

P. Assign (3)

find E. values & E. vectors

$$\textcircled{1}. \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix}$$

$$(-2-\lambda)[\lambda^2 - \lambda - 12] - 2[-2\lambda - 6] - 3[-4 + 1-\lambda]$$

$$\Rightarrow -2\lambda^2 + 2\lambda + (24) - \lambda^3 + \lambda^2 + 12\lambda + 4\lambda + (12) + 9 + 3\lambda = 0$$

$$\Rightarrow -\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$$

$$\hookrightarrow \lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$$\hookrightarrow (\lambda + 3)^2 (\lambda - 5) = 0$$

$$\hookrightarrow \lambda = -3/5$$

Eigen values = $-3/3/5$

for $\lambda = -3$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{cases} x + 2y - 3z = 0 \\ 2x + 4y - 6z = 0 \\ -x - 2y + 3z = 0 \end{cases} \rightarrow \text{all eqns same}$$

$$\hookrightarrow x = k$$

$$\hookrightarrow \text{Eigen vector} = \begin{bmatrix} k \\ k \\ k \end{bmatrix}$$

for $\lambda = 5$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} -7x + 2y - 3z = 0 \\ 2x - 4y - 6z = 0 \\ -x - 2y - 5z = 0 \end{cases}$$

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(2)

$$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

characteristic eqⁿ - $|A - \lambda I| = 0$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$$

$$(4-\lambda)[(1-\lambda)^2] + (2(1-\lambda)) = 0$$

$$(1-\lambda)[(4-\lambda)(1-\lambda) + 2] = 0$$

$$\lambda = 1 \quad 4 - 4\lambda - \lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda^2 - 3\lambda - 2\lambda + 6 = 0$$

$$\lambda(\lambda-3) - 2(\lambda-3) = 0$$

$$\lambda = 2, \lambda = 3$$

Eigen values = 1, 2, 3

for $\lambda = 1$,

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{aligned} 3x + z &= 0 \\ -2x &= 0 \\ -2y &= 0 \end{aligned} \rightarrow \text{eigen vector}$$

$$\begin{bmatrix} k \\ 0 \\ -3k \end{bmatrix}$$

for $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$2x + z = 0$$

$$-2x - y = 0$$

$$-2x - z = 0$$

$$x = k \Rightarrow z = -2k$$

$$y = -2k$$

eigen vector

$$\begin{bmatrix} k \\ -2k \\ -2k \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$x + z = 0$$

$$-2x - 2y = 0$$

$$-2x - 2z = 0$$

$$x = k \Rightarrow z = -k$$

$$y = -k$$

eigen vector

$$\begin{bmatrix} k \\ -k \\ -k \end{bmatrix}$$

③

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

characteristic Eqⁿ -

$$\begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix} = 0$$

$$(5-\lambda)(-\lambda)(3-\lambda) = 0$$

$$\lambda = 0$$

$$\lambda = 3$$

$$\lambda = 5$$

for $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$2x = 0$$

$$-3y = 0$$

$$-x = 0$$

$$\begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix}$$

eigen vector

for $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$5x = 0$$

$$-x + 3z = 0$$

$$z = k$$

$$x = 3k$$

eigen vector

$$\begin{bmatrix} 3k \\ 0 \\ k \end{bmatrix}$$

for $\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{aligned} -5y &= 0 \\ -x - 2z &= 0 \end{aligned}$$

$$\begin{aligned} z &= k \\ x &= -2k \end{aligned}$$

eigen vector

$$\begin{bmatrix} -2k \\ 0 \\ k \end{bmatrix}$$

(4)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

Characteristic Eqⁿ -

$$\begin{vmatrix} -\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$-\lambda(3-\lambda)(-2-\lambda) = 0$$

$$\begin{aligned} \lambda &= 0 \\ \lambda &= -2 \\ \lambda &= 3 \end{aligned}$$

for $\lambda = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{aligned} 3y + 4z &= 0 \\ -2z &= 0 \end{aligned}$$

$$y = z = 0$$

x must be non zero

$$x = k$$

eigen vector =

$$\begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{aligned} -3x &= 0 \\ 4z &= 0 \\ -5z &= 0 \end{aligned}$$

$$x = z = 0$$

y must be non zero

$$y = k$$

eigen vector =

$$\begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix}$$

for $\lambda = -2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$2x = 0 \Rightarrow x = 0$$

$$5y + 4z = 0$$

$$z = k \Rightarrow y = -\frac{4k}{5}$$

eigen vector

$$\begin{bmatrix} 0 \\ -\frac{4k}{5} \\ k \end{bmatrix}$$

⑤ $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} = A$

↓ when 2 rows are same in a matrix, its determinant value is zero.

$\Rightarrow |A| = 0 \checkmark$

$\lambda^3 + \text{cloud} \lambda^2 + \text{cloud} \lambda + \text{cloud} |A| = 0$

$\lambda [\lambda^2 + \text{cloud} \lambda + \text{cloud}] = 0$

and hence one value eigen value is zero.