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Practice Assignment - 01.

(a) Test for consistency & solve -

i) $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$

$$\begin{bmatrix} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - \frac{3}{2}R_1} \begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 1/3 & -\frac{23}{3} & \frac{29}{3} \\ 0 & 22 & -54 & 27 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_2 \cdot 66]{} \begin{bmatrix} 2 & -3 & 7 & 5 \\ 0 & 1/3 & -\frac{23}{3} & \frac{29}{3} \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$\rho(A) \neq \rho(AB)$

→ Inconsistent.

ii) $2x - y + 3z = 8$, $-x + 2y + z = 4$, $3x + y - 4z = 0$

$$\begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - \frac{3R_1}{2}]{R_2 \to R_2 + \frac{R_1}{2}} \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 3/2 & 5/2 & 8 \\ 0 & 5/2 & -\frac{15}{2} & -12 \end{bmatrix} \xrightarrow{R_3 \to R_3 - \frac{5}{3}R_2} \begin{bmatrix} 2 & -1 & 3 & 8 \\ 0 & 3/2 & 5/2 & 8 \\ 0 & 0 & -10/6 & -\frac{76}{3} \end{bmatrix}$$

$\rho(A) = \rho(AB) = n = 3$ ∴ unique sol^n

iii) $4x - y = 12$, $-x + 5y - 2z = 0$, $-2x + 4z = -8$

$$\begin{bmatrix} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 2R_2]{R_2 \to R_2 + R_1/4} \begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 19/4 & -2 & 3 \\ 0 & -10 & 8 & -8 \end{bmatrix} \xrightarrow{R_3 \to R_3 + \frac{40}{19}R_2} \begin{bmatrix} 4 & -1 & 0 & 12 \\ 0 & 19/4 & -2 & 3 \\ 0 & 0 & \frac{72}{19} & . \end{bmatrix}$$

$\rho(A) = \rho(AB) = n = 3$ ∴ unique sol^n

(d). $x + 3y - 2z = 0$, $2x - y + 4z = 0$, $x - 11y + 14z = 0$

$$\begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \rho(A) = \rho(AB) \neq n$$

⇓ ∞ sol^n

$x + 3y - 2z = 0$

$-7y + 8z = 0$

$\boxed{z = k}$ $\boxed{x = -\frac{10k}{7}}$

$\boxed{y = \frac{8}{7}k}$

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(b). $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix} \xrightarrow[R_2 \to R_2 - R_1]{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

i) no $sol^n \Rightarrow \rho(A) \neq \rho(AB)$

$\Rightarrow \boxed{\lambda = 3,\ \mu \neq 10}$

ii) uniq. $sol^n \Rightarrow \rho(A) = \rho(AB) = n = 3$

$\Rightarrow \boxed{\lambda \neq 3}$

iii) ∞ $sol^n \Rightarrow \rho(A) = \rho(AB) \neq 3$

$\boxed{\lambda = 3}\ \boxed{\mu = 10}$

(c) $x+y+z=1,\ x+2y+4z=\lambda,\ x+4y+10z=\lambda^2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix} \xrightarrow[R_2 \to R_2 - R_1]{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 2 & 6 & \lambda^2-\lambda \end{bmatrix}$$ have a sol^n

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix}$$

have a sol^n

$\underbrace{\rho(A)}_{2} = \underbrace{\rho(AB)}_{2} \neq n \Rightarrow \infty$ sol^n

$\lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda^2 - 2\lambda - \lambda + 2 = 0$

$= \lambda(\lambda-2) - 1(\lambda-2) = 0$

$\boxed{\lambda = 1}\ \boxed{\lambda = 2}$

for $\lambda = 1$

$x+y+z=1$

$y+3z=0$

$\boxed{z=k}$

$\boxed{y=-3k}$

$\boxed{x=1+2k}$

for $\lambda = 2$,

$x+y+z=1$

$y+3z=1$

$\boxed{z=k}$

$\boxed{y=1-3k}$

$\boxed{x=2k}$

Q. $3x + y - \lambda z = 0$, $4x - 2y + 3z = 0$, $2\lambda x + 4y + \lambda z = 0$

$\lambda = ? \Rightarrow eq^n \Rightarrow$ non-trivial sol^n

for this to be true, $|A| = 0$

$$\begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 3(-2\lambda + 12) - 1(4\lambda + 6\lambda) - \lambda(16 + 4\lambda) = 0$

$\lambda^2 + 8\lambda - 9 = 0$

$\lambda^2 + 9\lambda - \lambda - 9 = 0$

$\lambda(\lambda + 9) - 1(\lambda + 9) = 0$

$(\lambda - 1)(\lambda + 9) = 0$

$\boxed{\lambda = -9, \ \lambda = 1}$ Ans