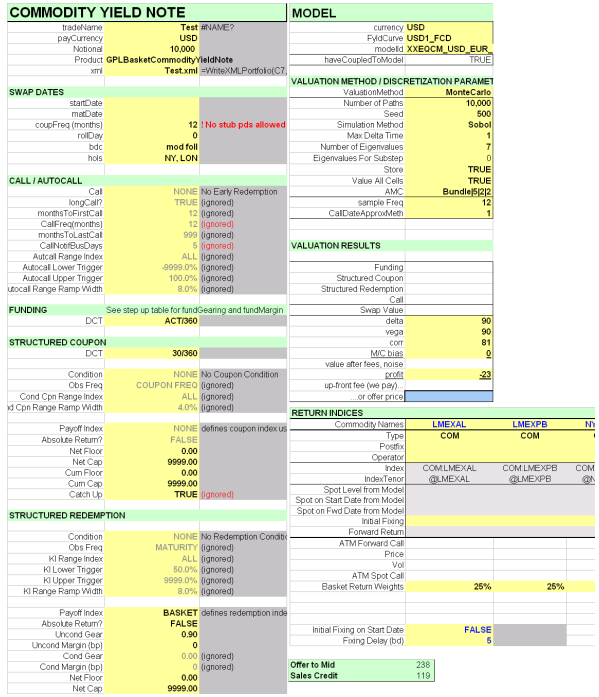
$$E_t\left[dW_t dW_t^T\right] = \rho dt$$

Geman, 2000

$$dS = k(L_t - lnS_t)S_t dt + \sqrt{V_t}dW_t^1$$

$$\frac{dL_t}{L_t} = \mu_t dt + \sigma_2 dW_t^2$$

$$dV_t = b(c - V_t)dt + \sigma_3\sqrt{V_t}dW_t^3$$



COMMODITY YIELD NOTE

tradeName	Test	#NAME?
payCurrency	USD	
Notional	10,000	
Product	GPLBasketCommodityYieldNote	
xml	Test.xml	=WriteXMLPortfolio(C7,
SWAP DATES		
startDate		
matDate		
coupFreq (months)	12	! No stub pds allowed
rollDay	0	
bdc	mod foll	
hols	NY, LON	
CALL / AUTOCALL		
Call	NONE	No Early Redemption
longCall?	TRUE	(ignored)
monthsToFirstCall	12	(ignored)
CallFreq(months)	12	(ignored)
monthsToLastCall	999	(ignored)
CallNotifBusDays	5	(ignored)
Autcall Range Index	ALL	(ignored)
Autocall Lower Trigger	-9999.0%	(ignored)
Autocall Upper Trigger	100.0%	(ignored)
utocall Range Ramp Width	8.0%	(ignored)
FUNDING	See step up table for fund	Gearing and fundMargin
DCT	ACT/360	
STRUCTURED COUPON		
DCT	30/360	
Condition	NONE	No Coupon Condition
Obs Freq	COUPON FREQ	(ignored)
Cond Cpn Range Index	ALL	(ignored)
d Cpn Range Ramp Width	4.0%	(ignored)
Payoff Index	NONE	defines coupon index us
Absolute Return?	FALSE	
Net Floor	0.00	
Net Cap	9999.00	
Cum Floor	0.00	
Cum Cap	9999.00	
Catch Up	TRUE	(ignored)
STRUCTURED REDEMPTION		
Condition	NONE	No Redemption Conditi
Obs Freq	MATURITY	(ignored)
KI Range Index	ALL	(ignored)
KI Lower Trigger	50.0%	(ignored)
KI Upper Trigger	9999.0%	(ignored)
KI Range Ramp Width	8.0%	(ignored)
Payoff Index	BASKET	defines redemption inde
Absolute Return?	FALSE	
Uncond Gear	0.90	
Uncond Margin (bp)	0	
Cond Gear	0.00	(ignored)
Cond Margin (bp)	0	(ignored)
Net Floor	0.00	
Net Cap	9999.00	

MODEL

currency	USD
FyldCurve	USD1_FCD
modelId	XXEQCM_USD_EUR_
haveCoupledToModel	TRUE
VALUATION METHOD / DISCRETIZATION PARAMET	
ValuationMethod	MonteCarlo
Number of Paths	10,000
Seed	500
Simulation Method	Sobol
Max Delta Time	1
Number of Eigenvalues	7
Eigenvalues For Substep	0
Store	TRUE
Value All Cells	TRUE
AMC	Bundle|5|2|2
sample Freq	12
CallDateApproxMeth	1
VALUATION RESULTS	
Funding	
Structured Coupon	
Structured Redemption	
Call	
Swap Value	
delta	90
vega	90
corr	81
M/C bias	0
value after fees, noise	
profit	-23
up-front fee (we pay)...	
...or offer price	

RETURN INDICES			
Commodity Names	LMEXAL	LMEXPB	N
Type	COM	COM	
Postfix			
Operator			
Index	COM:LMEXAL	COM:LMEXPB	COM
IndexTenor	@LMEXAL	@LMEXPB	@N
Spot Level from Model			
Spot on Start Date from Model			
Spot on Fwd Date from Model			
Initial Fixing			
Forward Return			
ATM Forward Call			
Price			
Vol			
ATM Spot Call			
Basket Return Weights	25%	25%	
Initial Fixing on Start Date	FALSE		
Fixing Delay (bd)	5		

Offer to Mid	238
Sales Credit	119

UBS FI Exotics Pricer: Front End

As in the Heston Model, c is the long term mean volatility with b as the speed of mean reversion. σ_3 is the volatility of volatility. dWt^1 and dWt^3 are correlated with a constant coefficient ρ.

Empirical Observations on Volatility

Steele argues that the volatility term structure shares similarities across equities, fixed income and commodities (excl. electricity)

Short-dated implied volatilities move more than long-dated volatilities

Shape and dynamics of the vol term structure imply mean reversion, that is, changes in vol are autocorrelated.

Yet empirically, there is volatility clustering.

- **High volatility begets high volatility**
- **Large negative shocks tend to produce a greater increase in volatility than positive shocks of comparable size**

As Volatility increases, so does:

- **Volatility on volatility**
- **Volatility skew**

J. Gatheral, “Modeling the Implied Volatility Surface”, Stanford Financial Math Seminar, Feb. 2003

J.M. Steele, “Martingale Markets”, Columbia APD, June 2008

Summary

- Tenor must be sufficiently long enough to offer a compelling participation rate (more option value from interest forgone on the zero coupon bond).
- In theory, principal protection is attractive to all investors- “who wouldn’t want it?”
- In practice, the protection element can limit upside participation with lower gearing and caps on maximum payouts: cost of insurance.
- Greater transparency in fees and secondary market pricing is needed to bolster liquidity and end-user demand, particularly from institutional clients
- Multi-commodity options are useful for portfolio diversification and investing with “hindsight”
- Dealer risks from exotics such as barrier options can be offset to a large extent by using vanilla options and then dynamically hedging them during the life of the trade. Risk is bucketed along a curve and stress reports (e.g. +10bp, -10bp in rates) allow dealers to examine trade characteristics under different hypothetical market scenarios.

Thank you for your time

1. No one knows what fair value of anything is, though one can begin to tell when it gets to be unfair. Fischer Black wrote somewhere that a price anywhere between half and twice of fair value is fair value. If you run this recursively, you get a range from zero to infinity.

2. People are strongly influenced by other people's opinions.

3. People try to figure out what other people will do.

4. People have a tendency to get on moving trains.

Is there a way to turn all this into a convincing model?

Emanuel Derman, Former Head of Goldman Sachs Quantitative Strategies, on Asset Bubbles, March 2008

"Structured product makers always used to battle to be the most innovative house and to come up with the most complex solutions, which was why we started seeing products that gave exposure to volatility smile, to forward smile and this kind of thing. The problem was that even the more sophisticated investors were not able to understand what was driving the behavior of these assets. That meant that if a strategy ended up not being successful than it was difficult to explain why, Investors want simplicity and transparency."

Pierre Bes, Barclays Capital, as quoted in *Euromoney*, May 2008

On Correlation (continued)

Empirical Observations on Correlation

1. Correlations are even more unstable than volatilities.

2. In terms of persistence and memory, correlation models generally display positive autocorrelation as the most recent 20-30 trading days have the highest predictive power (correlation "regimes", i.e. high correlations beget high correlations).

3. The rolling 6 month implied correlation for the S&P 500 and Nikkei 225 from the options market, 2000-2008 YTD, has tended to trade at a premium to historical (IC ρ = 0.7 vs. HC ρ = 0.2 sampled weekly, though HC ρ = 0.5 from 2004-2008 YTD).

4. Systemic risks are by definition, rare events. Pricing these events by expectations seems inadequate to capture the "jump risk" in the correlation structure. Rather, scenario-based approaches linking credit spreads, equity spot levels, volatilities and correlations appear to be more representative of actual market dynamics. In stressed environments, one would expect equity prices to fall sharply and correlations to jump (as investors indiscriminately sell shares). To model these considerations, one needs a robust equity/credit model (see Zhu, Zhang, Zhou, BIS Working Paper, 2005)

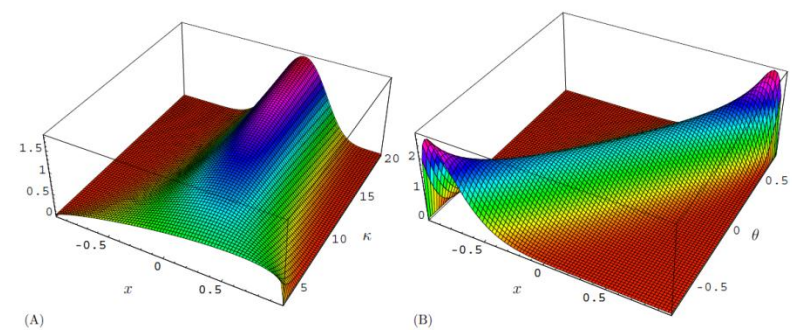
5. Covariance matrices are symmetric and positive semi definite. A negative definite matrix would imply that some portfolios have negative variances. In practice however, asynchronous data (different trading hours across global markets, holidays) may result in negative definite matrices (see S. Acomb, 2008).

6. **van Emmerich's model on stochastic correlation (2006):**

α , x_0 are parameters to calibrate to market data

$$dX_t = \alpha\sqrt{(1 - X_t)(1 + X_t)}dW_t \text{ , } \alpha \in \boldsymbol{R}^+, X_0 = x_0 \in (-1, 1)$$

$$E[X_t] = X_0 + E\left[\alpha \int_0^t \sqrt{(1 - X)(1 + X)}dW\right] = x_0$$



A drift term can be added; van Emmerich uses the Fokker-Planck equation to determine the transition density (plots above).

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