Advanced Image Processing: Assignment 3 (Due Mar 13, 2022)

Note: Answer all questions by supporting your answers with suitable plots and observations in a report. You will need to upload code that you may have written to solve the problems. Please provide detailed comments for any such code. The assignment will be evaluated not just based on the final results but also how you obtained them. Late submissions will be penalized.

Problem 1: MMSE estimation for Laplacian source (10 points)
Consider the noise model $Y = X + Z$, where X is distributed according to a Laplace distribution $f_X(x) = \frac{1}{2\sigma_X} \exp(-\frac{|x|}{\sigma_X})$ with $\sigma_X = 1$ and $Z \sim \mathcal{N}(0, \sigma_Z^2)$ with $\sigma_Z^2 = 0.1$. Compute the minimum mean squared estimate (MMSE) of x given y. If the MMSE estimate can not be written as a closed form expression of y, plot the estimate $\hat{x}$ as a function of y for the given parameters of the system (Ref: E. P. Simoncelli, and E. H. Adelson, "Noise removal via Bayesian wavelet coring," Proc. of IEEE International Conference on Image Processing, 1996).

Problem 2: Image denoising (30 points)
Take the lighthouse image provided to you, convert to greyscale and add white Gaussian noise with variance $\sigma_Z^2 = 100$ to it. Be sure to add noise in the grey scale domain where the range of pixel values is between 0 and 255. Compute and compare (subjectively and using mean squared error) the results of the following denoising methods

1. Low pass Gaussian filter: Vary the filter length in the set $\{3, 7, 11\}$ and standard deviations in the set $\{0.1, 1, 2, 4, 8\}$ to identify the filter with the best mean squared error (MSE).
2. MMSE filter on the high pass coefficients of the noisy image: You need to estimate the variance of the high pass coefficients of the original image and the variance of noise in the high pass image given the noise variance $\sigma_Z^2 = 100$ in the pixel domain.
3. Adaptive MMSE: Compute an adaptive version of the MMSE filter where the estimates are computed for patches of size 11×11 with overlap of 5 in the high pass domain. All pixels belonging to multiple patches need to be assigned the average values arising from the output due to multiple patches.

Problem 3: Image sharpening (20 points)

1. Sharpen the output of the image denoised using the low pass Gaussian filter in Problem 2 using a combination of a high pass filter and constant gain as discussed in class. Be sure to incorporate saturation of pixel values below 0 or above 255. Use the following high pass filter:

$$M = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Compute the gain that minimizes the mean squared error between the sharpened image and the original undistorted image.

2. Pick a smooth region, a region containing a single edge and a region containing texture in the sharpened image. Comment on whether the sharpened regions look visually good in terms of your perception of the quality. Also comment on whether a smaller gain, larger gain or the same gain would have been the best for the perceived quality for each region.