Unit-2

Continue

\# Let $A = \{a, b\}$

$R = \{(a,a)\ (b,a)\ (b,b)\}$

$S = \{(a,a)\ (b,a)\ (b,b)\}$

$(SOR)^{-1} = R^{-1} \circ S^{-1}$

Now to find S operation R.

$\Rightarrow (a,a) \in R$ and $(a,b) \in S$

$(a,b) \in SOR$

$\Rightarrow (b,a) \in R$ and $(b,a) \in S$

$(b,a) \in SOR$

$\Rightarrow (b,b) \in R$ and $(a,b) \in S$

$(b,b) \in SOR$

$SOR = \{(a,b)\ (b,a)\ (b,b)\}$

$(SOR)^{-1} = \{(b,a)\ (a,b)\ (b,b)\}$

$R^{-1} = \{(a,a)\ (a,b)\ (b,b)\}$

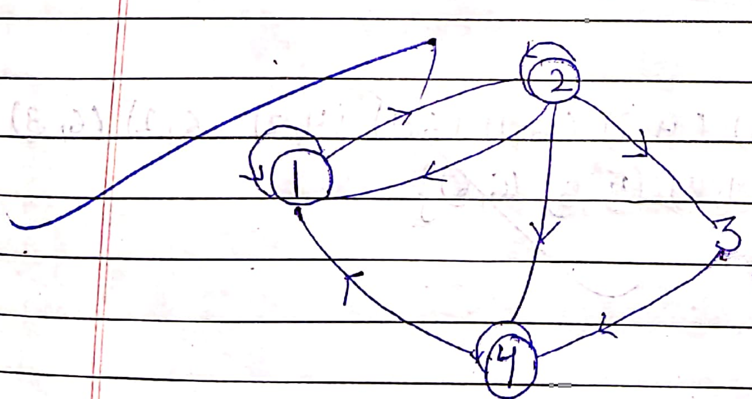
$S^{-1} = \{(a,a)\ (a,b)\ (b,b)\}$

$R^{-1} \circ S^{-1} = \{(a,b)\ (b,a)\ (b,b)\}$

\# Construct the graph of R

Let $A = \{1,2,3,4\}$

$R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (3,4)\ (4,1)\ (4,4)\}$



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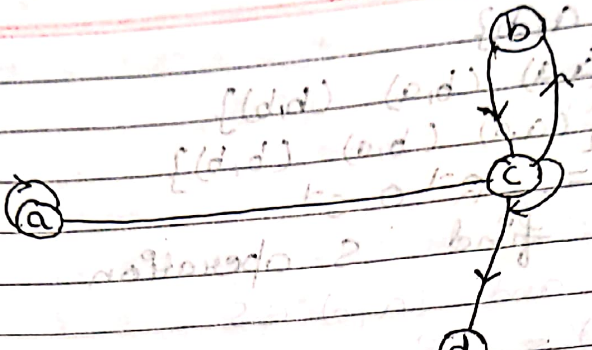
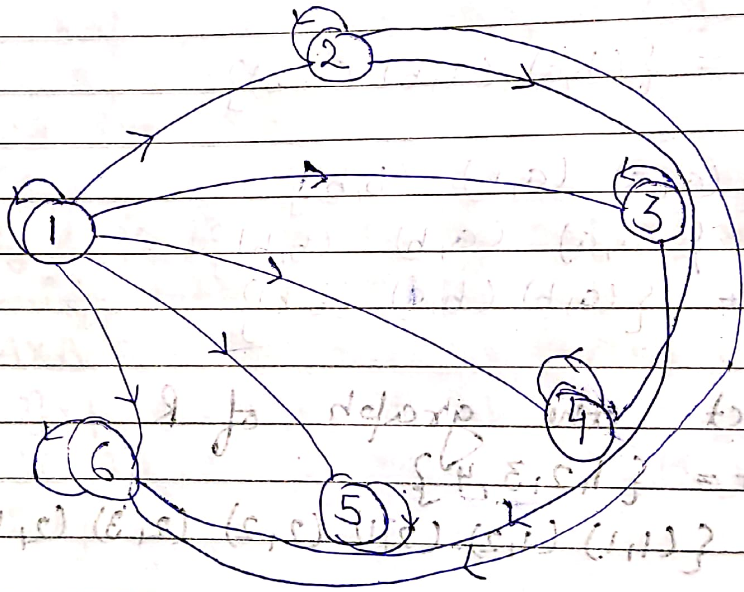


$A = \{a, b, c, d\}$

$R = \{(a,a)\ (a,c)\ (c,c)\ (b,c)\ (a,b)\ (c,d)\}$

Class-15 Let $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)\ (2,4)\ (2,6)\ (3,4)\ (1,1)$
$(2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\}$



$R^{-1} = \{(2,1)\ (3,1)\ (4,1)\ (5,1)\ (6,1)\ (4,2)\ (6,2)\ (6,3)$
$(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\}$

function :-

If A and B are two known empty set than a rule of f under which to every element x of the set A there correspondence one and only one elements of set capital B. The rule f is called the function from A to B it is denoted by $f : A \to B$.

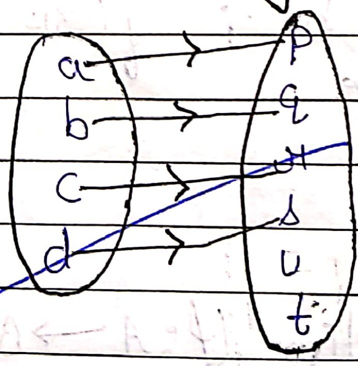
Domain :- Set of inputs is called Domain it is denoted by D_f. There the Domain is $N = \{1, 2, 3, 4 - - - - - -\}$

Codomain :- Set of possible outputs is called Codomain.

Range :- Set of actual outputs is called range it is denoted by R_f.

Methods of Representing a function :-

① Arrow Diagram :-



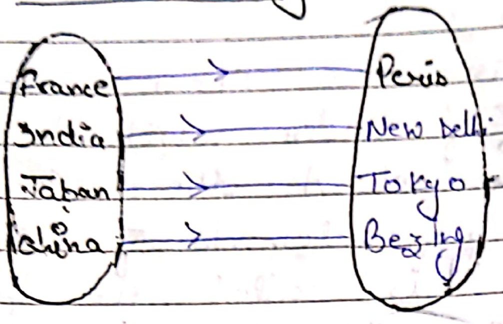
$D_f = \{a, b, c, d\}$

Codomain $= \{p, q, r, s, u, t\}$

$f(a) = p$, $f(b) = q$, $f(c) = r$

$f(d) = s$

(ii) Vertual Diagram :-



D_f = {france, India, Japan, china}
Codomain = {Peris, New delhi, Tokyo Bezing}
f(france) = Peris
f(India) = New Delhi
f(Japan) = Tokyo
f(china) = Bezing

Tabular form :-

A	x	1	2	3	4	5
B	y	a	b	c	d	e

$D_f = \{1, 2, 3, 4, 5\}$
Codomain = $\{a, b, c, d, e\}$

$f(1) = a$; $f(2) = b$; $f(3) = c$
$f(4) = d$; $f(5) = e$

Types of function :-

(1) **Real function :-** A function $(f: A \to B)$ is called real value if the image of every element of A under f is a real number.
$f(x) \in R \ \forall \ x \in A$

$y = f(x)$

(2) **Algebric function :-** The function ($f : A \to B$) consisting of finite number of terms involving different Power of independent variable x and the operations are called algebric function.

(3) **Polynomial function :-** A function whose domain and codomain both is the set of all real numbers and contains finite number of terms containing natural number power of x multiplied by the real constant is called polynomial function.

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots a_nx^n$$

(5) **Rational form :-** A function obtained by dividing a polynomial by the another polynomial is called a rational function.

$$y = f(x) = \frac{p(x)}{q(x)}$$

(6) **Irrational function :-** The Algebric function containing one more term having non-integeral relation Power of x are called Irrational function.

Ex- $$f(x) = \frac{x^{3/2} + 5x}{x^2+1}$$

$$= \frac{x \cdot x^{1/2} + \sqrt{x}}{x^2+1} - \frac{2x^2+5}{\sqrt{x}+2}$$

7- **Modulus function:-**

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

then $f(x)$ is called modulus function.

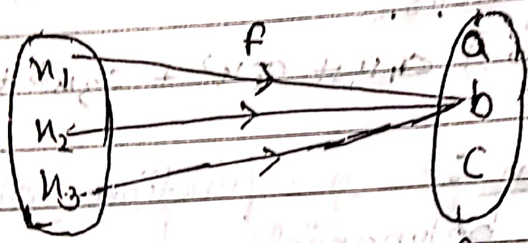
8- **Constant function:-** Let C be fixed real number. Then the function defined by $f(x) = C \; \forall x$ is called constant function C.

$D_f = R$ $\quad$ $R_f = C$

$A = \{x_1, x_2, x_3\}$ and $B = \{a, b, c\}$

and $f : A \to B$

$f(x_1) = b$ $\quad$ $f(x_2) = b$ $\quad$ $f(x_3) = b$



A $\qquad\qquad$ B

$D_f = \{x_1, x_2, x_3\}$ $\quad$ $R_f = \{b\}$ = Range of f

9- **Identity function:-** The function defined by $f(x) = x$ for all $x \in R$ is called the identity function clearly

$D_f = R$, $\quad R_f = R$

10- **Reciprocal function:-** The function defined by $f(x) = \frac{1}{x}$ is called the reciprocal function. The function $f(x) = \frac{1}{x}$ is not defined if $x = 0$

$D_f = R - \{0\}$, $\quad R_f = R - \{0\}$

11- **Exponential Function :-** The function $f(x) = e^x$ is called the exponential function.
Since $f(x) = e^x$ is defined for all real value of x. so
$D_f = R$. Also $y = e^x \Rightarrow x = \log y$
And, we know that $\log y$ is not defined when $y = 0$ or negative. So, $R_f =]0, \infty[$.

12- **Logarithmic function :-** The function $f(x) = \log x$ is called the logarithmic function. $\log x$ is not defined when x is zero or negative.
$D_f =]0, \infty[$
$R_f = \text{Range}(f) = \{\log x : x \in]0, \infty[\}$ = set of all real numbers =

13- **Trignometric function :-** The function $\sin x$, $\cos x$, $\tan$ $\cot x$, $\sec x$ are called the trignometric functions.
The domains and ranges of these functions are given below :-

1- $\sin x$:- $\sin x$ is defined for all value of $x \in R$ its domain $R = R$, As value of $\sin x$ lies between -1 and $+1$, so range $= [-1, 1]$

2- $\cos x$:- Since $\cos x$ is defined for all value of $x \in$ its domain $R = R$. As value of $\cos x$ lies between -1 and $+1$ so range $= [-1, 1]$

3- $\tan x$ We have $\tan x = \frac{\sin x}{\cos x}$ which is not defined when $\cos x = 0$:
$\cos x = 0 \Rightarrow \cos x = \cos(2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{2}$

Then domain $R = R - \{(2n+1)\frac{\pi}{2}, n \in I\}$

Since $\tan x$ take all value of x so its range $= R$

4) Secx :- We have $\sec x = \frac{1}{\cos x}$ is not defined when $\cos x = 0$ and $\cos x = 0 \Rightarrow \cos x = \cos(2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{2}$

$= R - \{(2n+1)\frac{\pi}{2}, n \in I\}$

$= R -]-1, 1[$

5) Cotx : We have $\cot x = \frac{\cos x}{\sin x}$ is not defined when

$\sin x = 0 \Rightarrow \sin x = \sin \pi$

$\Rightarrow x = n\pi \; ; \; n = 0, 1, 2, 3 \ldots$

Domain $= R - \{n\pi : n \in I\}$

Range $= R$

6) Cosecx : We know $\text{Cosec} = \frac{1}{\sin x}$ is not defined

When $\sin x = 0 \Rightarrow \sin x = \sin n\pi$

$\Rightarrow x = n\pi$

Domain $= R - \{n\pi : n \in I\}$

Range $= R -]-1, 1[$

If $f(x) = x^3 - \frac{1}{x^3}$ find the value of $f(x) + f(\frac{1}{x})$

$f(x) = x^3 - \frac{1}{x^3}$

$f(\frac{1}{x}) = \frac{1}{x^3} - \frac{1}{1/x^3}$

$f(\frac{1}{x}) = \frac{1}{x^3} - x^3$

Now, $f(x) + f\left(\frac{1}{x}\right)$

$\cancel{x^3} - \cancel{\frac{1}{x^3}} + \cancel{\frac{1}{x^3}} - \cancel{x^3}$

0 Ans.

If $f(x) = x^5 - \frac{1}{x^5}$ find the value of $f(x) + f\left(\frac{1}{x}\right)$

$f(x) = x^5 - \frac{1}{x^5}$

$f\left(\frac{1}{x}\right) = \frac{1}{x^5} - \frac{1}{1/x^5}$

$f(x) = f(x) + f\left(\frac{1}{x}\right)$

$= x^5 - \frac{1}{x^5} + \frac{1}{x^5} - x^5$

$f(x) = 0$

If $f(x) = \log\frac{(1+x)}{1-x}$ show the $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$

$f(x) = \log\frac{1+x}{1-x}$

$f(y) = \log\frac{(1+y)}{1-y}$

$f(x) + f(y) = \log\left(\frac{1+x}{1-x}\right) + \log\left(\frac{1+y}{1-y}\right)$

$f(x) + f(y) = \log\left[\left(\frac{1+x}{1-x}\right) \times \left(\frac{1+y}{1-y}\right)\right]$

$$f(x)+f(y) = \log\left(\frac{1+y+x+xy}{1-y-x+xy}\right)$$

R.H.S $\quad f\left(\frac{x+y}{1+xy}\right) = \log\left\{\frac{1+\frac{x+y}{1+xy}}{1-\left(\frac{x+y}{1+xy}\right)}\right\}$

$$= \log\left\{\frac{1+xy+x+y}{1+xy+x+y}\right\}$$

L.H.S = R.H.S

If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ show that

$$f\left(\frac{2x}{1+x^2}\right) = 2f(x)$$

$$f(x) = \log\left(\frac{1+x}{1-x}\right)$$

L.H.S $\quad f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$

$$= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$= \log\left(\frac{1+x}{1-x}\right)^2$$

$$= 2\log\left(\frac{1+x}{1-x}\right)$$

$= 2f(x)$ = R.H.S.

V.V.Imp

(1) Let $f(x) = (2x+5)$ and $g(x) = x^2 + 1$

(i) gof (2) fog (3) fof (4) ff

(1) gof

Let $gof(x) = g(2x+5)$

$gof(x) = (2x+5)^2 + 1$

$= 4x^2 + 25 + 2 \times 2x \times 5 + 1$

$= 4x^2 + 20x + 26$

(2) fog

Let $f(g(x)) = f(x^2+1)$

$= 2(x^2+1) + 5$

$= 2x^2 + 2 + 5$

$= 2x^2 + 7$

(iii) fof

Let $f(f(x)) = f(2x+5)$

$= 2(2x+5)+5$

$= 4x + 10 + 5$

$= 4x + 15$ Ans

(4) ff

Let $f(f(x))$

$f[f(x)] = f(x) \cdot f(x)$

$= (2x+5)(2x+5)$

$= (2x+5)^2$

$= 4x^2 + 25 + 20x$ Ans

Ques-1 $f(x) = \sqrt{x-1}$
$g(x) = \sqrt{4-x^2}$

① $(f-2g)(x)$ ② $f(g(x))$

(i) $(f-2g)(x)$
$f(x) - 2g(x)$
$\sqrt{x-1} - 2\sqrt{4-x^2}$

$f[g(x)]$
$f[\sqrt{4-x^2}]$
We know $f(x) = \sqrt{x-1} \Rightarrow \sqrt{\sqrt{4-x^2}-1}$

find the range of the function

(i) $f(x) = \dfrac{1}{2-\cos 3x}$

$f(x) = \dfrac{1}{2-\cos 3x}$

$f(x) = y$

We have

$y = \dfrac{1}{2-\cos 3x}$

$\dfrac{1}{y} = 2-\cos 3x$

$\cos 3x = 2-\dfrac{1}{y}$

$-1 \leq \cos 3x \leq 1$

$-1 \leq (2-\dfrac{1}{y}) \leq 1$

Range $\{\frac{1}{3}, 1\}$

L.H.S

$-1 \leq 2-\dfrac{1}{y}$

$\dfrac{1}{y} \leq 2+1$

$\boxed{y \geq \frac{1}{3}}$

R.H.S

$2-\dfrac{1}{y} \leq 1$

$2-1 \leq \dfrac{1}{y}$

$1 \leq \dfrac{1}{y}$

$y \geq 1$

Ques-1 If f: then f^{-1} We h... He...

Ques-1 If $f: R \to R$ is defined by $f(x) = x^2+1$ then then $f^{-1}(3)$, $f^{-1}(10, 37)$

We have

from ①

$f^{-1}(3)$

$f(x) = x^2+1$

$3 = x^2+1$

$3-1 = x^2$

$2 = x^2$

$\therefore \boxed{x = \pm\sqrt{2}}$

Hence $f^{-1}(3) = (-\sqrt{2}, +\sqrt{2})$

from ②

$f^{-1}(10) = x^2+1$

$10 = x^2+1$

$10-1 = x^2$ or $9 = x^2$ or $x = \pm\sqrt{9}$

$\boxed{x = \pm 3}$

Hence, $f^{-1}(10) = \{-3, +3\}$

Again

$f^{-1}(37) = x^2+1$

$37 = x^2+1$

$37-1 = x^2$

$36 = x^2$

$x = \pm\sqrt{36}$ $\boxed{x = \pm 6}$

Hence, $f^{-1}(37) = \{-6, +6\}$

Ans Ques-1 If $f: X \to Y$ and A & B are two subset of X then prove that

i) $f(A \cup B) = f(A) \cup f(B)$

= Let Y be any arbitrary element of a $f(A \cup B) = f(A) \cup f(B)$

Let Y be any arbitrary element of a $f(A \cup B) = f(A) \cup f(B)$

$y \in f(A \cup B) \Rightarrow y = f(x)$ when $x \in A \cup B$

$y = f(x)$, where $x \in A$ and $x \in B$

$y = f(x)$, $x \in A$ OR $y = f(x)$ and $x = B$

$y = f(A) = f(B)$

$y \in f(A \cup B)$

$y \in f(A) \cup f(B)$

$f(A \cup B) \subseteq f(A) \cup f(B)$

Again let arbitrary element of $f(A) \cup f(B)$ then $y \in f(A) \cup f(B)$

$y \in f(A)$ or $y \in f(B)$

$y = f(x)$ where $x \in A$ and $x \in B$

$y = f(x)$ where $x \in (A \cup B)$

$y = f(x)$ where $f(x) \in (A \cup B)$

$y \in f(A \cup B)$

$f(A) \cup f(B) \subseteq f(A \cup B)$

Ques-1 Is the inverse function $x^2 - 3 = y$ possible given reason.

Let $x \in R$ (Domain) & $y \in R$ (codomain)

$f : X \rightarrow Y$

$x^2 - 3 = y$

$f(x) = y$

$x^2 - 3 = y$

$x^2 = y + 3$

$x = (y+3)^{1/2}$

$f^{-1} : R \rightarrow R$

$f^{-1}(x) = (y+3)^{1/2}$ $\therefore$ $x \in R$

Ques

#

#

Ques Find the domain of the function
$$f(x) = \frac{x}{x^2-3x+2}$$

$$x^2 - 3x + 2 = 0$$
$$x^2 - 2x - x + 2 = 0$$
$$x(x-2) - 1(x-2) = 0$$
$$(x-2)(x-1) = 0$$
$$x = 1, 2$$

Domain $R - \{1, 2\}$

$f(x) = \frac{1}{x^2-1}$

$$x^2 - 1 = 0$$
$$x^2 = 1$$
$$x = \pm\sqrt{1}$$

domain Codomain

$R - \{1, -1\}$

Prove that if $f: A \to B$ is 1.1 on to maping then $f^{-1}: B \to A$ will be 1.1 maping

Since Prove f is 1.1 on to maping so we have

$$y = f(x)$$
$$f^{-1}(y) = x \quad x \in A \text{ and } y \in B$$
$$y_1 = f(x_1) \qquad x_1, x_2 \in A$$
$$y_2 = f(x_2) \qquad y_1, y_2 \in B$$
$$f^{-1}(y_1) = x_1$$

Similarly

$$f^{-1}(y_2) = x_2$$
$$\boxed{x_1 = x_2}$$
$$\boxed{y_1 = y_2}$$

Show the maping f^{-1} is 1.1

again Let x be an elementary of A for the maping f there exist an element y in B. such that $y = f(x)$

$f^{-1}(y) = x \quad \forall \; x \in A$

$\{f^{-1}(y) : B,\ y \in B\} = A$

Thus $f^{-1} : B \to A$ is on to

Hence $f^{-1} : B \to A$ is 11 on to matri's

\# If $f : A \to B$ such that $f(x) = x - 1$ and $g : B \to C$ such that $g(y) = y^2$ To find $f \circ g(y) = ?$

We have $f : A \to B$ such that

$f(x) = x - 1$

$g : B \to C$ such that $g(y) = y^2$

$f \circ g(y)$

$f[g(y)]$

$f[y^2]$

$\boxed{f[y^2] = y^2 - 1}$

\# If $f(x) = x^2 - 1$ and $g(x) = 3x + 1$

① $g \circ f$ ② $f \circ g$ ③ $f \circ f$ ④ $g \circ g$

① $g \circ f$

$g[f(x)]$

$g[x^2 - 1]$

$3[x^2 - 1] + 1$

$3x^2 - 3 + 1$

$\boxed{g \circ f = 3x^2 - 2}$

② $f \circ g$

$f[g(x)]$

$f[3x+1]$

$(3x+1)^2 - 1$

$9x^2 + 1 + 6x - 1$

$9x^2 + 6x$

$\boxed{f \circ g = 3x[3x + 2]}$

③ $f \circ f$

$f[f(\ldots$

$f(x^{\ldots}$

$(x^2 - \ldots$

$(x^2)^{\ldots}$

$x^4 \ldots$

$\boxed{f \circ f = x^4 \ldots}$

⑤ ff

$ff\ldots$

$= (\ldots$

$\boxed{ff \ldots}$

\#

of A for
element

atri's

nd g : B → C

(3) $f \circ f$

$f(f(x))$

$f(x^2-1)$

$(x^2-1)^2 - 1$

$(x^2)^2 + 1^2 - 2x^2 - 1$

$x^4 + \cancel{1} - 2x^2 - \cancel{1}$

$\boxed{f \circ f = x^4 - 2x^2}$

(4) $g \circ g$

$g[g(x)]$

$g(3x+1)$

$3(3x+1)+1$

$9x+3+1$

$\boxed{g \circ g = 9x+4}$

(5) $ff = (f(x))^2$

$f(x) \cdot f(x)$

$= (x^2-1) \cdot (x^2-1)$

$= (x^2-1)^2$

$\boxed{ff = x^4 + 1 - 2x^2}$

(6) $gg = [g(x)]^2$

$= g(x) \cdot g(x)$

$= (3x+1)^2$

$\boxed{gg = 9x^2 + 1 + 6x}$

#

Unit - 4

Partial Differentiation:

- $\frac{d}{dx} x^n = n x^{n-1}$
- $\frac{d}{dx} e^x = e^x$
- $\frac{d}{dx} \log x = \frac{1}{x}$

To find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = \log(x^2+y^2)$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left[\log(x^2+y^2)\right]$$

$$= \frac{1}{x^2+y^2} \frac{\partial}{\partial x} x^2+y^2$$

$$= \frac{1}{x^2+y^2} \cdot 2x = \boxed{\frac{2x}{x^2+y^2}}$$

or $$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left[\log(x^2+y^2)\right]$$

$$= \frac{1}{x^2+y^2} \frac{\partial}{\partial x}(x^2+y^2)$$

$$= \frac{1}{x^2+y^2} \cdot 2y = \boxed{\frac{2y}{x^2+y^2}}$$

To find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = \log(x^4+y^4+z^4)$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x} \log(x^4+y^4+z^4)$$

$$= \frac{1}{x^4+y^4+z^4} \frac{d}{dx} x^4+y^4+z^4$$

$$= \frac{1}{x^4+y^4+z^4} \cdot 4x^3$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{4x^3}{x^4+y^4+z^4}}$$

entation:-

g(x^2+y^2)

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\log(x^4+y^4+z^4)$$

$$= \frac{1}{x^4+y^4+z^4}\,\frac{\partial}{\partial y}\,x^4+y^4+z^4$$

$$\frac{\partial z}{\partial y} = \frac{1}{x^4+y^4+z^4}\,4y^3 = \boxed{\frac{4y^3}{x^4+y^4+z^4}}$$

To find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. $\log x^{10}+y^{10}+z^{10}$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\log(x^{10}+y^{10}+z^{10})$$

$$= \frac{1}{x^{10}+y^{10}+z^{10}}\,\frac{\partial}{\partial x}(x^{10}+y^{10}+z^{10})$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{10x^9}{x^{10}+y^{10}+z^{10}}}$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\log(x^{10}+y^{10}+z^{10})$$

$$= \frac{1}{x^{10}+y^{10}+z^{10}}\,\frac{\partial}{\partial y}\,x^{10}+y^{10}+z^{10}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{10y^9}{x^{10}+y^{10}+z^{10}}}$$

If $u = x^2+y^2+z^2$ show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = (x+y+z)$

We have $u = x^2+y^2+z^2$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2+y^2+z^2)$$

$$\frac{\partial u}{\partial x} = 2x$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(x^2+y^2+z^2)$$

$$\frac{\partial u}{\partial y} = 2y$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}(x^2+y^2+z^2)$$

$$\frac{\partial u}{\partial z} = 2z$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 2x + 2y + 2z$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 2(x+y+z)}$$ Ans

If $u = x^{12} + y^{12} + z^{12}$ Prove that
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 12(x^{11}+y^{11}+z^{11})$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^{12}+y^{12}+z^{12})$$

$$\frac{\partial u}{\partial x} = 12x^{11}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(x^{12}+y^{12}+z^{12})$$

$$\frac{\partial u}{\partial y} = 12y^{11}$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}(x^{12}+y^{12}+z^{12})$$

$$\frac{\partial u}{\partial z} = 12z^{11}$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 12x^{11} + 12y^{11} + 12z^{11}$$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 12(x^{11}+y^{11}+z^{11})}$$ Ans

If $z = \tan^{-1}\left(\frac{y}{x}\right)$ show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

We have $\quad z = \tan^{-1}\left(\frac{y}{x}\right)$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0 \quad \text{——— (1)}$$

We know $\quad \frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{\partial z}{\partial x}\right] \text{ — (2)}$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left[\tan^{-1}\left(\frac{y}{x}\right)\right]$$

$$\frac{\partial z}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \frac{\partial}{\partial x}\left(\frac{y}{x}\right)$$

$$\frac{\partial z}{\partial x} = \frac{1}{\frac{x^2+y^2}{x^2}} \; y \; (-1) x^{-2}$$

$$\frac{\partial z}{\partial x} = \frac{\cancel{x^2}}{x^2+y^2} \; \frac{-y}{\cancel{x^2}}$$

$$\frac{\partial z}{\partial x} = \frac{-y}{x^2+y^2}$$

Put the value of $\frac{\partial z}{\partial x}$ in eq (2)

$$\frac{\partial^2 z}{\partial x} = \frac{\partial}{\partial x}\left[\frac{-y}{x^2+y^2}\right]$$

$$\frac{\partial^2 z}{\partial x} = -\left\{\frac{x^2+y^2 \frac{\partial}{\partial x}(y) - y\frac{\partial}{\partial x}(x^2+y^2)}{(x^2+y^2)^2}\right\}$$

$$\frac{\partial^2 z}{\partial x} = -\left\{\frac{0 - y \times 2x}{(x^2+y^2)^2}\right\}$$

$$\frac{\partial^2 z}{\partial x} = \cancel{-}\; \frac{+2xy}{x^2+y^2}$$

$$\boxed{\frac{\partial^2 z}{\partial x} = \frac{2xy}{x^2+y^2}}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left[\frac{\partial z}{\partial y}\right] \text{ — } ③$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left[\tan^{-1}\left(\frac{y}{x}\right)\right]$$

$$\frac{\partial z}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{\partial}{\partial y}\left(\frac{y}{x}\right)$$

$$\frac{\partial z}{\partial y} = \frac{x^2}{x^2+y^2} \times \frac{1}{x}$$

$$\frac{\partial z}{\partial y} = \frac{x}{x^2+y^2}$$

Put the value of $\frac{\partial z}{\partial y}$ in eq ③

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{x^2+y^2\frac{\partial}{\partial y}x - x\frac{\partial}{\partial y}x^2+y^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{x^2+y^2 \times 0 - x\,2y}{(x^2+y^2)^2}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{-2xy}{(x^2+y^2)^2}$$

Put the value in eq ①

Taking L.H.L

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$$

$$\frac{2xy}{(x^2+y^2)^2} - \frac{2xy}{(x^2+y^2)^2} = 0 = \text{R.H.L}$$

Hence the Condition is satisfied

If $U = \sin^{-1}\frac{x}{y} + \tan^{-1}\left(\frac{y}{x}\right)$ $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

$\Rightarrow$ We have $u = \sin^{-1} x/y + \tan^{-1}\left(\frac{y}{x}\right)$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0 \quad \text{—— (1)}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\sin^{-1}\frac{x}{y} + \frac{\partial}{\partial x}\tan^{-1}\frac{y}{x}$$

$$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}}\frac{\partial}{\partial x}\frac{x}{y} + \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{\partial}{\partial x}\frac{y}{x}$$

$$\frac{\partial u}{\partial x} = \frac{1}{\frac{\sqrt{y^2-x^2}}{\cancel{y}}}\frac{1}{\cancel{y}} + \frac{1}{\frac{x^2+y^2}{\cancel{x^2}}}\frac{(-y)}{\cancel{x^2}}$$

$$\frac{\partial u}{\partial x} = \frac{1}{\sqrt{y^2-x^2}} - \frac{y}{x^2+y^2}$$

$$x\frac{\partial u}{\partial x} = \frac{x}{\sqrt{y^2-x^2}} - \frac{xy}{x^2+y^2} \quad \text{—— (2)}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\sin^{-1}\frac{x}{y} + \frac{\partial}{\partial y}\tan^{-1}\left(\frac{y}{x}\right)$$

$$\frac{\partial u}{\partial y} = \frac{1}{\sqrt{1-\left(\frac{x}{y}\right)^2}}\frac{\partial}{\partial y}\frac{x}{y} + \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{\partial}{\partial y}\frac{y}{x}$$

$$\frac{\partial u}{\partial y} = \frac{y}{\sqrt{y^2-x^2}}\, -y^{-2}x + \frac{x^2}{x^2+y^2}\,\frac{1}{x}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\sqrt{y^2-x^2}}\,\frac{-x}{y} + \frac{x}{x^2+y^2}$$

$$\frac{\partial u}{\partial y} = \frac{-x}{y\sqrt{y^2-x^2}} + \frac{x}{x^2+y^2}$$

$$y\frac{\partial u}{\partial y} = \frac{-xy}{y\sqrt{y^2-x^2}} + \frac{xy}{x^2+y^2}$$

$$y\frac{\partial y}{\partial y} = \frac{-x}{\sqrt{y^2-x^2}} + \frac{xy}{x^2+y^2} \quad \text{—— (3)}$$

eq ② + eq ③

$$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$$

$$\cancel{\frac{x}{\sqrt{y^2-x^2}}} - \cancel{\frac{xy}{x^2+y^2}} = \cancel{\frac{x}{\sqrt{y^2-x^2}}} + \cancel{\frac{xy}{x^2+y^2}}$$

0 Ans

Hence Proved.

Ques v.v.imp If $u = \log(x^3+y^3+z^3-3xyz)$ Prove that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

We have

$$u = \log(x^3+y^3+z^3-3xyz)$$

$$\frac{\partial u}{\partial x} = \frac{1}{x^3+y^3+z^3-3xyz}\,\frac{\partial}{\partial x}(x^3+y^3+z^3-3xyz)$$

$$\frac{\partial u}{\partial x} = \frac{1}{x^3+y^3+z^3-3xyz}\cdot 3x^2-3yz$$

$$\boxed{\frac{\partial u}{\partial x} = \frac{3x^2-3yz}{x^3+y^3+z^3-3xyz}} \quad \text{——} ①$$

$$\boxed{\frac{\partial u}{\partial y} = \frac{3y^2-3xz}{x^3+y^3+z^3-3xyz}} \quad \text{——} ②$$

$$\boxed{\frac{\partial u}{\partial z} = \frac{3z^2-3xy}{x^3+y^3+z^3-3xyz}} \quad \text{——} ③$$

adding eq ① ② and ③

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = \frac{3x^2-3yz}{(x^3+y^3+z^3-3xyz)} + \frac{3y^2-3xz}{(x^3+y^3+z^3-3xyz)} + \frac{3z^2-3xy}{(x^3+y^3+z^3-3xyz)}$$

$$= \frac{3x^2-3yz+3y^2-3xz+3z^2-3xy}{x^3+y^3+z^3-3xyz}$$

$$\frac{3(x^2+y^2+z^2-yz-xz-xy)}{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)}$$

$$= \frac{3}{x+y+z}$$

Taking eq (a) Left side

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right)$$

$$= \left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\left(\frac{3}{x+y+z}\right)$$

$$= 3\left\{\frac{\partial}{\partial x}(x+y+z)^{-1}+\frac{\partial}{\partial y}(x+y+z)^{-1}+\frac{\partial}{\partial z}(x+y+z)^{-1}\right\}$$

$$= 3\left\{(-1)(x+y+z)^{-2}+(-1)(x+y+z)^{-2}+(-1)(x+y+z)^{-2}\right\}$$

$$= 3\left\{\frac{-1}{(x+y+z)^2}-\frac{1}{(x+y+z)^2}-\frac{1}{(x+y+z)^2}\right\}$$

$$= 3\left\{\frac{-1-1-1}{(x+y+z)^2}\right\}$$

$$= \frac{3\times(-3)}{(x+y+z)^2}$$

$$\boxed{= \frac{-9}{(x+y+z)^2}}$$ Ans

<u>Homogeneous functions</u> :- Let us Consider the function

$$\boxed{f(x,y) = a_0 x^n + a_1 x^{n-1} y + a_2 x^{n-2} y^2 + \cdots a_n x^0 y^n}$$

<u>Euler's Theorem</u> :- If $u = f(x,y)$ be homogeneous function of x & y of degree n terms.

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu}$$

Ques If $u = \sin^{-1}\left[\frac{x^2+y^2}{x+y}\right]$ than Prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$

We have $u = \sin^{-1}\left[\frac{x^2+y^2}{x+y}\right]$

$$\sin u = \left[\frac{x^2+y^2}{x+y}\right]$$

thus, $f(x,y)$ is Homogeneous of degree $n = 2-1 = 1$

then by euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n \cdot u$$

$$x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) = 1 \cdot \sin u$$

$$x\cos u\frac{\partial u}{\partial x} + \cos u \, y\frac{\partial u}{\partial y} = \sin u$$

$$\cos u\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = \sin u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\sin u}{\cos u}$$

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u}$$

Ques If $u = \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$ show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

We have $u = \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$

$$\sin u = \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$$

thus $f(x,y)$ is Homogeneous of degree
$n = 10\left(\frac{1}{2} - \frac{1}{2}\right) = 0$

then by euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n \cdot u$$

$$x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) = 0 \times \sin u$$

$$x\frac{\partial}{\partial x}\cos u + y\frac{\partial}{\partial y}\cos u = 0$$

$$\cos u\left[x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right] = 0$$

$$\boxed{x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} = 0}$$

Ques- If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$

We have $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$$\tan u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$$

thus $f(x,y)$ is Homogeneous of degree
$n = 1 - \frac{1}{2} = \frac{1}{2}$

then by euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\,u$$

$$x\frac{\partial}{\partial x}(\tan u) + y\frac{\partial}{\partial y}(\tan u) = \frac{1}{2}\tan u$$

$$x\sec^2 u\frac{\partial u}{\partial x} + y\sec^2 u\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$

$$\sec^2 u\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right] = \frac{1}{2}\frac{\sin u}{\cos u}$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\frac{\sin u}{\cos u}\times\cos^2 u$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\times\frac{2}{2}\sin u\cos u$$

$$\boxed{x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{4}\sin 2u}$$

If $u=\log\left(\frac{x^3+y^3}{x+y}\right)$ show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2$

We have $u=\log\left(\frac{x^3+y^3}{x+y}\right)$

$$e^u=\frac{x^3+y^3}{x+y}$$

thus, $f(x,y)$ is Homogeneous of degree

$n=3-1=2$

then by euler theorem

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=n(u)$$

$$x\frac{\partial}{\partial x}e^u+y\frac{\partial}{\partial y}e^u=2e^u$$

$$x e^u\frac{\partial u}{\partial x}+y e^u\frac{\partial u}{\partial y}=2e^u$$

$$e^u\left[x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right]=2e^u$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{2\cancel{e^u}}{\cancel{e^u}}$$

$$\boxed{x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2}$$

If $u=\log\left[\frac{x^7+y^7}{x^2+y^2}\right]$ show $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=5$

We have $u=\log\left[\frac{x^7+y^7}{x^2+y^2}\right]$

$$e^u = \left[\frac{x^7+y^7}{x^2+y^2}\right]$$

Thus, $f(x,y)$ is Homogeneous of degree $n = (7-2) = 5$

then by euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n(u)$$

$$x\frac{\partial}{\partial x}(e^u) + y\frac{\partial}{\partial y}(e^u) = 5e^u$$

$$x\, e^u \frac{\partial u}{\partial x} + y e^u \frac{\partial u}{\partial x} = 5e^u$$

$$e^u\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x}\right] = 5e^u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = 5\frac{\cancel{e^u}}{\cancel{e^u}}$$

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = 5}$$

If $u = \frac{x^2y^2}{x^2+y^2}$ show $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial u}{\partial x}$

We have $u = \frac{x^2y^2}{x^2+y^2}$

then, $f(x,y)$ is Homogeneous of degree

$n = \cancel{2}\cdot 4 - 2 = 2$

then by euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u \longrightarrow (1)$$

Differen- w.r.t "x" (1) we get

$$x\cdot\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) + \frac{\partial u}{\partial x}\times 1 + y\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial u}{\partial y}\times\frac{\partial}{\partial x}(y) = 2\frac{\partial u}{\partial x}$$

$$x\frac{\partial^2 v}{\partial x^2} + \frac{\partial v}{\partial x} + y\frac{\partial^2 v}{\partial x \partial y} = 2\frac{\partial v}{\partial x}$$

$$x\frac{\partial^2 v}{\partial x^2} + y\frac{\partial^2 v}{\partial x \partial y} = 2\frac{\partial v}{\partial x} - \frac{\partial v}{\partial x}$$

$$\boxed{x\frac{\partial^2 v}{\partial x^2} + y\frac{\partial^2 v}{\partial x \partial y} = \frac{\partial v}{\partial x}}$$

Incomplete

If $u = \sec^{-1}\left[\frac{x^3+y^3}{x+y}\right]$ than Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

We have $u = \sec^{-1}\left[\frac{x^3+y^3}{x+y}\right]$

$$\sec u = \frac{x^3+y^3}{x+y}$$

thus, $f(x,y)$ is Homogeneous of degree

$n = 3-1 = 2$

then by euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

$$x\frac{\partial}{\partial x}(\sec u) + y\frac{\partial}{\partial y}(\sec u) = 2(\sec u)$$

$$x \sec u \tan u \frac{\partial u}{\partial x} + y \sec u \tan u \frac{\partial u}{\partial y} = 2 \sec u$$

$$\sec u \tan u\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right] = 2 \sec u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2 \sec u}{\sec u \tan u}$$

$\frac{1}{\tan u} = \cot u$

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u}$$

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