

Assignment - 04.

1). Rank = ?

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow{\begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \\ R_4 \rightarrow R_4 - 6R_1 \end{array}} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$\xrightarrow{\begin{array}{l} R_3 \rightarrow R_3 - R_4 \\ R_2 \leftrightarrow R_4 \end{array}} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -3 & 2 \end{bmatrix} \xleftarrow{\quad} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$\xrightarrow{R_4 \rightarrow R_4 + R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \# \text{Rank} = \text{No of non-zero rows}$$

 $\Rightarrow \text{Rank} = 3$ 2). $\omega \rightarrow$ vector space of all sym 2×2 matrices

$$T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + \begin{pmatrix} (b-c)x \\ (c-d)x \end{pmatrix} + (c-a)x^2.$$

dimension of $\omega = 3$: $(a-b) + (b-c)x + (c-a)x^2$ Symmetric 2×2 matrix has 2 independent values.
The diag. elem. & off-diag. elem. $[b=c][a][a]$

$$\therefore \dim(\omega) = 3$$

Rank of T :- T has all polynomials in P that can come by applying T mapping to all polynomial of degree 2.

$$\therefore \text{Rank of } T = 3$$

Using Rank-Nullity Theorem,

$$\begin{aligned} \text{Nullity} &= \dim(\omega) - \text{rank}(T) \\ &= 3 - 3 \\ &= 0. \end{aligned}$$

$$\textcircled{3} \quad A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Eigenvalues & E vectors of $A^{-1} \otimes A + 4I$

$$\text{char eq } r(A) = \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}$$

Eigen values of $A = 1, 3$

\Rightarrow Eigen values of $A^{-1} = 1, \frac{1}{3}$

$$(2-\lambda)^2 - 1 = 0$$

$$\bullet \lambda^2 + 4 - 4\lambda - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda^2 - 3\lambda - \lambda + 3 = 0$$

$$\lambda(\lambda-3) - 1(\lambda-3) = 0$$

$$(\lambda-1)(\lambda-3) = 0$$

$$\boxed{\lambda = 1 \\ \lambda = 3}$$

\rightarrow If A is invertible,

Eigen vector of A^{-1} = Eigen vector of A

\Rightarrow Eigen vector of $(A+4I)$ = Eigen vector of A

$$\lambda = 3,$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\text{for } \lambda = 1,$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{cases} -x - y = 0 \\ -x + y = 0 \end{cases}$$

$$\boxed{\begin{bmatrix} k \\ -k \end{bmatrix}}$$

$$\begin{cases} x - y = 0 \\ -x + y = 0 \end{cases} \rightarrow \boxed{\begin{bmatrix} k \\ k \end{bmatrix}}$$

$$\textcircled{4} \quad \begin{aligned} 3x - 0.1y - 0.2z &= 7.85 \\ 0.1x + 7y - 0.3z &= -19.3 \\ 0.3x - 0.2y + 10z &= 71.4 \end{aligned} \quad \rightarrow \text{diagonal dominance} \quad \checkmark$$

$$z = \frac{(-1.4 - 0.3x - 0.2y)}{10}$$

$$y = \frac{(-19.3 - 0.1x + 0.3z)}{7}$$

$$x = \frac{(7.85 + 0.1y + 0.2z)}{3}$$

$$\boxed{z = 6.67}$$

for first iterⁿ,

$$x = \frac{7.85}{3} = \boxed{2.6167}$$

$$\frac{-1.4 - 0.1(7.85) - 3.912}{7} = \boxed{z = -1.4}$$

$$y = \frac{(-19.3 - 0.1(2.6167))}{7} = \boxed{19.5617}$$

$$\Rightarrow z = \frac{-1.4 - 0.3(2.6167) - 0.2(19.5617)}{10} = \boxed{z = 6.67}$$

for second iteration,

$$\text{base case } (x, y, z) = (2.6167, 19.5617, 6.6703)$$

$$x = \underline{7.85 + 0.1(19.5617)} + 0.2(6.6703) = \boxed{7.716}$$

$$y = \underline{-19.3 + 0.1(7.7167)} + 0.3(6.6703) = \boxed{-2.36}$$

$$z = \underline{71.4 - 0.3(7.7167)} + 0.2(-2.36) = \boxed{6.956}$$

for third iteration,

$$\text{base case } (x, y, z) = \{7.716, -2.36, 6.956\}$$

$$x = \underline{7.85 + 0.1(-2.36)} + 0.2(6.956) \Rightarrow \boxed{x = 3.002}$$

$$y = \underline{-19.3 + 0.1(3.002)} + 0.3(6.956) \Rightarrow \boxed{y = -2.455}$$

$$z = \underline{71.4 - 0.3(3.002)} + 0.2(-2.455) \Rightarrow \boxed{z = 7.099}$$

(5). Consistent equations \rightarrow Equations which have solutions

Inconsistent equation \rightarrow Equations which don't have

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1 / R_3 \rightarrow R_3 - 3R_1$$

$$R_4 \rightarrow R_4 - R_1$$

$$\begin{cases} x + 3y + 2z = 0 \\ 2x - y + 3z = 0 \\ 3x - 5y + 4z = 0 \\ x + 17y + 4z = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - 2R_2} \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\beta(A) = \beta(AB) \neq n.$$

Inconsistent solution

(3)

$$\begin{aligned} 3x - 6y + 2z &= 23 \\ -4x + y - z &= -15 \\ x - 3y + 7z &= 16 \end{aligned}$$

$$\left. \begin{array}{l} x_0 = 1 \\ y_0 = 1 \\ z_0 = 1 \end{array} \right\} \text{initial value values}$$

$$x = \frac{23 + 6y - 2z}{3}$$

$$y = -15 + 4x + z$$

$$z = \frac{16 - x + 3y}{7}$$

iteration ①:

$$x_1 = \frac{23 + 6 - 27}{3} = 0.67$$

$$y_1 = -15 + 4\left(\frac{2}{3}\right) + 1$$

$$y_1 = -14 + \frac{8}{3} = -\frac{34}{3}$$

$$y_1 = -11.33$$

$$z_1 = \left(16 - \frac{2}{3} + (-34)\right) \frac{1}{3} \Rightarrow z_1 = -\frac{56}{3 \cdot 3} = -6.22$$

iteration ②:

$$x_2 = \left(23 + 4\left(-\frac{34}{3}\right) + 2\left(\frac{56}{3 \cdot 3}\right)\right) \frac{1}{3} \Rightarrow x_2 = -10.85$$

$$y_2 = \left(-15 + 4(-10.85) + -6.22\right) \Rightarrow y_2 = -64.62$$

$$z_2 = \left(16 + 10.85 + 3(-64.62)\right) \Rightarrow z_2 = -37.77$$

iteration ③

$$x_2 = -10.85$$

$$y_2 = -64.62$$

$$z_2 = -37.77$$

$$x_3 = \frac{23 + 6(-64.62) + 2(-37.77)}{3} \Rightarrow x_3 = -289.18$$

$$y_3 = -15 + 4(-289.18) - 37.77 \Rightarrow y_3 = -1209.49$$

$$z_3 = \frac{-516 - (-289.18) + 3(-1209.49)}{3}$$

$$z_3 = -3323.29$$

Q2. $T: W \rightarrow P$ $T: P_2 \rightarrow P_2$

$$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$$

Additivity : $T(u+v) = T(u) + T(v)$

Homogeneity : $T(cu) = c(T(u))$

①. $T(u) = (a+1) + (b+1)x + (c+1)x^2$

$$T(v) = (a'+1) + (b'+1)x + (c'+1)x^2 \quad \{ \text{Let} \}$$

$$T(u+v) = (a+a'+2) + (b+b'+2)x + (c+c'+2)x^2$$

\therefore satisfies additivity

② Let $du = da + dbx + dcx^2$

$$T(du) = T(da + dbx + dcx^2)$$

$$d(T(u)) = d(a+1) + d(b+1)x + d(c+1)x^2$$

\therefore satisfies homogeneity

Fun T satisfies both required conditions
to be a linear transformation.

⑩ Linear transformation for comp. vision for rotating images.

When rotating an image, a linear transformation matrix is applied to each and every pixel to achieve the desired rotation angle.

The transformation involves multiplying the original pixel coordinates by the rotation matrix, which alters the position of each pixel based on the rotation angle specified. This process preserves the image quality and structure while repositioning the pixels to create the rotated effect.

By utilizing linear transformation in computer vision tasks like image rotation, precise adjustments can be made to images without distorting their content.

(9)
 Q. Explain one application of matrix operations in image processing & example.

Let's consider a real-life example where you want to rotate a digital photograph. The photograph is slightly tilted. To correct this, you want to rotate the image by a certain angle. This task can be accomplished by using matrix operations.

Given an angle of rotation θ , the rotation matrix for a 2D image rotation is:

$$R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

To rotate an image represented by a matrix I (where each element of the matrix corresponds to a pixel value), you perform matrix multiplication with the rotation matrix R :

$$I_{\text{rotated}} = R \cdot I$$

Let us say image has to rotate anti cw by 30° , then

$$R = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \Rightarrow R = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$$

we then multiply each pixel value in the original image matrix by this rotation matrix to obtain the rotated image.

This process effectively applies a geometric transformation to the image, resulting in the desired rotation. Once rotated, the image will appear corrected, with the horizon level.